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ADVANCES IN ESTIMATION OF SENSITIVE ISSUES ON SUCCESSIVE OCCASIONS

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1. INTRODUCTION

In surveys we often gather information related to issues that people like to hide from their fellow human beings. Finding HIV tests positive, frequent drunken driving, abortions indiscreetly induced, misconduct with the spouse, false claim for social benefits, under reporting of income tax, etc., are some of the properties that may involve unethical stigmas to many delinquents. People generally dislike their revelations.

But in practice, the collection of truthful and reliably accurate data relating to sensitive issues seems crucial and highly challenging as respondents often provide untrue responses or even refuse to respond due to social stigma and or fear.

Hence, in such circumstances the methods that protect anonymity are a solution. Two widely practised ways has been noticed that protect the anonymity of respondents. One is randomised response technique and other is scrambled response technique. Warner (1965) initiated a technique to deal with sensitive issues which is to obtain responses through a randomized response (RR) survey where every sampled unit is asked to give a response through a RR device as per instruction from the investigator. One can refer to Greenberg *et al.* (1971), Bar-lev *et al.* (2004), Diana and Perri (2011) and Arcos *et al.* (2015) etc. for a comprehensive review of such RR procedure. However, there is another approach to deal with sensitive issue called scrambled response technique introduced by Pollock and Bek (1976). Many researchers such as Eichhorn and Hayre (1983), Saha (2007) and Diana and Perri (2010), etc. considered the scrambled response models to deal with sensitive issues.

There are many sensitive issues which need to be monitored over time as they may change over time. To address the character changing over time Jessen (1942) initiated the sampling procedure. His marvelous ideas was carried forward by many others such as

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Patterson (1950), Sen (1973), Feng and Zou (1997), Singh and Priyanka (2008), Priyanka and Mittal (2014, 2015a,b) and Priyanka *et al.* (2015, 2018a) etc. However, if the variable which opt to change over time is also sensitive in nature, then their arises a need to apply randomized/scrambled response techniques on successive occasions. Arnab and Singh (2013), Yu *et al.* (2015), Naeem and Shabbir (2018), Singh *et al.* (2018) and Priyanka *et al.* (2018b) have put their efforts to deal with sensitive issues on successive occasions.

In the present work an improved class of estimators have been proposed for estimating sensitive population mean at current occasion in two occasion successive sampling using an innocuous auxiliary variable. The behaviour of proposed improved class of estimator are discussed for scrambled response models. Many existing estimators in successive sampling literature have been modified to work under the considered scrambled response model for dealing with sensitive issues. The proposed improved class of estimators have been compared with recent estimators such as modified Singh and Pal (2015, 2017). Theoretical considerations are integrated with empirical and simulation studies to ascertain the efficiency gain derived from the proposed improved class of estimators.

2. Survey strategies and analysis

2.1. *Notations and preliminaries*

A finite population U of N units has been considered for sampling over two successive occasions. The sensitive study variable be referred as x at the first occasion and y at second occasion. Whereas z is assumed to be innocuous auxiliary variable which is available at both the successive occasions. At first occasion a simple random sample without replacement of size n is drawn and at the second occasion two independent samples are drawn by considering the partial overlapping case, one is matched sample of size $m = n\lambda$ drawn as sub sample from the sample of size n from first occasion and another is unmatched simple random sample of size $u = (n - m) = n\mu$ drawn afresh at the second (current) occasion so that the sample size at both the occasion is n. On first (second) occasion the sensitive variables $x(y)$ are coded to $g(h)$ with the aid of scrambling variables W_1 and W_2. The scrambling variable are so considered that they may follow any distribution. The following notations to be considered here after.

$\bar{X}$, $\bar{Y}$, $\bar{Z}$, $\bar{G}$, $\bar{H}$, $\bar{W}_1$, $\bar{W}_2$: Population means of the variables x, y, z, g, h, W_1 and W_2, respectively.

$\bar{h}_u$, $\bar{h}_m$, $\bar{g}_m$, $\bar{h}_n$, $\bar{g}_n$: Sample mean of the variables based on sample sizes shown in suffices.

$\bar{z}_u$, $\bar{z}_m$, $\bar{z}_n$: Sample mean of the innocuous auxiliary variate based on sample sizes shown in suffice.

ρ_{yx}, ρ_{xz}, ρ_{yz}, ρ_{hg}, ρ_{hz}, ρ_{gz}: Correlation coefficient between the variables shown in suffices.

C_x, C_y, C_z: Coefficient of variation of variables shown in suffices.

S_x^2, S_y^2, S_z^2: Population mean squared error of x, y and z, respectively.

σ_x^2, σ_y^2, σ_z^2: Population variance of x, y and z, respectively.

2.2. *Scrambled response techniques on successive occasions*

Scrambling the true response increases the participation of respondents. A recent paper by Diana and Perri (2010) developed efficient scrambled response model for one time survey. In this paper we intend to modify their scrambled response model for two occasion successive sampling with the proposed improved class of estimators. The coded response at first and second occasion under modified Diana and Perri (2010) model (say M_{DP}) are given as

$$G = \phi_x(X + W_1) + (1 - \phi_x)W_2X \tag{1}$$

$$H = \phi_y(Y + W_1) + (1 - \phi_y)W_2Y, \tag{2}$$

where $0 \leq \phi_x, \phi_y \leq 1$.

