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ESTIMATION OF THE PARAMETERS OF THE NEW WEIBULL-PARETO DISTRIBUTION USING RANKED SET SAMPLING

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1. Introduction

1.1. *Ranked set sampling*

McIntyre (1952) introduced the concept of ranked set sampling (RSS) as a new sampling scheme for data collection. Due to its importance for a variety of applications in statistics, it is republished in McIntyre (2005) to estimate the mean of Australian pasture and forage yields. As claimed by McIntyre (1952, 2005), the mean of the RSS is an unbiased estimator of the population mean. Also, the variance of the RSS mean is smaller than that of the simple random sampling (SRS) with equal number of measurement elements. This sampling scheme is useful when it is difficult to measure large number of elements but visually (without inspection) ranking some of them is easier. For example, in McIntyre's experiment the yields of pasture plots can be assessed without the actual laborious process of weighing and mowing the hay for a lots of plots. Moreover, the RSS scheme is also highly applicable in instances where measuring a variable of interest is difficult and risky to measure. For example, in studying some diseases such as the yellowing of the body of an infant, one of the main steps is to measure the bilirubin level of the infant by taking their blood samples. However, it is risky and excruciating to take the blood samples. It is rather easy to rank the babies and take the measurement of the bilirubin level on their urine samples (Paul and Thomas, 2017).

The RSS consists of the following steps:

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1. randomly select m sets each of size m elements from the population under study (typically m is in the range 2 to 5);

2. the elements for each set in Step (1) are ranked visually or by any negligible cost method that does not require actual measurements;

3. select and quantify the i^{th} minimum from the i^{th} set, $i = 1, 2, ..., m$, to get a new set of size m, which is called the ranked set sample;

4. repeat Steps (1)-(3) h times (cycles) until obtaining a sample of size $n = mh$.

The i^{th} data point (measured unit) acquired in the j^{th} cycle is denoted by $Y_{ji} = X_{j(i)}$, $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, h$. This version of RSS is a balanced RSS, in the sense that in each cycle the number of data points is fixed. The following matrices clarify the procedure of RSS.

$$\text{Step 1: } \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1m} \\ X_{21} & X_{22} & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mm} \end{bmatrix} \qquad \text{Step 2: } \begin{bmatrix} \mathbf{X_{1(1)}} & X_{1(2)} & \cdots & X_{1(m)} \\ X_{2(1)} & \mathbf{X_{2(2)}} & \cdots & X_{2(m)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m(1)} & X_{m(2)} & \cdots & \mathbf{X_{m(m)}} \end{bmatrix}$$

$$\text{Step 3: } \{X_{1(1)}, X_{2(2)}, \ldots, X_{m(m)}\} \qquad \text{Step 4: } \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1m} \\ Y_{21} & Y_{22} & \cdots & Y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{h1} & Y_{h2} & \cdots & Y_{hm} \end{bmatrix}$$

Note that although $h \times m^2$ elements are sampled, only $h \times m$ of them are selected for measurement. In case of perfect ranking (no error was made in the ranking mechanism) the measured elements are called the *order statistics* and they are not ordered (see Navarro *et al.*, 2007, for some examples where order statistics are not ordered). In case of imperfect ranking the measured elements are called the *judgment order statistics*.

Dell and Clutter (1972) and Takahasi and Wakimoto (1968) explained the mathematical theory behind the claims of McIntyre (1952, 2005) by showing that the efficiency of the RSS mean with respect to SRS, defined by the ratio of the variances of the two sample means, is bounded by 1 and $(m+1)/2$. Moreover, Dell and Clutter (1972) also proved that the RSS mean is at least as efficient as the SRS mean even when there are ranking errors. For more about RSS see Al-Saleh and Samuh (2008), Samuh and Al-Saleh (2011), Al-Nasser and Al-Omari (2015), Al-Omari and Haq (2015), Al-Omari (2016), Amro and Samuh (2017), and Samuh (2017).

In the literature, some authors estimate the parameters of specific distributions using RSS and some of its modifications. Bhoj (1997) obtained the estimates of the location and scale parameters of the extreme value distribution using RSS. Abu-Dayyeh *et al.* (2004) proposed some estimators for estimating the location and the scale parameters of the logistic distribution using SRS, RSS and some of its other modifications. Parameter estimation for the generalized logistic distribution is studied within the context of RSS by Khamnei and Abusaleh (2017). Esemen and Gürler (2017) investigated the method of maximum likelihood of the shape and scale parameters of the generalized Rayleigh distribution within the context of RSS. For more results and references, see Lam *et al.* (1994), Bhoj and Ahsanullah (1996), Chacko and Thomas (2008), Al-Saleh and Diab (2009), Sarikavanij *et al.* (2014), and Samuh and Qtait (2015).

The purpose of this paper is to study the maximum likelihood estimation of the parameters concerning the new Weibull-Pareto distribution within the context of SRS, RSS, median RSS (MRSS), and extreme RSS (ERSS).

The organization of the paper is the following. The new Weibull-Pareto distribution is introduced in Section 2. Maximum likelihood estimation and Fisher information are discussed in Section 3. Interval estimates for the parameters are constructed in Section 4. A simulation study is carried out in Section 5. A real data application is presented in Section 6. Finally, Section 7 concludes the paper.

2. The new Weibull-Pareto distribution

The new Weibull-Pareto (NWP) distribution is defined by Nasiru and Luguterah (2015) as a generalization of the Pareto distribution. It is of great interest and is popularly used in analyzing lifetime data. For example, it is used by Nasiru and Luguterah (2015) to model the exceedances of flood peaks (in m^3/s) of the Wheaton River near Carcross in Yukon Territory, Canada. Aljarrah *et al.* (2015) used the NWP distribution to model the remission times (in months) of a random sample of 128 bladder cancer patients.

The probability density function (pdf) of the NWP distribution, with shape parameter γ and scale parameters β and λ, is given by

$$f(x;\beta,\gamma,\lambda)=\frac{\beta\gamma}{\lambda}\left(\frac{x}{\lambda}\right)^{\gamma-1}e^{-\beta\left(\frac{x}{\lambda}\right)^{\gamma}}, \quad x>0, \beta,\gamma,\lambda>0. \tag{1}$$

The corresponding cumulative distribution function (cdf) is given by

$$F(x;\beta,\gamma,\lambda)=1-e^{-\beta\left(\frac{x}{\lambda}\right)^{\gamma}}. \tag{2}$$

The mean of the NWP distribution is

$$\mu=E(X)=\lambda\beta^{-1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right) \tag{3}$$

with variance defined as

$$\sigma^2=\mathrm{Var}(X)=\lambda^2\beta^{-2/\gamma}\left(\Gamma\left(\frac{\gamma+2}{\gamma}\right)-\Gamma\left(1+\frac{1}{\gamma}\right)^2\right) \tag{4}$$

The pdf curves and their corresponding cdf curves of the NWP distribution for different values of β, γ, and λ are shown in Figure 1. For more details about the statistical properties of the distribution see Nasiru and Luguterah (2015).

