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THE CONCEPT OF MEASURE, FROM PLATO TO MODERN STATISTICS. ART OF LIVING OR INTELLECTUAL PRINCIPLE?

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1. INTRODUCTION

There is nothing more reassuring in the uncertainty of our everyday life than the possibility of measuring something: an operation which renders objective our sensory perceptions of the world by overcoming the ambiguous divide between personal assumption and the reality of phenomena. In fact, if everything that can be measured exists, the factual existence is made evident and transmissible by a measure, that is by a number which, in its meagre essence, appears to leave no room for subjective interpretations. But what is measure? If we are still today looking for an answer it is because a univocally accepted general definition does not exist.

In effect, one of the most curious aspects of this topic is indeed the existence of innumerable and often contradictory definitions of measure and of a measurable phenomenon. As we move forward in time, the more we approach this age of high scientific and technological specialisation, the more do these definitions confront new realities which cannot be compared with known models.

Paradoxically, it is the very physics of our century, heir of the knowledge which historically provided the foundation of metrology, that developes a conception both rigorous and also strongly reductive of measure as an operation which allows the determination of how many times a physical quantity contains another physical quantity of reference, homogeneous with it, assumed as a standard unit, (International Committee of Weights and Measures, 1950-60). It is a definition both limited and inadequate because it excludes from the measurable a lot of phenomena, which since immemorial time have been the object of quantitative analysis, some of which are in fact widely dealt with in physics manuals, such as acoustics, thermodynamics and photometry. Besides, which role should be assigned to the operation of counting: perhaps that of the first quantitative criterion devised by man to understand the world. Which meaning should be attributed to the probability of an event: the last refined rational strategy for the measurement of the uncertainty of events, the indeterminism intrinsic to natural phenomena.

2. PLATO'S ART OF MEASUREMENT

One has to look back long in time in order to rediscover the cultural breadth of a science yet to be constructed and free from preconceptions of the technology on which it would then be founded. One has to return to the speculative power of the most ancient philosophy, in which all the knowledge of the world is cogently represented by the art of measurement. One must read again Plato when Socrates says, in the Protagoras (357 a, b): "Well, my fellow men, as the salvation of our life has shown itself to be in the correct choice of pleasure and pain, of the many and the few, of the greater and the smaller, of the farther and the nearer, will it not be shown with equal clarity to be, above all, art in measurement, because it is, in fact, a search for excess and for defect and reciprocal equality. Not only this, but will such an art of measurement not necessarily result in an art and a science" [Our translation]

What did Socrates' friends measure, and what had their fathers measured: Everything. The ancient architects, by building for the Gods the majestic temples which history still preserves for us, offered to man the perfection of their measurements. By observing the birds, the augurs measured the uncertainty of events, and the astrologers foresaw - again by measuring - the motion of celestial bodies.

In ancient Egypt the art of measurement had already reached refined levels: the quadrant and the astrolabe were already known and used for determining the positions of stars, the hourglass for measuring time, and the gnomon for following the passage of the seasons. Astronomical knowledge and the measurement instrumentation for the forecasting of mysterious events laden with esotericism, such as eclipses, gave the caste of priests who guarded their secrets a great political and social power.

Thus, the use of numbers and of geodetic apparatus were tools of the trade for the ancient engineers, just as notched, rules and scales for the measure of lengths and weights were commonplace for shopkeepers and merchants. And money was already the measure used for the relationship of exchange for goods and services.

3. PYTHAGORAS RELATIONSHIP

Historiography claims that Pythagoras discovered the numerical proportions relating to sound after having observed that the pitch produced by the beating of an anvil with four hammers varied inversely with the weight of the hammer. Thus goes the tale recounted by the mathematician Nicomachus of Gerasa (I-II century) in his 'Enchiridion Harmonices' [Our translation, from Nicomaque de Gèrase, Manuel d'Harmonique et autres texts relatifs a la Musique, par Ch. Em. Ruelle, Paris, Bauer éd.. 1881, p. 19-22]:

> "27. One day while he [Pythagoras] was out walking, completely absorbed in his thinking and meditations suggesting various combinations, trying to imagine an aid for the ear, one that would be certain and without error, such as the eye has in the form of the compass, ruler or dioptre, and touch has in the form of scales or the in-