For estimating sensitive population mean at current occasion, Equation (2) can be solved for $\bar{Y}$ as

$$\bar{Y} = \frac{\bar{H} - \phi_y \bar{W}_1}{\phi_y + (1 - \phi_y)\bar{W}_2}, \tag{3}$$

with

$$\rho_{hg} = \frac{\phi_x\phi_y[A_1] + [A_2]\left(\phi_x + \phi_y\right) + A_3}{\sqrt{A_4}\sqrt{A_5}}, \quad \rho_{hz} = \frac{\rho_{yz}\sigma_y\left[\phi_y(1 - \bar{W}_2) + \bar{W}_2\right]}{\sqrt{A_4}},$$

$$\rho_{gz} = \frac{\rho_{xz}\sigma_x\left[\phi_x(1 - \bar{W}_2) + \bar{W}_2\right]}{\sqrt{A_5}}, \quad C_h^2 = \frac{A_4}{\bar{H}^2} \text{ and } C_g^2 = \frac{A_5}{\bar{G}^2},$$

where

$$A_1 = \rho_{yx}\sigma_y\sigma_x + \sigma_{W_1}^2 - \rho_{yx}2\bar{W}_2\sigma_y\sigma_x + \sigma_{W_2}^2\left[\rho_{yx}\sigma_y\sigma_x + \bar{X}\bar{Y}\right] + \rho_{yx}\bar{W}_2^2\sigma_y\sigma_x,$$

$$A_2 = \bar{W}_2\rho_{yx}\sigma_y\sigma_x - \sigma_{W_2}^2\left[\rho_{yx}\sigma_y\sigma_x + \bar{X}\bar{Y}\right] - \bar{W}_2^2\rho_{yx}\sigma_y\sigma_x,$$

$$A_3 = \sigma_{W_2}^2\left[\rho_{yx}\sigma_y\sigma_x + \bar{X}\bar{Y}\right] + \bar{W}_2^2\rho_{yx}\sigma_y\sigma_x,$$

$$A_4 = (\phi_y)^2\left[\sigma_y^2 + \sigma_{W_1}^2\right] + \left(1 - \phi_y\right)^2\left[\sigma_{W_2}^2(\bar{Y}^2 + \sigma_y^2) + \sigma_y^2\bar{W}_2^2\right] + 2\phi_y(1 - \phi_y)\bar{W}_2\sigma_y^2,$$

$$A_5 = (\phi_x)^2\left[\sigma_x^2 + \sigma_{W_1}^2\right] + (1 - \phi_x)^2\left[\sigma_{W_2}^2(\bar{X}^2 + \sigma_x^2) + \sigma_x^2\bar{W}_2^2\right] + 2\phi_x(1 - \phi_x)\bar{W}_2\sigma_x^2.$$

REMARK 1. *The choice of $\phi_x(\phi_y) = 1$ in scrambled response model M_{DP}, generates modified additive scrambled response model by Pollock and Bek (1976). However, the choice of $\phi_x(\phi_y) = 0$ in M_{DP}, yields the modified multiplicative model by Pollock and Bek (1976) which is elaborated in details by Eichhorn and Hayre (1983).*

REMARK 2. *In the above model $E(W_1) = \bar{W}_1$, $E(W_2) = \bar{W}_2$, $V(W_1) = \sigma^2_{W_1}$, $V(W_2) = \sigma^2_{W_2}$.*

REMARK 3. *Suitable estimator of population mean of coded response variable $\bar{H}$ need to be investigated and replaced in Equation (3) in order to obtain appropriate estimator of sensitive population mean at current occasion under two occasion successive sampling.*

2.2.1. *Design of the proposed improved class of estimators*

To get maximum utilization of innocuous auxiliary variable, the availability of variance of auxiliary variable may be utilized to estimate the coded response variable which lead to improve the estimator for sensitive variable. In this section we intend to propose an improved general class of estimator for the population mean of coded response variable, $\bar{H}$ at current occasion as

$$T = \Omega T_u + (1-\Omega) T_m, \tag{4}$$

where

$$T_u = F_u(\bar{h}_u, a_u,\ b_u) \ \text{with}\ a_u = \frac{\bar{z}_u}{\bar{Z}},\ b_u = \frac{s^2_{zu}}{S^2_Z}$$

and

$$T_m = F_m(\bar{h}_m, t_1, t_2),$$

where $t_1 = K_1(\bar{g}_m, a_m, b_m)$ and $t_2 = K_2(\bar{g}_n, a_n, b_n)$, $a_m = \frac{\bar{z}_m}{\bar{z}_n}$, $b_m = \frac{s^2_{zm}}{s^2_{zn}}$, $a_n = \frac{\bar{z}_n}{\bar{Z}}$, $b_n = \frac{s^2_{zn}}{S^2_z}$ and $\Omega \in [0,1]$ is a scalar quantity to be chosen suitably.

Therefore, substituting the improved class of estimator T of the coded response variable $\bar{H}$ in Equation (3), the estimator of sensitive population mean at current occasion is obtained as

$$\hat{\bar{Y}} = \frac{T - \phi_y \bar{W}_1}{\phi_y + (1-\phi_y)\bar{W}_2}. \tag{5}$$

2.2.2. *Regularity conditions*

Following Srivastava (1980) and Tracy *et al.* (1996) the following regularity conditions has been assumed.

(i) The points $(\bar{h}_u, a_u, b_u)$ and $(\bar{h}_m, t_1, t_2)$ assumes the values in a closed convex subset $\mathbb{R}^3$ of three dimensional real spaces containing the point $(\bar{H}, 1, 1)$ and $(\bar{H}, \bar{G}, \bar{G})$ respectively.