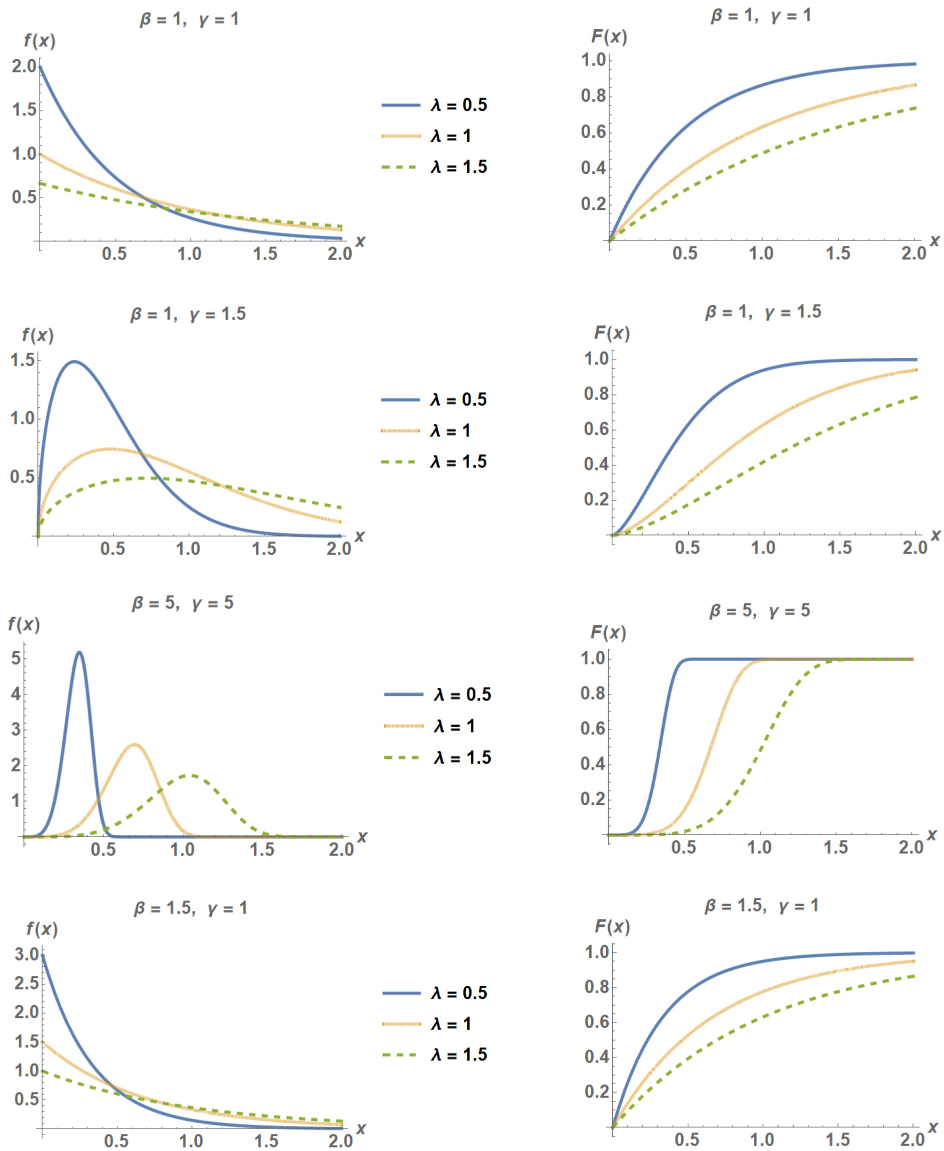

Figure 1 – The pdf curves (left panel) and their corresponding cdf curves (right panel) of the NWP distribution for different values of β, γ, and λ.

3. Maximum likelihood estimation and Fisher information

In this section, the maximum likelihood estimation of the shape parameter γ and scale parameters β and λ for NWP distribution based on SRS, RSS, MRSS, and ERSS is investigated.

3.1. *Using SRS*

Suppose $X_1, X_2, \ldots, X_n$ be a random sample of size n selected from the NWP distribution $f(x;\beta,\gamma,\lambda)$, where the values of β, γ, and λ are unknown. The likelihood function $L_{SRS}(\beta,\gamma,\lambda)$ is given by

$$\begin{aligned} L_{SRS}(\beta,\gamma,\lambda) &= \prod_{i=1}^{n} f(x_i;\beta,\gamma,\lambda) \\ &= \frac{\beta^n \gamma^n \prod_{i=1}^{n} \left(\frac{x_i}{\lambda}\right)^{\gamma} e^{-\beta \sum_{i=1}^{n} \left(\frac{x_i}{\lambda}\right)^{\gamma}}}{\prod_{i=1}^{n} x_i}. \end{aligned} \tag{5}$$

Thus, the log likelihood function $l_{SRS}(\beta,\gamma,\lambda)$ is

$$\begin{aligned} l_{SRS}(\beta,\gamma,\lambda) &= \log L_{SRS}(\beta,\gamma,\lambda) \\ &= n\log(\beta) + n\log(\gamma) - n\gamma\log(\lambda) + \gamma\sum_{i=1}^{n}\log(x_i) \\ &\quad -\beta\lambda^{-\gamma}\left(\sum_{i=1}^{n} x_i^{\gamma}\right) - \sum_{i=1}^{n}\log(x_i). \end{aligned} \tag{6}$$

Differentiating the log likelihood function with respect to β, γ, and λ, respectively, yields

$$\frac{\partial l_{SRS}(\beta,\gamma,\lambda)}{\partial\beta} = \frac{n}{\beta} - \lambda^{-\gamma}\left(\sum_{i=1}^{n} x_i^{\gamma}\right), \tag{7}$$

$$\begin{aligned} \frac{\partial l_{SRS}(\beta,\gamma,\lambda)}{\partial\gamma} &= \frac{n}{\gamma} - n\log(\lambda) + \sum_{i=1}^{n}\log(x_i) - \beta\lambda^{-\gamma}\left(\sum_{i=1}^{n} x_i^{\gamma}\log(x_i)\right) \\ &\quad + \beta\lambda^{-\gamma}\log(\lambda)\left(\sum_{i=1}^{n} x_i^{\gamma}\right), \end{aligned} \tag{8}$$

and

$$\frac{\partial l_{SRS}(\beta,\gamma,\lambda)}{\partial\lambda} = \beta\gamma\lambda^{-\gamma-1}\sum_{i=1}^{n} x_i^{\gamma} - \frac{\gamma n}{\lambda}. \tag{9}$$