> vention of measures, he found himself passing, by pure fortuitous coincidence, a blacksmith's forge, and very clearly heard iron hammers beating the anvil and producing a range of sounds which, the though mixed, were completely consonant with one another, except for one pair alone. He recognised among them the consonance of diapason (octave), diapente (fifth) and diatessaron (fourth). Regarding the intermediate interval between fourth and fifth, he realised it was by itself dissonant, but on the other hand complementary with the greater of these two consonances.
> 28. Overjoyed, he entered the forge as though a god had granted his wish and, by means of varied experiments, after having realised that it was the difference in weight that caused the difference in sound and not the strength of blacksmith's blows, nor the shape of the hammer, nor the shifting of the iron being beaten, he measured with great accuracy the weight of the hammers and their impulsive strengths, and after finding the latter perfectly identical, he went home.
> 29. (...) He hung four ropes on some nails, similar in quality, number of threads, thickness and torsion, and to each of these hung a weight, fixed at the lower end. In addition he made the ropes of exactly the same length and then by hitting the ropes together two by two, he respectively identified the said consonances which varied for each pair of ropes."

There follows in the Nichomachean text the detailed formulation of the numerical relationships calculated by Pythagoras between the length of the ropes and the weights hung on them.

It is of little consequence how much truth lies in such a narration. It is, though, true that the Pythagorean school saw in those relationships the essence of a universe guided by the rigorous harmony of numbers. For the first time the concept of number detaches itself from the numbered entity, the geometric shape from the physical object: numbers and shapes become abstract concepts, and the relationships among them are valid beyond their concrete verification in the real world: they become relationships between ideas. Thus, art becomes science and measure finds its first axiomatic relationships.

4. GALILEO'S INSTRUMENTS

One has, in fact, to pass through natural Platonic philosophy, through the myth of a natural science which finds its highest expression in the mathematics of forms and reaches its extreme rationalisation by removing any meaning from the capricious contingency of phenomena, in order to assign to the idea-number the task of liberating speculative thought from comparison with reality. This evolution of Platonic philosophy finds in Euclidean geometry its most refined expression and the first realised axiomatic base of spatial measurements.

Perhaps the taking root over the centuries, albeit with ups and downs, of astronomy and of geodesy as guiding sciences and models of reference for many other areas of research is due to this very interaction between the art of measurement and the rational abstraction of forms.

However, the decline of classical civilisation and the subsequent weakening of philosophical thinking leads to the consolidation of the subordinate role of science in relation to philosophy, with the latter mainly dealing with theology and alchemy. This period of European scientific backwardness which stretches till the XVII century, and which was only completely resolved by the Galilean revolution, had not however interrupted the development of business, commerce and industry. Technology continued to progress: watches substituted hourglasses, craftsmen built machines and tools of greater and greater complexity. The arrival of the pendulum, binoculars, and the precision scale acted to disseminate the Galilean imperative of measuring the measurable in mechanics, astronomy and chemistry, paving the way for modern science.

With Galileo, the practice of measurement in fact leaves the environment of commerce and craftsmanship, in which it had been enclosed as a practical although refined activity of daily life, when experimental science, discovered in the art of measurement an essential aid. Neither Galileo's measures (lengths, weights and times) nor his mathematical games of combinatorial calculus were of use in constructing buildings or trading goods, but were instead useful in translating into simplified formal models the phenomena of nature. If the great book of nature is written in mathematical language - as Galileo thought - it belongs to science the task of searching for the numerical relationships between observed quantities.

It is in this spirit that the Royal Society of London, since its foundation in 1662, took on the task of defining a universal measure of length related to the dimension of a pendulum, the oscillation of which lasts exactly one second. The arm of the pendulum would have taken the role of the unit of measurement of a universal spatio-temporal system. A fascinating and ambitious aspiration, but one which did not take into account the different effects of gravity on the terrestrial globe and which made the lengths of pendula of equal oscillation times vary.

Once the myth of a universal measure had been abolished, then science turned to the perfecting of instruments and to the search for measures suitable for the new phenomena which were becoming established in modern thought. Thus, the concept of measure as a speculative process to gather knowledge about the world had in any case been lost.

5. KANT DISTINCTION

As it is known, the first attempt to give a global vision of the measurable was due to Kant (Kritik der reinen Vernunft, 1781) when, starting from the principle that "...the pure scheme of quantity as a concept of the intellect is the number" he differentiates between extensive quantities, which are susceptible to decomposition into parts, and intensive quantities unitarily perceived as sensations describable by ordered relationships.