(ii) The functions F_u and F_m are continuous and bounded in $\mathbb{R}^3$.

(iii) The first and second order partial derivatives of $F_u(\bar{h}_u, a_u, b_u)$ and $F_m(\bar{h}_m, t_1, t_2)$ exists and are continuous and bounded in $\mathbb{R}^3$.

(iv) t_1 and t_2 are two different classes of estimators of $\bar{G}$ through samples of sizes m and n respectively such that $K_1(\bar{G}, 1, 1) = K_2(\bar{G}, 1, 1) = \bar{G}$.

(v) $F_u(\bar{H}, 1, 1) = \bar{H}$ and $F_{1u}(Q) = \frac{\partial F_u(\cdot)}{\partial \bar{h}_u}|_Q = 1$ with $Q = (\bar{H}, 1, 1)$.

(vi) $F_m(\bar{H}, \bar{G}, \bar{G}) = \bar{H}$ and $F_{1m}(R) = \frac{\partial F_m(\cdot)}{\partial \bar{h}_m}|_R = 1$ with $R = (\bar{H}, \bar{G}, \bar{G})$.

3. FEATURES OF THE PROPOSED IMPROVED CLASS OF ESTIMATORS

3.1. *Bias and mean squared error of T*

It may be observed from Section 2 that the proposed improved class of estimator T depends on T_u and T_m which are biased for $\bar{H}$. This indicates that the combined improved class of estimators T is also biased for $\bar{H}$. Hence, following transformations has been assumed in order to derive bias and mean squared error of the estimator T:
$\bar{h}_u = \bar{H}(1+e_1)$, $\bar{h}_m = \bar{H}(1+e_2)$, $\bar{g}_n = \bar{G}(1+e_3)$, $\bar{g}_m = \bar{G}(1+e_4)$, $\bar{z}_u = \bar{Z}(1+e_5)$,
$\bar{z}_m = \bar{Z}(1+e_6)$, $\bar{z}_n = \bar{Z}(1+e_7)$, $s_{zu}^2 = S_z^2(1+e_8)$, $s_{zm}^2 = S_z^2(1+e_9)$,
$s_{zn}^2 = S_z^2(1+e_{10})$ such that, $E(e_j) = 0$, $|e_j| < 1$, where $j = 1, 2, 3, \ldots, 10$.

3.1.1. *Bias and mean squared error of T_u and T_m*

The bias and mean squared error of class of estimators T_u are derived up to first order approximations using the above transformations as:

$$T_u = F_u(\bar{h}_u, a_u, b_u).$$

Expanding $F_u(\bar{h}_u, a_u, b_u)$ about the point $Q = (\bar{H}, 1, 1)$ in a first order Taylor series, we have

$$\begin{aligned} T_u = [&F_u(Q) + (\bar{h}_u - \bar{H})F_1 + (a_u - 1)F_2 + (b_u - 1)F_3 \\ &+ \frac{1}{2}[(\bar{h}_u - \bar{H})^2 F_{11} + (a_u - 1)^2 F_{22} + (b_u - 1)^2 F_{33} + (\bar{h}_u - \bar{H})(a_u - 1)F_{12} \\ &+ (\bar{h}_u - \bar{H})(b_u - 1)F_{13} + (a_u - 1)(b_u - 1)F_{23} + \ldots]], \end{aligned} \tag{6}$$

where

$$\begin{aligned} &F_1 = \frac{\partial F_u}{\partial \bar{h}_u}|_Q = 1, \ F_2 = \frac{\partial F_u}{\partial a_u}|_Q, \ F_3 = \frac{\partial F_u}{\partial b_u}|_Q, \ F_{11} = \frac{\partial^2 F_u}{\partial \bar{h}_u^2}|_Q = 0, \\ &F_{22} = \frac{\partial^2 F_u}{\partial a_u^2}|_Q, \ F_{33} = \frac{\partial^2 F_u}{\partial b_u^2}|_Q, \ F_{12} = \frac{\partial^2 F_u}{\partial \bar{h}_u \partial a_u}|_Q, \ F_{13} = \frac{\partial^2 F_u}{\partial \bar{h}_u \partial b_u}|_Q, \\ &F_{23} = \frac{\partial^2 F_u}{\partial a_u \partial b_u}|_Q, \ Q = (\bar{H}, 1, 1). \end{aligned}$$

Now, assuming $F_2 = -F_3$ as F_2 and F_3 are based on sample size u and $(a_u - 1)$ and $(b_u - 1)$ assumes same value $\bar{H}$ at $(\bar{H}, 1, 1)$. Hence, the expression of T_u up to first order approximation becomes

$$\begin{aligned} [T_u - \bar{H}] = [&(\bar{h}_u - \bar{H}) + [(a_u - 1) - (b_u - 1)]F_2 + \frac{1}{2}[(a_u - 1)^2 F_{22} \\ &+ (b_u - 1)^2 F_{33} + (\bar{h}_u - \bar{H})(a_u - 1)F_{12} + (\bar{h}_u - \bar{H})(b_u - 1)F_{13} \\ &+ (a_u - 1)(b_u - 1)F_{23} + \ldots]]. \end{aligned} \tag{7}$$