The MLE of β as a function of γ and λ, say $\hat{\beta}(\gamma,\lambda)$, can be obtained as

$$\hat{\beta}(\gamma,\lambda)=\frac{n\lambda^{\gamma}}{\sum_{i=1}^{n}x_i^{\gamma}}, \tag{10}$$

and the MLE of λ as a function of β and γ, say $\hat{\lambda}(\beta,\gamma)$, can be obtained as

$$\hat{\lambda}(\beta,\gamma)=\left(\frac{\beta\sum_{i=1}^{n}x_i^{\gamma}}{n}\right)^{1/\gamma}. \tag{11}$$

The MLE of γ cannot be written in explicit form. So, estimates for γ can be obtained by using numerical methods. The `mle2` function in the `bbmle` package in R (R Core Team, 2018) is used.

The Fisher information (FI) number is used to measure the amount of information that an observable sample carries about the parameter(s). The FI number for the parameter θ is defined as

$$FI(\theta)=-E\left(\frac{\partial^2\log L(\theta)}{\partial\theta^2}\right).$$

For a random sample $X_1,X_2,\dots,X_n$ from the NWP distribution, the FI numbers of β, γ, and λ are, respectively, given by

$$FI_{SRS}(\beta)=-E\left(\frac{\partial^2 l_{SRS}(\beta,\gamma,\lambda)}{\partial\beta^2}\right)=\frac{n}{\beta^2}, \tag{12}$$

$$FI_{SRS}(\lambda)=-E\left(\frac{\partial^2 l_{SRS}(\beta,\gamma,\lambda)}{\partial\lambda^2}\right)=\frac{n\gamma^2}{\lambda^2}, \tag{13}$$

$$\begin{aligned}FI_{SRS}(\gamma)&=-E\left(\frac{\partial^2 l_{SRS}(\beta,\gamma,\lambda)}{\partial\gamma^2}\right)\\&=\frac{n\left(6\log(\beta)(\log(\beta)+2C-2)+6(C-1)^2+\pi^2\right)}{6\gamma^2},\end{aligned} \tag{14}$$

where $C=-\Gamma'(1)$ is the Euler's constant. The observed FI numbers are evaluated at the maximum likelihood estimates.

3.2. *Using RSS*

Suppose $\{Y_{ji},\ j=1,2,\dots,h,\ i=1,2,\dots,m\}$ be a RSS from an NWP distribution, where h is the number of cycles and m is the set size. It can be seen from the structure of RSS that the data are all mutually independent and, in addition, for each $i=1,2,\dots,m$ the data are identically distributed. The distribution of the i^{th} data point, for each $j=$

$1,2,\ldots,h$, is the same as the distribution of the i^{th} order statistic of the random sample $X_1,X_2,\ldots,X_m$, that is

$$\begin{aligned} f_{Y_{ji}}(y) = f_{X_{(i)}}(y) &= \frac{m!}{(i-1)!(m-i)!} f(y)(F(y))^{i-1}(1-F(y))^{m-i} \\ &= \frac{m!\beta\gamma\lambda^{-\gamma}y^{\gamma-1}\left(1-e^{\beta\lambda^{-\gamma}(-y^{\gamma})}\right)^{i}e^{-\beta\lambda^{-\gamma}(m-i)y^{\gamma}}}{(i-1)!(m-i)!\left(e^{\beta\lambda^{-\gamma}y^{\gamma}}-1\right)}. \end{aligned} \tag{15}$$

The likelihood function of RSS $\{Y_{ji},\, j=1,2,\ldots,h,\, i=1,2,\ldots,m\}$ is given by

$$\begin{aligned} L_{RSS}(\beta,\gamma,\lambda) &= \prod_{i=1}^{m}\prod_{j=1}^{h} f_{Y_{ji}}(y_{ji}) \\ &= \prod_{i=1}^{m}\prod_{j=1}^{h} \frac{m!\beta\gamma\lambda^{-\gamma}y_{ji}^{\gamma-1}\left(1-e^{\beta\lambda^{-\gamma}\left(-y_{ji}^{\gamma}\right)}\right)^{i}e^{-\beta\lambda^{-\gamma}(m-i)y_{ji}^{\gamma}}}{(i-1)!(m-i)!\left(e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-1\right)} \\ &= \frac{\beta^{mh}\gamma^{mh}\lambda^{\gamma(-mh)}\prod_{i=1}^{m}\prod_{j=1}^{h}\left(1-e^{-\beta\lambda^{-\gamma}y_{ij}^{\gamma}}\right)^{i}}{\prod_{i=1}^{m}\prod_{j=1}^{h}\left(e^{\beta\lambda^{-\gamma}y_{ij}^{\gamma}}-1\right)} \\ &\quad \times \prod_{i=1}^{m}\prod_{j=1}^{h} y_{ij}^{\gamma-1} e^{-\beta\lambda^{-\gamma}\sum_{i=1}^{m}\sum_{j=1}^{h}(m-i)y_{ij}^{\gamma}}. \end{aligned} \tag{16}$$

Thus, the log likelihood function is

$$\begin{aligned} l_{RSS}(\beta,\gamma,\lambda) =& mh\log(\beta) - \gamma mh\log(\lambda) + mh\log(\gamma) \\ & - \sum_{i=1}^{m}\sum_{j=1}^{h}\log\left(e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-1\right) + \sum_{i=1}^{m}\sum_{j=1}^{h}(\gamma-1)\log(y_{ji}) \\ & + \sum_{i=1}^{m}\sum_{j=1}^{h} i\log\left(1-e^{-\beta\lambda^{-\gamma}y_{ji}^{\gamma}}\right) - \beta\lambda^{-\gamma}\left(\sum_{i=1}^{m}\sum_{j=1}^{h}(m-i)y_{ji}^{\gamma}\right). \end{aligned} \tag{17}$$