In the Analysis of Principles, under the title "Systematic Representation of all the Synthetic Principles of the Pure Intellect", Kant defines as extensive all the "quantities wherein the representations of parts makes possible the representation of the whole..." and as intensive, the quantities "...as unit, and in which multiplicity can be represented only by approximating to negation". The firsts are the answer to the question "how big is a thing" and they refer to representable phenomena in space and time by that very synthesis by which space and time are determined. The seconds express "a certain degree of effect on the senses" and they refer to all the phenomena in which "reality is the object of sensation".

The firsts correspond to additivity criteria and the units of elementary measure add up, the seconds are only susceptible to graduation in relation to the limit condition of absence of the very phenomenon. "Hence - states Kant - between the reality in the phenomenon and its negation there is a continuous chain of many possible intermediate sensations...". "Hence any sensation,...has a grade, that is an intensive quantity, ...and between the reality and the negation there is a continuous chain of possible realities".

Nowadays the Kantian differentiation between extensive and intensive quantities no longer finds many reasons to continue to exist, even though in the 1920s it provoked one of the most fertile debates amongst metrologists. Coherently, physicists continued to state that a quantity belongs to scientific research if, and only if, it may be described by a metric ordering. That is, by a unit sample measure on which to operate additivity, even if by these rigid definitions a vast class of measures, lacking immediate additivity and tied to psychophysical research and then emerging, were excluded by scientific research. Perhaps this traditional opposition was due to an excessive mistrust of social, behavioural and psychological sciences which tried to find a first quantitative foundation in psychometrics. And perhaps it has been this very attitude which stimulated quantitative psychology to give itself a highly formalised order, till it reached in those years a very high level of precision, which is today largely lost.

Nevertheless, the most orthodox scientific bodies are still inclined to reject any definition of an extensive quantity which does not respect the requisite of additivity, leaving to the fringes of science the so-called intensive quantities linked to photometric, audiometric, psychometric, and biochemical measurements which still have a long measurable tradition. It suffices to think of the ordinal scales of mineral hardness, nowadays highly refined and with their own specific theory of errors, of so many non-additive chemical measurements aimed at the identification of the class to which an element belongs, or of the nuclear measures based on the counting of certain pulses: here, in fact, the measurement unit appears to be missing.

6. AXIOMATIC FORMULATION

The paradigms of classical mechanics which claimed to describe reality only in terms of mass, length, time, and force have led to the erection of barriers within science by creating a hierarchic order among disciplines on the basis of a totally arbitrary criterion of comparison: the presence or absence of additive measures. However, those ambiguous distinctions are destined to die out, superseded by the more recent axiomatic formulation of the theory of measure and by the more general epistemological assessments of the categories of the measurable.

A different way of tackling the problem turns, in fact, to the axiomatic approach to the foundation of measure, that is to the criteria associated to the assignment of a number to a quantity independently from pre-set techniques for such assignments (D.H. Krantz, *et al.* 1971).

Among the numerous attempts to reduce to axioms the systems of relationships between the physical world and metrical description, the simplest formulation of a system of measures, and one which is apparently less contradictory, is based on the concept of numerical representation of a physical structure conceived as theoretical model, i. e. as a collection of elements linked by one or more empirical relationships.

As an example of an assiomatic formulation of the concept of measure, we have: (i) A is a class of objects, (ii) $\Re$ is a relational structure and (iii) $\odot$ is a logic operation of concatenation. Besides it is possible to define, for example, an order preserving additive function φ, by means of a homomorphism that maps (i) A into R_0 (a set of real numbers), (ii) $\Re$ in >, and (iii) $\odot$ in +, while maintaining all the properties of the reference set.

Given the conditions of equivalence (transitive, symmetric, reflexive) and strict order and the association, monotonicity and Archimedean axioms, a measure φ is determined by constructing a corresponding linkage of the relations inside the empirical structures and the relations among numbers, such that, for a, b, $c \in A$, we have:

$$a \sim b \Leftrightarrow \varphi(a) = \varphi(b)$$
$$a \odot b \sim c \Leftrightarrow \varphi(a) + \varphi(b) = \varphi(c)$$

where ~ denotes the condition of equivalence.