Taking expectations on both sides in the above equation, we get bias of T_u up to first order approximation as

$$B(T_u) = \frac{1}{2u}[F_{22}C_z^2 + H_{33}(\eta_{04} - 1) + \bar{H}F_{12}\rho_{hz}C_hC_z + \bar{H}F_{13}C_h + \eta_{12} + F_{23}C_z\eta_{03}]. \tag{8}$$

Now, squaring both sides of Equation (7) and retaining terms up to first order of approximations, we have

$$\left(T_u - \bar{H}\right)^2 = [\bar{H}^2 e_1^2 + F_2^2(e_5^2 + e_8^2 - 2e_5e_8) + 2\bar{H}F_2(e_1e_5 - e_1e_8)]. \tag{9}$$

Taking expectations on both sides of Equation (9), the mean squared error of T_u is obtained as

$$M(T_u) = \frac{1}{u}[S_h^2 + F_2^2C_z^2 + F_2^2(\eta_{04} - 1) - 2F_2^2C_z\eta_{03} + 2F_2S_h\rho_{hz}C_z - 2S_hF_2\eta_{12}],$$

which is optimized for $F_2 = \left(\frac{S_h(\eta_{12}-\rho_{hz}C_z)}{C_z^2+(\eta_{04}-1)-2C_z\eta_{03}}\right) = F_2^*$ (say).

Further, substituting optimum value of F_2 in the above equation we obtain the required optimum mean squared error of T_u as

$$M(T_u)_{opt.} = \frac{1}{u}[S_h^2 + (F_2^*)^2 C_z^2 + (F_2^*)^2(\eta_{04}-1) - 2(F_2^*)^2 C_z\eta_{03} + 2F_2^* S_h\rho_{hz}C_z - 2S_hF_2^*\eta_{12}], \quad (10)$$

where $\nu_{rs} = \frac{1}{N-1}\Sigma(G_i-\bar{G})^r(Z_i-\bar{Z})^s$, $\eta_{rs} = \frac{\nu_{rs}}{\nu_{20}^{\frac{r}{2}}\nu_{02}^{\frac{s}{2}}}$, $\nu_{rs}^o = \frac{1}{N-1}\Sigma(X_i-\bar{X})^r(Z_i-\bar{Z})^s$, $\eta_{rs}^o = \frac{(\nu_{rs})^o}{(\nu_{20}^{\frac{r}{2}})^o(\nu_{02}^{\frac{s}{2}})^o}$.

Similarly, the bias and mean squared error of class of estimators T_m are derived up to first order approximations using the above transformations and are obtained as:

$$B(T_m) = \frac{1}{2}\Bigg[\frac{1}{m}[S_g^2M_{22} + S_hS_g\rho_{hg}H_2M_{12}] + \frac{1}{n}[S_g^2M_{33} + J_2^2M_{33} + S_g^2M_{23} + S_gJ_2M_{23}Q_3] + \left(\frac{1}{m}-\frac{1}{n}\right)[Q_1H_2^2M_{22} + 2H_2M_{22}S_gQ_3 + H_2M_{12}S_hQ_2 + M_{23}H_2\rho_{gz}C_zS_g]\Bigg], \quad (11)$$

where

$$M_1 = \frac{\partial F_m}{\partial \bar{h}_m}|_R = 1,\ M_2 = \frac{\partial F_m}{\partial t_1}|_R,\ M_3 = \frac{\partial F_m}{\partial t_2}|_R,\ M_{11} = \frac{\partial^2 F_m}{\partial \bar{h}_m^2}|_R = 0,$$
$$M_{22} = \frac{\partial^2 F_m}{\partial t_1^2}|_R,\ M_{33} = \frac{\partial^2 F_m}{\partial t_2^2}|_R,\ M_{12} = \frac{\partial^2 F_m}{\partial \bar{h}_m \partial t_1}|_R,\ M_{13} = \frac{\partial^2 F_m}{\partial \bar{h}_m \partial t_2}|_R,$$
$$M_{23} = \frac{\partial^2 M}{\partial t_1 \partial t_2}|_R,\ R = (\bar{H}, \bar{G}, \bar{G})$$

and

$$M(T_m)_{opt.} = \frac{1}{m}S_h^2 + \left(\frac{1}{m}-\frac{1}{n}\right)[(M_2^*)^2S_g^2 + (M_2^*)^2(H_2^*)^2Q_1 + 2S_hS_gM_2^*\rho_{hg} + 2S_hM_2^*H_2^*Q_2 - 2(M_2^*)^2S_gH_2^*\eta_{12}^o] + \frac{1}{n}\left[(M_2^*)^2(J_2^*)^2Q_1 - 2M_2^*J_2^*S_hQ_2\right], \quad (12)$$

which is optimized for $M_2 = \frac{-(\rho_{hg}Q_1+\eta_{12}^oQ_2^o)}{Q_1-(\eta_{12}^o)^2} = M_2^*$ (say), $J_2 = \frac{S_hQ_2}{M_2Q_1} = J_2^*$ (say) and $H_2 = \frac{M_2S_g\eta_{12}^o - S_hQ_2}{M_2Q_1} = H_2^*$ (say), where $Q_1 = C_z^2 + (\eta_{04}-1) - 2C_z\eta_{03}$, $Q_2 = \rho_{hz}C_z - \eta_{12}$, $Q_2^o = \rho_{hz}C_z - \eta_{12}^o$ and $Q_3 = \rho_{gz}C_z - \eta_{12}^o$.