Differentiating $l_{RSS}(\beta,\gamma,\lambda)$ with respect to β, γ, and λ, respectively, we get

$$\begin{aligned} \frac{\partial l_{RSS}(\beta,\gamma,\lambda)}{\partial\beta} =& \frac{mh}{\beta} - \sum_{i=1}^{m}\sum_{j=1}^{h}\frac{\lambda^{-\gamma}y_{ji}^{\gamma}\left(e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-i\right)}{e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-1} \\ & - \lambda^{-\gamma}\sum_{i=1}^{m}\sum_{j=1}^{h}(m-i)y_{ji}^{\gamma}, \end{aligned} \tag{18}$$

$$
\begin{aligned}
\frac{\partial l_{RSS}(\beta,\gamma,\lambda)}{\partial\gamma} = & \frac{mh}{\gamma} - mh\log(\lambda) + \sum_{i=1}^{m}\sum_{j=1}^{h}\log(y_{ji}) \\
& + \beta\lambda^{-\gamma}\sum_{i=1}^{m}\sum_{j=1}^{h}(m-i)y_{ji}^{\gamma}(\log(\lambda)-\log(y_{ji})) \\
& + \sum_{i=1}^{m}\sum_{j=1}^{h}\frac{\beta\lambda^{-\gamma}y_{ji}^{\gamma}(\log(\lambda)-\log(y_{ji}))\left(e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-i\right)}{e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-1},
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\frac{\partial l_{RSS}(\beta,\gamma,\lambda)}{\partial\lambda} = & -\frac{\gamma mh}{\lambda} + \beta\gamma\lambda^{-\gamma-1}\sum_{i=1}^{m}\sum_{j=1}^{h}(m-i)y_{ji}^{\gamma} \\
& + \sum_{i=1}^{m}\sum_{j=1}^{h}\frac{\beta\gamma\lambda^{-\gamma-1}y_{ji}^{\gamma}\left(e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-i\right)}{e^{\beta\lambda^{-\gamma}y_{ji}^{\gamma}}-1}.
\end{aligned} \tag{20}
$$

The solutions of these equations give the MLEs of the parameters β, γ and λ. However, the solutions are not in closed forms, and hence the estimates for β, γ and λ, can be obtained by solving the equations numerically. Let us denote them by $\hat{\beta}_{RSS}$, $\hat{\gamma}_{RSS}$ and $\hat{\lambda}_{RSS}$, respectively.

3.3. *Using ERSS*

ERSS was proposed by Samawi *et al.* (1996). The procedure of the ERSS is described as follows.

1. Randomly select m sets of size m elements each from the study population. These may be denoted as set $1=\{Z_{11}^*, Z_{12}^*, \ldots, Z_{1m}^*\}$, set $2=\{Z_{21}^*, Z_{22}^*, \ldots, Z_{2m}^*\}$, and so on till the last set, set $m=\{Z_{m1}^*, Z_{m2}^*, \ldots, Z_{mm}^*\}$. It is assumed that the largest and lowest elements in each set can be determined virtually or by any negligible cost method. This is of course, a simple and practical approach.

2. If m is even, measure the lowest ranked element in set 1. Repeat this procedure for set 2 till set $(m/2)$. Represent the measured elements as $Z_1, Z_2, \ldots, Z_{(m/2)}$. Furthermore, measure the largest ranked element in set $(m/2+1)$. Repeat this procedure for set $(m/2+2)$ till the last set, set m. Represent the measured elements as $Z_{(m/2+1)}, Z_{(m/2+2)}, \ldots, Z_m$.

3. If m is odd, measure the lowest ranked element in set 1. Repeat this procedure for set 2 till set $((m-1)/2)$. Represent the measured elements as $Z_1, Z_2, \ldots, Z_{((m-1)/2)}$. Furthermore, measure the largest ranked element in set $((m+1)/2)$. Repeat this

procedure for set $((m+3)/2)$ till set $(m-1)$. Represent the measured elements as $Z_{((m+1)/2)}, Z_{((m+3)/2)}, \ldots, Z_{(m-1)}$. Elements in the last set can be measured in two different ways:

- select the average of the measures of the lowest and the largest ranked elements, or
- measure the median ranked element, say Z_m. In this paper, we consider this way. The acquired sample, $\{Z_1, Z_2, \ldots, Z_m\}$, is called an ERSS of size m.

4. Independently repeat the steps h cycles, if needed, to acquire an ERSS of size $n = h \times m$.

To this end, the ERSS scheme produces a data set as follows

$$\mathbf{Z} = \{Z_{ji}\} = \begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{h1} & Z_{h2} & \cdots & Z_{hm} \end{pmatrix}.$$

Accordingly, the likelihood function of the parameters depends on whether m is odd or even. Let us denote the pdf of the i^{th} order statistic by $g_i(z_{ji})$. For even set size m, the likelihood function of ERSS $\{Z_{ji},\, j = 1,2,\ldots,h,\, i = 1,2,\ldots,m\}$ is given by

$$\begin{aligned} L_{ERSS_e}(\beta,\gamma,\lambda) &= \prod_{i=1}^{\frac{m}{2}}\prod_{j=1}^{h} g_1(z_{ji}) \prod_{i=\frac{m}{2}+1}^{m}\prod_{j=1}^{h} g_m(z_{ji}) \\ &= \prod_{i=1}^{\frac{m}{2}}\prod_{j=1}^{h} m\beta\gamma\lambda^{-\gamma} z_{ji}^{\gamma-1} e^{-m\beta\lambda^{-\gamma} z_{ji}^{\gamma}} \\ &\times \prod_{i=\frac{m}{2}+1}^{m}\prod_{j=1}^{h} \frac{m\beta\gamma\lambda^{-\gamma} z_{ji}^{\gamma-1}\left(1-e^{-\beta\lambda^{-\gamma} z_{ji}^{\gamma}}\right)^m}{e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1} \\ &= \left(m\beta\gamma\lambda^{-\gamma}\right)^{hm} \prod_{i=1}^{m}\prod_{j=1}^{h} z_{ji}^{\gamma-1} \prod_{i=1}^{\frac{m}{2}}\prod_{j=1}^{h} e^{-m\beta\lambda^{-\gamma} z_{ji}^{\gamma}} \\ &\times \prod_{i=\frac{m}{2}+1}^{m}\prod_{j=1}^{h} \frac{\left(1-e^{-\beta\lambda^{-\gamma} z_{ji}^{\gamma}}\right)^m}{e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1}. \end{aligned} \tag{21}$$