The uniqueness theorem is essential for this formulation: given two measures φ, φ' homomorphic on the same empirical structure, the function $f(\varphi) = \varphi'$ identifies the admissible transformation among measures and allows to define procedures of equivalent measurements. In the class of extensive attributes, given a comparative quantity k, chosen as a standard, unit, and a quantity a, with measure $\varphi(a)$, the ratio $f(a) / f(k)$ is univocally determined whatever k. In fact, if φ' is a numerical function built on a standard unit $a \neq k$ it must be $\varphi(a) / \varphi(k) = \varphi'(a) / \varphi'(k)$, that is $\varphi'(a) = \alpha\varphi(a)$ by setting $\varphi(k) = \alpha$, with $\alpha > 1$. So the similarity transformations $\varphi'(a) = \alpha\varphi(a)$ are admissible and define scales of univocally determined ratios.

On the contrary, when the admissible transformation is of the kind $\varphi' = \alpha\varphi + \beta$, with $\alpha > 0$, an interval scale is obtained where the following ratio stays invariant:

$$\frac{\phi'(a)-\phi'(b)}{\phi'(c)-\phi'(d)} = \frac{\phi(a)-\phi(b)}{\phi(c)-\phi(d)}$$

Here we have two arbitrary choices - the origin and the measurement unit - and their variation leads to similar transformations.

Generally, a classification of measures in terms of admissible transformations requires a non ambiguous identification of the measurement procedure selected. When ratio scales are defined, we often think that only the choice of the comparison unit must be taken as free, but other non restricted aspects could have been neglected. In setting up a standard sequence, when, for example, a length a is measured, it is usual to count the number n of elementary units k, which are needed to define a by introducing the operation of addition.

However, this is an arbitrary choice as well. In fact, the elementary units could be multiplied according to exponential k^n. But the transformed function $\phi' = k^n$ is not additive and it allows only an homomorfism in (R_0, >, ⊗), where ⊗ is now any binary operation instead of addition.

These examples illustrate the need for further thoughts on arbitrary assumptions which are implicit in the numerical measurement scales. Furthermore, there is a need to verify the admissibility of some transformations. Non-admissibility implies non-equivalence of measurement procedures, hence the non comparability of the results. What remains invariant is the set of empirical relationships, some of which are intuitively taken or assumed as axioms.

Hence, any phenomenal occurrence at the moment of observation and measurement is ideally described by a model, whith is developed from a conventional set of invariant qualitative relations which are defined *a priori*. These axioms are collections of qualitative empirical laws and as such they must be interpreted and verified by homomorphic numerical structures from which to draw concrete subsets of quantitative measurements.

The deeper analysis of the epistemological foundations of the theory of measure in modern scientific research carried out by philosophers of the Circle of Vienna - Mach and Carnap in particular - and by the major exponents of Berlin neopositivism - Reichenbach and Hempel - has put at the base of a correct definition of measure every procedure of classification which allows the assignment of an object to a class and the construction within the class of a relation of order, not necessarily a quantitative one, linked for instance to evaluations of the kind "more than...", "less than...", that is relations of precedence or coincidence. So before any measure comes a representation of the world in phenomenal categories linked to one another by a set of relationships which can be referred back to numerical language.

It is therefore convenient to further enlarge the concept of measure so as to include not only the rational quantities, where there is a standard unit that can be added up, but also the phenomenic entities which are representable by numerical structures not bound to additivity: for instance, relationships of classification, ordering or, generally, any indirect instrumental measure.

The translation process of a natural event to a number is always a product of the mind, a theoretical construction. On the basis of this knowledge a fruitful exchange started between science and technology, one which has often made ambiguous the difference between measure and measured quantity, starting an operational concept of metrology expressed in the Bridgemanian operationalism (Bridgeman, 1927). Besides, the interaction between a physical phenomenon and an instrument of measure, or similarly, between a social phenomenon and the procedure of observation, has always threatened the nineteenth century pretension of objectivity of an observation independent of the observer.

The risk however is of falling into an operative concept that favours measurement over the phenomenon, by simply attributing the qualification of quantity to all that is measurable. An operational criterion to which Bridgeman himself would have been opposed. The essence of measurement is always characterized by a semantic link between an idealisation of reality and a symbolic mathematical structure. In every measure a model is always implicit; even in the simplest measure of length an abstraction takes place from a three dimensional reality to an ideal class of unidimensional objects.