THEOREM 4. *Bias of the improved class of estimators T to the first order of approximations are obtained as*

$$B(T) = \Omega B(T_u) + (1-\Omega)B(T_m), \tag{13}$$

where $B(T_u)$ and $B(T_m)$ are given in Equations (8) *and* (11) *respectively.*

PROOF. The bias of the improved class of estimators T is given by

$$\begin{aligned} B(T) &= E\left(T - \bar{H}\right) \\ &= E\left[\Omega(T_u - \bar{H}) + (1-\Omega)(T_m - \bar{H})\right] \\ &= \Omega B(T_u) + (1-\Omega)B(T_m) \end{aligned}$$

Substituting the values of $B(T_u)$ and $B(T_m)$ from the Equations (8) and (11) in the above equation, we have the expression for the bias of the general class of estimators T given in Equation (13). □

THEOREM 5. *Mean squared error of the improved class of estimators T to first order of approximations are obtained as*

$$M(T) = \Omega^2 M(T_u)_{opt.} + (1-\Omega)^2 M(T_m)_{opt.}, \tag{14}$$

where $M(T_u)_{opt.}$, $M(T_m)_{opt.}$ are given in Equations (10) *and* (12) *respectively.*

PROOF. The mean squared error of the improved class of estimators T is given by

$$\begin{aligned} M(T) &= E\left(T - \bar{H}\right)^2 \\ &= E\left[\Omega(T_u - \bar{H}) + (1-\Omega)(T_m - \bar{H})\right]^2 \\ &= \Omega^2 M(T_u) + (1-\Omega)^2 M(T_m) + 2\Omega(1-\Omega)cov(T_u,\ T_m) \end{aligned} \tag{15}$$

The optimum values of $M(T_u)$ and $M(T_m)$ are computed in Equations (10) and (12) respectively as the estimators T_u and T_m are based on two independent samples of sizes u and m respectively. So, $cov(T_u, T_m) = 0$. Hence, substituting the optimum values of $M(T_u), M(T_m)$ and $cov(T_u, T_m)$ in the above Equation (15) we have the expression for the mean squared error of the general class of estimators T as in Equation (14). □

3.2. *Minimum mean squared error of the proposed improved class of estimator T*

The mean squared error of improved class of estimators T in Equation (14) is a function of unknown constant Ω therefore, it is optimized with respect to Ω and subsequently the optimum value of Ω is obtained as

$$\Omega_{opt.} = \frac{M[T_m]_{opt.}}{M[T_u]_{opt.} + M[T_m]_{opt.}}. \tag{16}$$

Substituting the value of $\Omega_{opt.}$ from Equation (16) in Equation (14), we get the optimum mean squared error of the class of estimator T as

$$M[T]_{opt.} = \frac{M[T_u]_{opt.} \times M[T_m]_{opt.}}{M[T_u]_{opt.} + M[T_m]_{opt.}}. \tag{17}$$

Further, substituting the values of $M[T_u]_{opt.}$ and $M[T_m]_{opt.}$ from Equations (10) and (12) respectively in Equation (17), the simplified values of $M[T]_{opt.}$ is derived as

$$M[T]_{opt.} = \frac{L_1\mu - L_2}{(\mu)^2 K_3 - \mu L_3 - K_1}\left(\frac{S_h^2}{n}\right), \tag{18}$$

where

$$K_1 = 1 + K^2 Q_1 + 2KQ_2,$$
$$K_2 = 1 + M_2^2 + M_2^2\tilde{H}_2^2 Q_1 + 2M_2\rho_{hg} + 2M_2\tilde{H}_2 Q_2 - 2M_2^2\tilde{H}_2\eta_{12}^o,$$
$$K_3 = M_2^2\tilde{J}_2^2 Q_1 - 2M_2\tilde{J}_2 Q_2 - M_2^2 - M_2^2\tilde{H}_2^2 Q_1 - 2M_2\rho_{hg} - 2M_2\tilde{H}_2 Q_2 + 2M_2^2\tilde{H}_2\eta_{12}^o,$$
$$\tilde{J}_2 = \frac{Q_2}{M_2 Q_1}, \;\; \tilde{H}_2 = \frac{M_2\eta_{12}^o - Q_2}{M_2 Q_1},$$
$$L_1 = K_1K_3, \;\; L_2 = K_1K_2 + K_1K_3, \;\; L_3 = K_2 + K_3 - K_1, \;\; K = \frac{-Q_2}{Q_1}.$$

3.3. *Optimum replacement strategy*

The mean squared error of improved class of estimator T from Equation (18) is a function of μ which are the rotation rates or the fractions of sample to be drawn afresh at current occasion. It is also an important factor in reducing the cost of the survey, hence to estimate population mean with maximum precision and minimum cost, mean squared error of the class of estimators T derived in Equation (18) have been optimized with respect to μ. The optimum value of μ say $\hat{\mu}_f$ is given by

$$\hat{\mu}_f = \min\left\{\frac{I_2 \pm \sqrt{(I_2)^2 - I_1 I_3}}{I_1}\right\} \in [0,1], \tag{19}$$

where

$$I_1 = L_1K_3, \;\; I_2 = L_2K_3, \;\; I_3 = L_1K_1 + L_3L_2.$$

Replacing the optimum value of μ from Equation (19) in Equation (18), the minimum mean squared error of T is obtained as

$$M[T]_{\min} = \frac{L_1\hat{\mu}_f - L_2}{(\hat{\mu}_f)^2 K_3 - \hat{\mu}_f L_3 - K_1}\left(\frac{S_h^2}{n}\right). \tag{20}$$