For odd set size m, the likelihood function is

$$\begin{aligned}
L_{ERSS_o}(\beta,\gamma,\lambda) &= \prod_{i=1}^{\frac{m-1}{2}}\prod_{j=1}^{h} g_1(z_{ji}) \prod_{i=\frac{m+1}{2}}^{m-1}\prod_{j=1}^{h} g_m(z_{ji}) \prod_{i=m}^{m}\prod_{j=1}^{h} g_{\frac{m+1}{2}}(z_{ji}) \\
&= \prod_{i=1}^{\frac{m-1}{2}}\prod_{j=1}^{h} m\beta\gamma\lambda^{-\gamma} z_{ji}^{\gamma-1} e^{-m\beta\lambda^{-\gamma} z_{ji}^{\gamma}} \\
&\times \prod_{i=\frac{m+1}{2}}^{m-1}\prod_{j=1}^{h} \frac{m\beta\gamma\lambda^{-\gamma} z_{ji}^{\gamma-1}\left(1-e^{-\beta\lambda^{-\gamma} z_{ji}^{\gamma}}\right)^{m}}{e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1} \\
&\times \prod_{i=m}^{m}\prod_{j=1}^{h} \frac{\beta\gamma m!\lambda^{-\gamma} z_{ji}^{\gamma-1} e^{-\frac{1}{2}\beta(m-1)\lambda^{-\gamma} z_{ji}^{\gamma}}\left(1-e^{\beta\lambda^{-\gamma}\left(-z_{ji}^{\gamma}\right)}\right)^{\frac{m+1}{2}}}{\Gamma\left(\frac{m+1}{2}\right)^{2}\left(e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1\right)} \\
&= \frac{m!^{h} m^{h(m-1)}\left(\beta\gamma\lambda^{-\gamma}\right)^{hm}}{\Gamma\left(\frac{m+1}{2}\right)^{2h}} \prod_{i=1}^{m}\prod_{j=1}^{h} z_{ji}^{\gamma-1} \prod_{i=1}^{\frac{m-1}{2}}\prod_{j=1}^{h} e^{-m\beta\lambda^{-\gamma} z_{ji}^{\gamma}} \\
&\times \prod_{i=\frac{m+1}{2}}^{m-1}\prod_{j=1}^{h} \frac{\left(1-e^{-\beta\lambda^{-\gamma} z_{ji}^{\gamma}}\right)^{m}}{e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1} \\
&\times \prod_{i=m}^{m}\prod_{j=1}^{h} \frac{e^{-\frac{1}{2}\beta(m-1)\lambda^{-\gamma} z_{ji}^{\gamma}}\left(1-e^{\beta\lambda^{-\gamma}\left(-z_{ji}^{\gamma}\right)}\right)^{\frac{m+1}{2}}}{\left(e^{\beta\lambda^{-\gamma} z_{ji}^{\gamma}}-1\right)}. \qquad (22)
\end{aligned}$$

As there are no closed form of the MLEs under even and odd set size m, the MLEs $\hat{\beta}_{ERSS}$, $\hat{\gamma}_{ERSS}$ and $\hat{\lambda}_{ERSS}$ are obtained numerically.

3.4. Using MRSS

MRSS was proposed by Muttlak (1997). The procedure of the MRSS is described as follows.

1. Randomly select m random samples, each of size m elements, from a target population.

2. The elements of each random sample in Step 1 are ranked visually with regards to the variable of interest.

3. From each sample in Step 2, if the set size m is odd select the $[(m+1)/2]$-th smallest rank element i.e the median of each sample. While if the set size m is even select

from the first $m/2$ samples the $(m/2)$-th smallest rank element and from the second $m/2$ samples the $[(m+2)/2]$-th smallest rank element. This step yields m sample elements which is the median RSS.

4. Repeat Steps 1-3 h cycles until obtaining a sample of size $n = mh$.

Suppose $X_1, X_2, \ldots, X_n$ be a random sample of size n selected from the NWP distribution $f(x;\beta,\gamma,\lambda)$. Let $V = \{V_{ji}, j = 1,2,\ldots,h, i = 1,2,\ldots,m\}$ be a MRSS; that is

$$V_{ji} = \begin{cases} X_{\left(\frac{m+1}{2}\right)} & \text{if } m \text{ is odd }, i = 1,\ldots,m \,\&\, j = 1,\ldots,h \\ X_{\left(\frac{m}{2}\right)} & \text{if } m \text{ is even }, i = 1,\ldots,\frac{m}{2} \,\&\, j = 1,\ldots,h \\ X_{\left(\frac{m+2}{2}\right)} & \text{if } m \text{ is even }, i = \frac{m+2}{2},\ldots,m \,\&\, j = 1,\ldots,h. \end{cases}$$

The pdf of V_{ji} is

$$g_{V_{ji}}(v) = \begin{cases} g_{\frac{m+1}{2}}(v_{ji}) = f_{X_{\left(\frac{m+1}{2}\right)}}(v) & \text{if } m \text{ is odd }, i = 1,\ldots,m \,\&\, j = 1,\ldots,h \\ g_{\frac{m}{2}}(v_{ji}) = f_{X_{\left(\frac{m}{2}\right)}}(v) & \text{if } m \text{ is even }, i = 1,\ldots,\frac{m}{2} \,\&\, j = 1,\ldots,h \\ g_{\frac{m+2}{2}}(v_{ji}) = f_{X_{\left(\frac{m+2}{2}\right)}}(v) & \text{if } m \text{ is even }, i = \frac{m+2}{2},\ldots,m \,\&\, j = 1,\ldots,h. \end{cases}$$

The likelihood function of the parameters depends on whether m is odd or even. For even set size m, the likelihood function of MRSS $\{V_{ji}, j = 1,2,\ldots,h, i = 1,2,\ldots,m\}$ is given by