7. THE ROLE OF ERRORS

The formal systems of measure, however axiomatised, cannot be evaluated solely in mathematical terms. The internal coherence and elegance of the theory must be compared with the measurable reality. Hence, any judgement regarding the validity of a system of measure cannot avoid taking into account also empirical criteria. The majority of the definitions of measure and of the theories developed around those definitions have ignored the fundamental role of error, trivially reduced to the status of technical problem of the metrologists or handed over to the statisticians to be dealt with. However, it is the very refining of techniques of measurement, combined with the results of the advanced development of the physical sciences, that has demonstrated how the immanence of error cannot be eliminated and how it becomes more noticeable the greater the precision of the instruments and the degree of rigour involved. But which type of error is it?

The first and more neglected component of uncertainty derives from the inadequacy of the model in describing the reality to be measured. This may happen when the theoretical framework is weaker (social and economic phenomena), but it appears frequently even in the more advanced areas of physics (subatomic physics and astrophysics).

It must be said that this first neglected component historically relegated to the less developed sciences, appear often to be more aware of uncertainties due to problems that have been badly posed, models which have been mispecified. Every empirical system can be described by a set of numerical relations if, and only if, it is reduced to a model through a simplification of the phenomenically relevant relationships: for example, keeping certain parameters constant and overlooking others, or modifying the form of relation (making linear or orthogonalising the equations) or, again, introducing a convenient variable, nearly always of the stochastic type, with the important job of representing "the rest of the world" not explicitated by the model. The measurements which result have a strictly limited field of validity linked to the model: when they are extended outside this ambit, they generate unexplained variability.

The second source of error, which has long since stopped troubling researchers, relates to the variability of results generated by the contingent characteristics of the experimental situation. The awareness of instrumental error which can create a difference between the results obtained from a measurement repeated several times on the same object has given rise, from the 18th century, to a consolidated theory founded on refined probabilistic solutions.

The third component of error is induced by the instrument and, in macroscopic phenomena, can be summarised in the so-called systematic distortion present in an instrument which is not well balanced; this is a disturbing factor which is amply discussed in the techniques for quality control and the reliability of instrumental apparatus, in which statistical methodology has an essential role. In this field, however, a new component of indeterminacy comes from more complex instruments in the phase of transition from the instrumental signals to the fields of numbers.

As long as the measures are length, weight and mass, the problem of correspondence between signal and measurement apparently does not arise. When the signal is photoelectric, ecographic, Doppler, ... the conversion of the signal, which is not solely a technical matter, leads to new logical difficulties regarding the criteria for assigning a (not necessarily biunivocal) correspondence, from the field *S* of signals to the field R_0 of real numbers, consistent with all the empirical relationships to be described.

This problem, raised by physicists, is also of great methodological relevance for the social sciences. For instance, when one uses a statistical indicator of a population's welfare, is the researcher really certain to have built a syntactically consistent chain in the passage between the model of reality, field of signals and numerical structure? The answer should make one very cautious in accepting those measures as objective, that is comparable, and hence subjectively transferable.

In the microscopic and submicroscopic phenomena of physics and biology, the effect of the instrument has an altogether different impact and finds its most synthetic formulation in the uncertainty principle. As Heisenberg wrote: that which derives from an observation is only a function of probability.

The possibility of making a systematic error more or less irrelevant, and to use probabilistic models for handling accidental error, does not change the nature of random variables of every quantity described by a measurement. The fundamental consequence of this knowledge, which has conquered sectors of physics which are more traditionally deterministic, is the necessity of expressing a measurement by an interval. This is also true for the ideal measurement, the one virtually without error, in a deterministic conception of the world. This conceptual position challenges one of the fundamentals on which the definition of measurement is based - strongly sustained by Bertrand Russell - that is the biunivocal correspondence between physical structure and numerical structure. i.e. that in microphysics we cannot measure at the same time the position and the velocity of a given unit.

A measurement, therefore, is no longer an exact value, rather it is an interval of uncertainty, an unavoidable moment of indeterminacy. So here is another myth which fades. According to modern theoreticians of measure, for every measurement one can only associate an interval of indifference within which there is no reason to choose one value rather then another. This band of uncertainty should not be seen as a term of residual error, which is not susceptible to further correction, but as a logical definition of measurement in a physical system.