3.4. *Mean squared error of sensitive population mean estimator*

The mean squared error of the estimator for sensitive population mean $\hat{\hat{Y}}$ is obtained as

$$M[\hat{\hat{Y}}] = \frac{M[T]_{\min}}{[\phi_y + (1-\phi_y)\bar{W}_2]^2} \tag{21}$$

4. Comparison of the estimators

To judge the performance of the proposed improved class of estimators for sensitive population mean $\hat{\hat{Y}}$, it has been compared with modified (Singh and Pal, 2015, 2017) which are described below.
(i) The recent estimator by Singh and Pal (2015) have been modified for estimating sensitive population mean at current occasion which is given as

$$\hat{\hat{Y}}_{s1} = \frac{T_{s1} - \phi_y \bar{W}_1}{\phi_y + (1-\phi_y)\bar{W}_2}, \tag{22}$$

where

$$T_{s1} = \Omega_{s1} \bar{h}_u \exp\left(\frac{\bar{Z} + C_z}{\bar{z}_u + C_z}\right) + (1-\Omega_{s1})\bar{h}_m \left(\frac{\bar{g}_m}{\bar{g}_n}\right)\left(\frac{\bar{Z} + C_z}{\bar{z}_n + C_z}\right) \text{ with } \Omega_{s1} \in (0,1).$$

(ii) The estimator by Singh and Pal (2017) have also been modified for estimating sensitive population mean at current occasion and is given as

$$\hat{\hat{Y}}_{s2} = \frac{T_{s2} - \phi_y \bar{W}_1}{\phi_y + (1-\phi_y)\bar{W}_2}, \tag{23}$$

where

$$T_{s2} = \Omega_{s2} \bar{h}_u \exp\left(\frac{\bar{Z} - \bar{z}_u}{\bar{Z} + \bar{z}_u}\right) + (1-\Omega_{s2})\bar{h}_m \exp\left(\frac{\bar{g}_n - \bar{g}_m}{\bar{g}_n + \bar{g}_m}\right) \exp\left(\frac{\bar{Z} - \bar{z}_n}{\bar{Z} + \bar{z}_n}\right),$$

with $\Omega_{s2} \in (0,1)$. Further the mean squared error of $\hat{\hat{Y}}_{s1}$ and $\hat{\hat{Y}}_{s2}$ are computed and presented in Table 1, where

$$K_{11} = 1 + \theta(\theta - 2\rho_{hz}),\ K_{12} = 2 - 2\rho_{hg},\ K_{13} = \theta(1 - 2\rho_{hz}) + (2\rho_{hg} - 1),$$
$$K_{21} = \frac{5}{4} - \rho_{hz},\ K_{22} = \frac{5}{4} - \rho_{hg},\ K_{23} = \rho_{hg} - \rho_{hz},\ L_{j1} = K_{j1}K_{j3},$$
$$L_{j2} = K_{j1}K_{j2} + K_{j1}K_{j3},\ L_{j3} = K_{j2} + K_{j3} - K_{j1},\ I_{j1} = L_{j1}K_{j3},$$
$$I_{j2} = L_{j2}K_{j3},\ I_{j3} = L_{j1}K_{j1} + L_{j3}L_{j2},\ \theta = \left(\frac{\bar{Z}}{\bar{Z} + C_z}\right);\quad j = 1 \,\&\, 2.$$

TABLE 1
Mean squared error.

Estimator	Mean squared error
$\hat{\hat{Y}}_{s1}$	$M[\hat{\hat{Y}}_{s1}] = \frac{M[T_{s1}]_{\min}}{[\phi_y+(1-\phi_y)\bar{W}_2]^2}$
	$M[T_{s1}]_{\min} = \frac{L_{11}\hat{\mu}_{s1}-L_{12}}{(\hat{\mu}_{s1})^2K_{13}-\hat{\mu}_{s1}L_{13}-K_{11}}\left(\frac{S_h^2}{n}\right)$, with $\hat{\mu}_{s1} = \frac{I_{12}\pm\sqrt{(I_{12})^2-I_{11}I_{13}}}{I_{11}}$
$\hat{\hat{Y}}_{s2}$	$M[\hat{\hat{Y}}_{s2}] = \frac{M[T_{s2}]_{\min}}{[\phi_y+(1-\phi_y)\bar{W}_2]^2}$
	$M[T_{s2}]_{\min} = \frac{L_{21}\hat{\mu}_{s2}-L_{22}}{(\hat{\mu}_{s2})^2K_{23}-\hat{\mu}_{s2}L_{23}-K_{21}}\left(\frac{S_h^2}{n}\right)$, with $\hat{\mu}_{s2} = \frac{I_{22}\pm\sqrt{(I_{22})^2-I_{21}I_{23}}}{I_{21}}$

5. Special cases

Following Remark 1, if we consider $\phi_x(\phi_y) = 1$ in mean squared errors of $\hat{\hat{Y}}$, $\hat{\hat{Y}}_{s1}$ and $\hat{\hat{Y}}_{s2}$ given in Equation (21) and Table 1 respectively, we get the mean squared errors of these estimators under modified additive model (say M_{PB1}). Similarly by substituting $\phi_x(\phi_y) = 0$ in mean squared errors of $\hat{\hat{Y}}$, $\hat{\hat{Y}}_{s1}$ and $\hat{\hat{Y}}_{s2}$ given in Equation (21) and Table 1 respectively, the mean squared errors are obtained under modified multiplicative model (say M_{PB2}) which are given in Table 2.