$$\begin{aligned} L_{MRSS_e}(\beta,\gamma,\lambda) &= \prod_{i=1}^{\frac{m}{2}}\prod_{j=1}^{h} g_{\frac{m}{2}}(v_{ji}) \prod_{i=\frac{m}{2}+1}^{m}\prod_{j=1}^{h} g_{\frac{m+2}{2}}(v_{ji}) \\ &= \prod_{i=1}^{\frac{m}{2}}\prod_{j=1}^{h} \frac{\beta\gamma m!\lambda^{-\gamma} v_{ji}^{\gamma-1} e^{-\frac{1}{2}\beta m\lambda^{-\gamma} v_{ji}^{\gamma}} \left(1-e^{\beta\lambda^{-\gamma}\left(-v_{ji}^{\gamma}\right)}\right)^{m/2}}{\Gamma\left(\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}\right)\left(e^{\beta\lambda^{-\gamma} v_{ji}^{\gamma}}-1\right)} \\ &\times \prod_{i=\frac{m}{2}+1}^{m}\prod_{j=1}^{h} \frac{\beta\gamma m!\lambda^{-\gamma} v_{ji}^{\gamma-1} e^{-\frac{1}{2}\beta m\lambda^{-\gamma} v_{ji}^{\gamma}} \left(1-e^{\beta\lambda^{-\gamma}\left(-v_{ji}^{\gamma}\right)}\right)^{m/2}}{\Gamma\left(\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}\right)} \\ &= \left(\frac{m!\beta\gamma\lambda^{-\gamma}}{\Gamma\left(\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}\right)}\right)^{hm} \prod_{i=1}^{m/2}\prod_{j=1}^{h} \frac{1}{\left(e^{\beta\lambda^{-\gamma} v_{ji}^{\gamma}}-1\right)} \\ &\times \prod_{i=1}^{m}\prod_{j=1}^{h} e^{-\frac{1}{2}\beta m\lambda^{-\gamma} v_{ji}^{\gamma}} \left(1-e^{\beta\lambda^{-\gamma}\left(-v_{ji}^{\gamma}\right)}\right)^{m/2}. \end{aligned} \tag{23}$$

For odd set size m, the likelihood function is

$$
\begin{aligned}
L_{MRSS_o}(\beta,\gamma,\lambda) &= \prod_{i=1}^{m}\prod_{j=1}^{h} g_{\frac{m+1}{2}}(v_{ji})\\
&= \prod_{i=1}^{m}\prod_{j=1}^{h} \frac{\beta\gamma m!\lambda^{-\gamma} v_{ji}^{\gamma-1} e^{-\frac{1}{2}\beta(m-1)\lambda^{-\gamma} v_{ji}^{\gamma}} \left(1-e^{\beta\lambda^{-\gamma}\left(-v_{ji}^{\gamma}\right)}\right)^{\frac{m+1}{2}}}{\Gamma\left(\frac{m+1}{2}\right)^2 \left(e^{\beta\lambda^{-\gamma} v_{ji}^{\gamma}}-1\right)}\\
&= \left(\frac{m!\beta\gamma\lambda^{-\gamma}}{\Gamma\left(\frac{m+1}{2}\right)^2}\right)^{hm} \prod_{i=1}^{m}\prod_{j=1}^{h} \frac{v_{ji}^{\gamma-1} e^{-\frac{1}{2}\beta(m-1)\lambda^{-\gamma} v_{ji}^{\gamma}} \left(1-e^{\beta\lambda^{-\gamma}\left(-v_{ji}^{\gamma}\right)}\right)^{\frac{m+1}{2}}}{\left(e^{\beta\lambda^{-\gamma} v_{ji}^{\gamma}}-1\right)}.
\end{aligned} \tag{24}
$$

Again, there are no closed form of the MLEs under even and odd set size m, the MLEs $\hat{\beta}_{MRSS}$, $\hat{\gamma}_{MRSS}$ and $\hat{\lambda}_{MRSS}$ are obtained numerically.

The comparison between different estimators of a specific parameter θ can be done using the asymptotic efficiency (see Basu, 1956). The asymptotic efficiency of $\hat{\theta}_1$ with respect to $\hat{\theta}_2$ for estimating θ is defined by

$$
\text{Aeff}(\hat{\theta}_1;\hat{\theta}_2) = \lim_{n\to\infty} \text{eff}(\hat{\theta}_1;\hat{\theta}_2) = \frac{FI_1(\theta)}{FI_2(\theta)}.
$$

Since the FI numbers of β, γ, and λ cannot be obtained in closed form under RSS, ERSS, and MRSS, their values will be obtained through a simulation study.

4. Interval estimates

Let $X_1,\ldots,X_n$ be a random sample from $f(x;\theta)$, where θ is an unknown quantity. A confidence interval for the parameter θ, with confidence coefficient $1-\alpha$, is an interval with random endpoints $[L(X_1,\ldots,X_n), U(X_1,\ldots,X_n)]$. It is given by

$$
P\left(L(X_1,\ldots,X_n) \le \theta \le U(X_1,\ldots,X_n)\right) = 1-\alpha.
$$

The interval $[L(X_1,\ldots,X_n), U(X_1,\ldots,X_n)]$ is the well-known $100(1-\alpha)\%$ confidence interval for θ. Moreover, let $\hat{\theta}$ be the MLE of θ. It is well known that, under some mild regularity conditions (see Davison, 2008, p. 118), the MLE has the following properties:

1. $\hat{\theta}$ is asymptotically consistent;
2. $\hat{\theta}$ is asymptotically unbiased;
3. the sampling distribution of $\hat{\theta}$ is asymptotically normal with its variance obtained from the inverse Fisher information number of sample size 1 at the unknown parameter θ; that is, $\hat{\theta}_{MLE} \to N\left(\theta, FI^{-1}(\theta)\right)$ as $n\to\infty$.

Accordingly, the approximate $100(1-\alpha)\%$ confidence limits for $\hat{\theta}$ of θ can be constructed as

$$P\left(-z_{\frac{\alpha}{2}} \leq \frac{\hat{\theta}-\theta}{\sqrt{FI^{-1}(\hat{\theta})}} \leq z_{\frac{\alpha}{2}}\right) = 1-\alpha,$$

where z_α is the α^{th} upper percentile of the standard normal distribution.

Therefore, the approximate $100(1-\alpha)\%$ confidence limits for the parameters β, γ and λ of the NWP distribution are given, respectively, by

$$P\left(\hat{\beta}-z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\beta})} \leq \beta \leq \hat{\beta}+z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\beta})}\right) = 1-\alpha,$$

$$P\left(\hat{\gamma}-z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\gamma})} \leq \gamma \leq \hat{\gamma}+z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\gamma})}\right) = 1-\alpha,$$

$$P\left(\hat{\lambda}-z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\lambda})} \leq \lambda \leq \hat{\lambda}+z_{\frac{\alpha}{2}}\sqrt{FI^{-1}(\hat{\lambda})}\right) = 1-\alpha.$$