The need to approximate measurements with intervals of indifference takes on a different meaning if it is seen as an operational limitation, or if it becomes an intrinsic factor of the definition of measure. To admit an "intrinsic" uncertainty means that no object can be expressed by a sole number, that a "true" measurement does not exist, that measurement is the numerical approximation of an interval, that error must not be seen any longer as a difference from a "true" and an observed value, but as a disparity between measurements. The mathematics to be used is no longer the algebra of real numbers, but instead the algebra of sets. To establish a class of measurement means therefore, establishing a class of compatible intervals, which however do not possess the property of transitivity. In order to give a mathematical justification to these new concepts, there is an attempt at extending the transitive property to "sets of classes of mutually compatible values". This means that different extensive measurements can correspond to intervals of compatible and therefore equivalent measurements, so admitting the existence of measurements which are logically equal for diverse objects. This is a contradiction in reference to the classical theory of continuous measures in which we have been versed, which does not admit the conceptual possibility of identical values; the eventual equality must be attributed exclusively to numerical approximation. But this is certainly not a contradiction for statistics, which has founded its methods on equivalent classes of measurement (eterograde intervals) and homogeneous classification of different objects.

A solution from the world of statistical methodology could be given involving the intervals of uncertainty and the sets of indifference in random numbers to be treated in the language of probability. Physics has had to borrow from this new syntax to give a coherent description of many microprocesses, giving rise to statistical physics in which probability has assumed a privileged semantic role (Woolf, 1961).

8. INTENSIVE QUANTITIES

A family of atypical measures, developed covertly and unable to emerge because charged with not having a supporting theory and moreover with not having objective quantities to be translated into numbers, has managed, in our century, to impose its own results on scientific research. These measures concern quantitative evaluations of (intensive) quantities based on the relationship between physical stimulus and sensorial answers, in which the so-called personal equation unavoidably intervenes. Quantitative physiology and psychometrics have reached a formal theoretical framework sometimes able to offer hints to the physical sciences for dealing with unresolved photometric questions, and to the social sciences in order to propose new methods of analysis. In this context, a prevalent role is taken by the subjective component, conditioned by the assimilation of experience, which removes from the quantitative relationships the requisite of independence and hence of additivity, typical of instrumental observations.

In the 17th century Galilei had already animated a lively debate regarding the best criterion for evaluating an unknown quantity by anticipating the difference between instrumental errors of measurement and sensorial errors of valuation: a differentiation of which science has become aware of in dealing with the quantitative determination of external reality, where the components of error arising from the instruments of observation overlap with those arising from the observer's senses. The first results, experimentally based on an idea advanced by Fechner (1851), who proposed a logarithmic relationship between subjective valuation and intensity of stimulus, were subsequently taken up again and integrated by the first methodologists of statistics (Galton, McAlister, 1879) who was able to formulate the theoretical distribution of these specific sets of measure and their related errors. On the basis of these results two fundamental lines of research have been developed: quantitative physiology, thanks to which we are today able to measure hearing capacity or pain level, times of reaction or sensitivity to drugs, etc., and the behavioural sciences, which started at the beginning of the century with the first metric scales of intelligence and have reached the most sophisticated aptitude scales based on the analysis of factors (L.L. Thurstone, 1927) and similarity scales for the study of behaviour (C.H. Coombs, 1964) where the procedures of measurement are strictly linked to the statistical techniques of information synthesis. In all of these researches, the a priori existence of an interpretative model which is able to create a correspondence between a numerical system and a set of phenomenic relationships is essential.

If measurement has played an essential role in science, the search for an explanation of the fundamental axioms on which it is based has always attracted the interest of scientists. One of the recurrent temptations when an attribute has to be measured is to avoid the empirical and theoretical obstacles set by the search for a fundamental quantity to which to correlate the others, by substituting it with some simple physical quantity that one believes to be strongly linked with the attribute under scrutiny: the family income in place of welfare, the hours of fasting in place of hunger, the number of correct answers in place of intelligence, ... These indirect measures may be appropriate when deeper analyses are not available, and the initial variables to be interpreted are sufficiently known; however to accept them as objective definitions of nonanalysed concepts is a misplaced form of operationism.

9. PROBABILITIES

One cannot conclude this brief tour of the complex world of measurement without questioning the role to be assigned to probability. Almost always neglected by metrology, it is dealt with separately, even in the sacred books of measurement theory. However, since the origins of modern science, probability has established itself in its dual fundamental aspect of measuring an event and measuring the measurement error. As an essential instrument of astronomers and physicists of the 18th and 19th centuries in their search for the most plausible estimation of an unknown quantity, probability has found in the theory of errors a highly formalised, autonomous methodological body, a basic knowledge for all the quantitative disciplines. Equal to this has been its importance in the explanation of natural and social phenomena.