TABLE 2
Mean squared error of the estimators $\hat{\hat{Y}}, \hat{\hat{Y}}_{s1}$ and $\hat{\hat{Y}}_{s2}$ under M_{PB1} and M_{PB2} models.

Estimators	Mean squared error under the modified additive scrambled model (M_{PB1})
$[\hat{\hat{Y}}]_{(\phi_x=\phi_y=1)} = \hat{\hat{Y}}_1$	$M[\hat{\hat{Y}}_1] = M[T_1]_{\min}$
$[\hat{\hat{Y}}_{s1}]_{(\phi_x=\phi_y=1)} = \hat{\hat{Y}}_{s11}$	$M[\hat{\hat{Y}}_{s11}] = M[T_{s11}]_{\min}$
$[\hat{\hat{Y}}_{s2}]_{(\phi_x=\phi_y=1)} = \hat{\hat{Y}}_{s21}$	$M[\hat{\hat{Y}}_{s21}] = M[T_{s21}]_{\min}$
Estimators	**Mean squared error under the modified multiplicative scrambled model (M_{PB2})**
$[\hat{\hat{Y}}]_{(\phi_x=\phi_y=0)} = \hat{\hat{Y}}_2$	$M[\hat{\hat{Y}}_2] = \frac{M[T_2]_{\min}}{\bar{W}_2^2}$
$[\hat{\hat{Y}}_{s1}]_{(\phi_x=\phi_y=0)} = \hat{\hat{Y}}_{s12}$	$M[\hat{\hat{Y}}_{s12}] = \frac{M[T_{s12}]_{\min}}{\bar{W}_2^2}$
$[\hat{\hat{Y}}_{s2}]_{(\phi_x=\phi_y=0)} = \hat{\hat{Y}}_{s22}$	$M[\hat{\hat{Y}}_{s22}] = \frac{M[T_{s22}]_{\min}}{\bar{W}_2^2}$

6. Efficiency comparison

The percent relative efficiency of the proposed improved class of estimators, with respect to the estimators due to modified Singh and Pal (2015, 2017) under three models M_{PB1},

M_{PB2} and M_{DP} have been computed as

$$E_{11} = \frac{M[\hat{Y}_{s11}]}{M[\hat{Y}_1]} \times 100,$$

$$E_{21} = \frac{M[\hat{Y}_{s21}]}{M[\hat{Y}_1]} \times 100,$$

$$E_{12} = \frac{M[\hat{Y}_{s12}]}{M[\hat{Y}_2]} \times 100,$$

$$E_{22} = \frac{M[\hat{Y}_{s22}]}{M[\hat{Y}_2]} \times 100,$$

$$E_{13} = \frac{M[\hat{Y}_{s1}]}{M[\hat{Y}]} \times 100,$$

$$E_{23} = \frac{M[\hat{Y}_{s2}]}{M[\hat{Y}]} \times 100.$$

REMARK 6. *The two scrambling variables W_1 and W_2 used to code the true response through scrambled response models may follow any distribution. But following Pollock and Bek (1976) and Eichhorn and Hayre (1983), we consider scrambling variable W_1 to follow normal distribution with mean* 0 *and variance* 1. *However, the scrambling variable W_2 has been assumed to follow normal distribution with mean* 1 *and variance* 1.

7. NUMERICAL ILLUSTRATION

In order to validate the theoretical results, numerical illustrations has been supplemented. A sensitive population with a non-sensitive auxiliary variable have been considered from Statistical Abstracts of United States as:

X: Rate of abortions in 2004
Y: Rate of abortions in 2007
Z: Number of residents in 2004.

The artificial data for W_1 and W_2 have also been generated as per assumption in Remark 6. The optimum value of $\hat{\mu}$'s and percent relative efficiencies E_{ij}; $i, j = 1$ and 2 have been computed for the above data and are presented in Table 3.

TABLE 3
Numerical results.

i	Scrambled response model		$\hat{\mu}_{s1i}$	$\hat{\mu}_{s2i}$	$\hat{\mu}_{fi}$	E_{1i}	E_{2i}
1	M_{PB1}		0.7811	0.6677	0.7230	205.8170	147.8605
2	M_{PB2}		0.8461	0.6812	0.7934	197.0943	152.0356
3	M_{DP}	p					
		0.1	0.8428	0.6807	0.7894	197.4820	151.7487
		0.2	0.8161	0.6754	0.7620	198.7214	148.8066
		0.3	0.8030	0.6725	0.7455	199.8596	147.7941
		0.4	0.7951	0.6706	0.7354	200.8924	147.4103
		0.5	0.7898	0.6694	0.7287	201.9435	147.3774
		0.6	0.7859	0.6685	0.7239	202.9990	147.5471
		0.7	0.7832	0.6679	0.7205	203.9954	147.8217
		0.8	0.7813	0.6676	0.7182	204.8293	148.1151
		0.9	0.7802	0.6674	0.7168	205.3659	148.3393

TABLE 4
Simulation results.

j	Scrambled response model		Sets*	E_{s1j}	E_{s2j}
1	M_{PB1}		I	211.8754	145.1343
			II	207.7061	147.3438
2	M_{PB2}		I	204.3984	144.5748
			II	201.4420	149.1114
3	M_{DP}	p			
		0.1	I	204.7290	144.8387
			II	201.5453	149.5322
		0.2	I	205.3721	143.7815
			II	201.9164	147.2712
		0.3	I	206.2934	144.1131
			II	202.6177	146.7898
		0.4	I	207.1571	144.4481
			II	203.4838	146.5415
		0.5	I	208.1681	144.7478
			II	204.1736	146.8231
		0.6	I	209.1362	145.1926
			II	204.9386	147.2050
		0.7	I	209.8869	145.8146
			II	205.9717	147.4142
		0.8	I	210.4815	146.3900
			II	206.7884	147.7016
		0.9	I	210.8246	146.8191
			II	207.3104	147.9268

* Set I: $n = 20$, $u = 12$, $m = 8$ and Set II: $n = 20$, $u = 15$, $m = 5$.