5. SIMULATION STUDY

To investigate the properties of the proposed MLEs of the parameters β, γ and λ of the NWP distribution a simulation study is conducted. Monte Carlo simulation is applied for different sample sizes, $m = \{4,5\}$ and $h = \{10,50,100\}$, for the parameter values $(\beta = 1.5, \gamma = 1, \lambda = 0.5)$ and $(\beta = 0.5, \gamma = 2, \lambda = 1.5)$. Biases and MSEs of the MLEs of β, γ and λ are computed over 10000 replications under SRS, RSS, ERSS, and MRSS, where

$$\text{Bias}(\hat{\theta}) = E(\hat{\theta}-\theta),$$

and

$$\text{MSE}(\hat{\theta}) = E(\hat{\theta}-\theta)^2.$$

The results are shown in Tables 1 and 2. It can be seen from these results that estimates of β, γ and λ based on RSS, ERSS, and MRSS have smaller biases and MSEs than the corresponding estimates based on SRS. Biases and MSEs in RSS, ERSS, and MRSS decrease as set sizes and/or number of cycle increase. Moreover, one can see from Tables 1 and 2 that the MLEs derived by RSS, ERSS, and MRSS are more efficient than SRS estimators. Furthermore, a 95% asymptotic confidence interval of β, γ and λ under SRS, RSS, ERSS, and MRSS are calculated and the results are displayed in Tables 3 and 4. It can be concluded that the widths of the intervals constructed by RSS, ERSS, and MRSS are narrower than the one constructed by SRS.

TABLE 1
The bias, MSE, and efficiency values of estimating the parameters ($\beta = 1.5, \gamma = 1, \lambda = 0.5$) under SRS, RSS, ERSS, and MRSS.

		$\beta = 1.5$			$\gamma = 1$			$\lambda = 0.5$		
$n = m \times h$	Sampling	Bias($\hat{\beta}$)	MSE($\hat{\beta}$)	Eff	Bias($\hat{\gamma}$)	MSE($\hat{\gamma}$)	Eff	Bias($\hat{\lambda}$)	MSE($\hat{\lambda}$)	Eff
4×10	SRS	0.056	0.012		0.038	0.019		0.015	0.004	
	RSS	0.058	0.007	1.58	0.026	0.012	1.58	0.015	0.003	1.75
	ERSS	0.059	0.008	1.45	0.020	0.010	1.97	0.018	0.002	1.82
	MRSS	0.057	0.007	1.60	0.034	0.016	1.21	0.014	0.003	1.49
4×50	SRS	0.062	0.006		0.007	0.003		0.020	0.001	
	RSS	0.052	0.004	1.60	0.005	0.002	1.53	0.017	0.001	1.60
	ERSS	0.054	0.004	1.45	0.005	0.002	1.80	0.018	0.001	1.48
	MRSS	0.053	0.004	1.60	0.005	0.003	1.18	0.017	0.001	1.49
4×100	SRS	0.058	0.005		0.004	0.002		0.019	0.001	
	RSS	0.046	0.003	1.61	0.003	0.001	1.57	0.015	0.001	1.83
	ERSS	0.049	0.003	1.45	0.003	0.001	1.86	0.016	0.001	1.67
	MRSS	0.046	0.003	1.61	0.003	0.001	1.18	0.015	0.001	1.65
5×10	SRS	0.060	0.010		0.027	0.015		0.016	0.004	
	RSS	0.058	0.006	1.67	0.017	0.008	1.81	0.017	0.002	1.85
	ERSS	0.060	0.007	1.52	0.015	0.007	2.10	0.017	0.002	1.95
	MRSS	0.059	0.006	1.64	0.026	0.013	1.13	0.015	0.003	1.39
5×50	SRS	0.061	0.006		0.006	0.003		0.019	0.001	
	RSS	0.049	0.003	1.74	0.003	0.001	1.76	0.016	0.001	1.80
	ERSS	0.050	0.004	1.62	0.003	0.001	2.07	0.016	0.001	1.76
	MRSS	0.050	0.004	1.63	0.005	0.002	1.13	0.016	0.001	1.54
5×100	SRS	0.056	0.004		0.003	0.001		0.018	0.001	
	RSS	0.043	0.003	1.71	0.002	0.001	1.66	0.014	0.000	1.90
	ERSS	0.044	0.003	1.62	0.001	0.001	1.94	0.015	0.000	1.88
	MRSS	0.046	0.003	1.54	0.003	0.001	1.07	0.015	0.000	1.58

TABLE 2

The bias, MSE, and efficiency values of estimating the parameters ($\beta = 0.5, \gamma = 2, \lambda = 1.5$) under SRS, RSS, ERSS, and MRSS.

		$\beta = 0.5$			$\gamma = 2$			$\lambda = 1.5$		
$n = m \times h$	Sampling	Bias($\hat{\beta}$)	MSE($\hat{\beta}$)	Eff	Bias($\hat{\gamma}$)	MSE($\hat{\gamma}$)	Eff	Bias($\hat{\lambda}$)	MSE($\hat{\lambda}$)	Eff
4×10	SRS	0.033	0.014		0.070	0.075		0.048	0.011	
	RSS	0.031	0.007	2.05	0.051	0.048	1.57	0.050	0.007	1.59
	ERSS	0.033	0.008	1.92	0.043	0.039	1.91	0.055	0.007	1.57
	MRSS	0.030	0.007	2.14	0.056	0.061	1.22	0.049	0.008	1.42
4×50	SRS	0.035	0.004		0.014	0.013		0.052	0.004	
	RSS	0.030	0.002	1.81	0.010	0.008	1.55	0.044	0.003	1.51
	ERSS	0.032	0.002	1.59	0.006	0.007	1.80	0.047	0.003	1.34
	MRSS	0.026	0.002	2.26	0.012	0.011	1.21	0.040	0.002	1.71
4×100	SRS	0.032	0.002		0.007	0.006		0.048	0.003	
	RSS	0.027	0.001	1.78	0.004	0.004	1.56	0.040	0.002	1.46
	ERSS	0.029	0.002	1.56	0.003	0.003	1.80	0.044	0.003	1.29
	MRSS	0.025	0.001	2.19	0.007	0.005	1.20	0.037	0.002	1.68
5×10	SRS	0.034	0.012		0.058	0.060		0.052	0.009	
	RSS	0.031	0.005	2.49	0.036	0.032	1.86	0.050	0.005	1.81
	ERSS	0.034	0.005	2.24	0.029	0.027	2.18	0.052	0.005	1.80
	MRSS	0.027	0.004	2.91	0.052	0.052	1.15	0.046	0.006	1.68
5×50	SRS	0.035	0.003		0.010	0.010		0.051	0.004	
	RSS	0.028	0.002	2.11	0.007	0.006	1.73	0.042	0.002	1.57
	ERSS	0.030	0.002	1.79	0.005	0.005	1.98	0.045	0.003	1.40
	MRSS	0.025	0.001	2.62	0.009	0.009	1.15	0.038	0.002	1.85
5×100	SRS	0.031	0.002		0.006	0.005		0.046	0.003	
	RSS	0.024	0.001	2.02	0.004	0.003	1.61	0.037	0.002	1.54
	ERSS	0.027	0.001	1.68	0.003	0.003	1.86	0.040	0.002	1.38
	MRSS	0.023	0.001	2.38	0.005	0.004	1.08	0.034	0.002	1.75

TABLE 3
A 95% asymptotic confidence interval for ($\beta = 1.5, \gamma = 1, \lambda = 0.5$) under SRS, RSS, ERSS, and MRSS.