Since the natural sciences of matter and life have adopted an indeterministic explanation of phenomena, wherein constants change with the changing of the referral system and measurements become stochastic variables, the only rational answer is the statistical distribution of the probabilities of the possible states of a system (Scardovi, 1981). Well-established in this new way of dealing with science, probability as a measure of random events has become an essential component of statistical mechanics and genetics of populations. And in the behavioural sciences it has offered models of rational choice in situations of uncertainty: one thinks, to give an example, of the reduction of economic strategy to the theory of games (von Neumann e Morgenstern, 1944). This theory, founded on the principle of utility, in which probability becomes an intrinsic element of measurement, has its roots in the 19th century concept of measurement of marginal utility and in the Paretian's intuition of the ordered scale of preferences in consumer behaviour; and subsequently it is found in the formalisation of decision procedures, whose most relevant axioms deal with the maximisation of the expected utility (Savage, 1954).

From this decision-making vision of knowledge, typical of the forties, combined with the most recent informatic conquests in the elaboration of large quantities of information, so-called expert systems devised for the generation of chains of rules aimed at the solution of diagnostic problems have emerged.

Attempts to measure the reliability of a diagnosis, in the widest meaning of the term, through the recombination of various types of uncertainty internal to the mental process simulated by an expert system have led to original proposals. But the most coherent and most consolidated solution for treating uncertainty - supported by three centuries of experience - remains probability: like a length or a temperature, writes Lindley (1987), it too can be expressed in standard units.

Today probability permeates all the sectors of research which are required to have criteria of induction and decision, and has rightly entered into the most complex systems where a measurement must be assigned to each and every uncertainty. Why then is there so much reticence in considering the probability of an event as of the same epistemic nature as the estimation of force, length or weight; and why should the measurement of a probability give rise to so many more philosophical and methodological controversies than the measurement of a mass, of a sound, of a velocity or any other attribute?

Perhaps the answer is only a psychological one and must be read on a psycho-sensorial scale. The thickness of uncertainty does not possess the same solidity as our writing desk. It is better then, as suggested by Plato, to return to the ancient classics and rediscover in a philosophical speculation, new ways in the "art of measurement", free from the bonds of an irritating formalisation.

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RIASSUNTO

Il concetto di misura da Platone alla statistica moderna. Arte del vivere o principio dell'intelletto?

L'articolo si propone di dimostrare che il concetto di misura si è affermato nella sua dimensione statistica fin dalle origini del pensiero: dalle speculazioni filosofiche di Platone ai rapporti armonici di Pitagora. Per arrivare a una definizione moderna della misura occorre attendere il XVII secolo, quando Galilei ha fatto uscire la misura dalle botteghe e dai mercati per riportarla agli onori della scienza sperimentale. E' ancora con Galilei che comincia a delinearsi la nozione di errore che conoscerà il suo pieno sviluppo nei secoli successivi. Infine, nel XIX secolo, Kant distingue le grandezze estensive dalle grandezze intensive. Se queste ultime non hanno la "materialità" richiesta dalle scienze fisiche, nondimeno anch'esse possono essere misurate. Nasce la psicometria, gli orizzonti della ricerca si allargano e la ricerca scientifica ammette che tutto può essere misurato, compresa l'incertezza, il rischio, l'indeterminazione. La scienza si arricchisce quindi della probabilità come strumento potente di misura del vasto mondo dell'incerto.

SUMMARY

The concept of measure, from Plato to modern statistics. Art of living or intellectual principle?

This paper purports to show that the statistical concept of "measure" was advanced right from the origins of western thought: from the philosophical speculation of Plato, to the harmonic numerical relationships of Pythagoras. We must wait until the sixteenth century for a modern definition, when Galileo takes measurement away from the domain of

shops and merchants and brings it to the realm of experimental science. It is Galileo too, who begins to delineate the statistical theory of errors which was further developed in the following centuries. Finally, in the nineteenth century, Kant distinguishes between extensive and intensive quantities; the letter do not possess the materiality which is required by physics, but which can still be measured. And so "psycomentrics" was conceived; the horizons of research were widened, and science recognised that everything can be measured, uncertainty, risk and indeterminismess too. So, science conquers probability as a powerful means of measurement in the vast world of uncertainty.