	SRS		RSS	
Parameter	Interval	Width	Interval	Width
β	(1.426, 1.685)	0.259	(1.444, 1.643)	0.198
γ	(0.936, 1.071)	0.135	(0.950, 1.054)	0.104
λ	(0.466, 0.570)	0.104	(0.477, 0.552)	0.075
	ERSS		MRSS	
Parameter	Interval	Width	Interval	Width
β	(1.443, 1.646)	0.204	(1.442, 1.650)	0.208
γ	(0.953, 1.050)	0.097	(0.938, 1.068)	0.130
λ	(0.477, 0.552)	0.076	(0.473, 0.556)	0.083

TABLE 4
A 95% asymptotic confidence interval for ($\beta = 0.5, \gamma = 2, \lambda = 1.5$) under SRS, RSS, ERSS, and MRSS.

	SRS		RSS	
Parameter	Interval	Width	Interval	Width
β	(0.445, 0.617)	0.173	(0.464, 0.585)	0.122
γ	(1.871, 2.141)	0.270	(1.898, 2.110)	0.213
λ	(1.440, 1.652)	0.212	(1.452, 1.622)	0.170
	ERSS		MRSS	
Parameter	Interval	Width	Interval	Width
β	(0.460, 0.593)	0.133	(0.467, 0.579)	0.112
γ	(1.904, 2.102)	0.198	(1.875, 2.135)	0.260
λ	(1.450, 1.630)	0.180	(1.454, 1.614)	0.160

6. Real data application

In this section, a real-life data set is analyzed for the purpose of illustration to show the usefulness of the RSS, MRSS, and ERSS schemes in reducing the MSEs of the estimators comparing with the traditional SRS scheme. Data set includes the 72 exceedances of flood peaks (in m^3/s) of the Wheaton River near Carcross in Yukon Territory, Canada, for the year 1958-1984, rounded to one decimal place (Choulakian and Stephens, 2001). The observations are given in Table 5.

TABLE 5
Exceedances (in m^3/s) of Wheaton river flood data.

1.7	2.2	14.4	1.1	0.4	20.6	5.3	0.7	1.4	18.7	8.5	25.5
11.6	14.1	22.1	1.1	0.6	2.2	39.0	0.3	15.0	11.0	7.3	22.9
0.9	1.7	7.0	20.1	0.4	2.8	14.1	9.9	5.6	30.8	13.3	4.2
25.5	3.4	11.9	21.5	1.5	2.5	27.4	1.0	27.1	20.2	16.8	5.3
1.9	10.4	13.0	10.7	12.0	30.0	9.3	3.6	2.5	27.6	14.4	36.4
1.7	2.7	37.6	64.0	1.7	9.7	0.1	27.5	1.1	2.5	0.6	27.0

To assess whether this data set is well-modeled by a NWP distribution, Kolmogorov-Smirnov test is applied. The MLEs of the parameters β, γ, λ and the p-value of the Kolmogorov-Smirnov test are $0.238, 0.883, 2.24$ and 0.3526, respectively. The p-value is not significant, and thus, the data can be modeled by the NWP distribution.

For purposes of comparison, a SRS of size 15 is drawn from this data set, and in RSS and its modifications $m = 3$ and $h = 5$ are chosen. The NWP distribution is fitted to each of these samples (SRS, RSS, MRSS, and ERSS). For each of them, the MLEs, the Akaike information criterion (AIC), and Bayesian information criterion (BIC) are evaluated and the results are reported in Table 6. According to the values of AIC and BIC, data obtained by RSS is the best data fitted by the NWP distribution.

TABLE 6
The MLEs, -2LL, AIC, and BIC values for Exceedances of Wheaton river flood data under SRS, RSS, MRSS, and ERSS.

Sampling	Parameter Estimates ($\hat{\beta}, \hat{\gamma}, \hat{\lambda}$)	-2LL	AIC	BIC
SRS	(0.199, 0.886, 2.487)	-56.629	119.258	121.382
RSS	(0.311, 0.776, 2.334)	**-51.498**	**108.996**	**111.120**
MRSS	(0.064, 1.696, 3.408)	-53.418	112.837	114.961
ERSS	(0.269, 0.843, 2.411)	-52.515	111.030	113.154

7. Concluding remarks and further work

In this paper, maximum likelihood estimation for estimating the unknown parameters of new Weibull Pareto distribution is studied in the RSS framework and some of its modifications. Since solutions of these estimators have no closed forms, the new obtained estimators are compared via a simulation study with the conventional estimators obtained by SRS. Two criteria are used for comparison, the mean squared errors and the bias values. It is found that the biases and MSEs of the estimators under RSS, ERSS, and MRSS are smaller than the corresponding estimators obtained by SRS. Thus, estimation based on RSS scheme and its modification are more efficient than estimation under the SRS scheme. Also, to evaluate the precision of the estimators, confidence intervals for the unknown parameters are constructed. It is found that the estimators obtained by RSS, ERSS, and MRSS are more precise than the corresponding estimators obtained by SRS.

Finally, it is worth mentioning that in this paper perfect RSS is investigated, and it is of great interest to study how information can be lost due to imperfect ranking. Thus, this study can be extended to include imperfect RSS schemes provided that a multivariate (or bivariate) version of the NWP distribution must be derived, however this is left as a future work.

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SUMMARY

The method of maximum likelihood estimation based on ranked set sampling (RSS) and some of its modifications is used to estimate the unknown parameters of the new Weibull-Pareto distribution. The estimators are compared with the conventional estimators based on simple random sampling (SRS). The biases, mean squared errors, and confidence intervals are used to the comparison. The effect of the set size and number of cycles of the RSS schemes are addressed. Monte Carlo simulation is carried out by using R. The results showed that the RSS estimators are more efficient than their competitors using SRS.

Keywords: New Weibull-Pareto distribution; Fisher information; Maximum likelihood estimation; Ranked set sampling.