8. Illustrative simulation based findings

Simulation studies has been carried out to show the applicability of the proposed improved class of estimators using Monte Carlo simulation for data mentioned in Section 8. Under the simulation study 5,000 different samples have been examined and the process have been repeated for different combination of constants termed as different sets. The simulated percent relative efficiency of proposed improved class of estimator under considered three scrambled response models M_{PB1}, M_{PB2} and M_{DP} with respect to recent estimator by Singh and Pal (2015, 2017) modified to work for sensitive issues respectively have been computed and are denoted as E_{s1j} and E_{s2j}, respectively, where $j = 1,2,3$ denote the three considered models. The simulation results are presented in Table 4.

9. A vivid illustration of the strategy including true and masked responses

It is well known that the estimators under scrambled response techniques are less efficient than the estimators obtained using direct questioning method. Hence, a comparison has been done for the scrambled response technique with respect to direct questioning method. In absence of scrambling mechanism at any occasion, the similar estimator under direct method is proposed as

$$T_D = \chi F_{uD} + (1-\chi)F_{mD}\ ; \quad \chi \in [0,1], \tag{24}$$

where

$$T_{uD} = F_{ud}(\bar{y}_u, a_u, b_u), \tag{25}$$

$$T_{mD} = F_{md}(\bar{y}_m, t_1^*, t_2^*) \text{ with } t_1^* = K_{1d}(\bar{x}_m, a_m, b_m),\ t_2^* = K_{2d}(\bar{x}_n, a_m, b_m), \tag{26}$$

where F_{ud}, F_{md}, K_{1d} and K_{2d} follow similar regularity conditions as stated in Section 2.2.2. The minimum mean squared error of T_D is obtained as

$$M[T_D]_{\min} = \frac{L_{d1}\hat{\mu}_d - L_{d2}}{(\hat{\mu}_d)^2 K_{d3} - \hat{\mu}_d L_{d3} - K_{d1}} \left(\frac{S_y^2}{n}\right), \tag{27}$$

with

$$\hat{\mu}_d = \frac{I_{d2} \pm \sqrt{(I_{d2})^2 - I_{d1} I_{d3}}}{I_{d1}}, \tag{28}$$

where

$$K_{d1} = 1 + K_d^2 Q_1 + 2K_d Q_{d2},$$
$$K_{d2} = 1 + M_{d2}^2 + M_{d2}^2 \tilde{H}_{d2}^2 Q_{d1} + 2M_{d2}\rho_{yx} + 2M_{d2}\tilde{H}_{d2} Q_{d2} - 2M_{d2}^2 \tilde{H}_{d2}\eta_{12}^o,$$
$$K_{d3} = M_{d2}^2 \tilde{J}_{d2}^2 Q_{d1} - 2M_{d2}\tilde{J}_{d2} Q_{d2} - M_{d2}^2 - M_{d2}^2 \tilde{H}_{d2}^2 Q_{d1} - 2M_{d2}\rho_{yx} - 2M_{d2}\tilde{H}_{d2} Q_{d2} + 2M_{d2}^2 \tilde{H}_{d2}\eta_{12}^o,$$
$$\tilde{J}_{d2} = \frac{Q_{d2}}{M_{d2} Q_{d1}}, \quad \tilde{H}_{d2} = \frac{M_{d2}\eta_{12}^o - Q_{d2}}{M_{d2} Q_{d1}},$$
$$L_{d1} = K_{d1}K_{d3},\ L_{d2} = K_{d1}K_{d2} + K_{d1}K_{d3},\ L_{d3} = K_{d2} + K_{d3} - K_{d1},$$
$$K_d = \frac{-Q_{d2}}{Q_{d1}},\ I_{d1} = L_{d1}K_{d3},\ I_{d2} = L_{d2}K_{d3},\ I_{d3} = L_{d1}K_{d1} + L_{d3}L_{d2}.$$

To examine the scrambling effect, the percent relative efficiencies with respect to direct method of the proposed estimators under three scrambled response models have been computed as

$$E_{D1} = \frac{M[T_D]_{\min}}{M[\hat{\hat{Y}}_1]} \times 100,\ E_{D2} = \frac{M[T_D]_{\min}}{M[\hat{\hat{Y}}_2]} \times 100,\ E_{D3} = \frac{M[T_D]_{\min}}{M[\hat{\hat{Y}}]} \times 100. \tag{29}$$

From the data mentioned in Section 7, the percent relative efficiencies have been scrutinized for different choices of $p = 0.1, 0.2, 0.3, 0.4, \ldots, 0.9$ where $p \in \{\phi_y, \phi_x\}$. The results obtained under three different models are presented in Figure 1. The optimum value of fraction of sample to be drawn afresh at current occasion for proposed general class of estimator under the three scrambled response models for varying model parameter with respect to direct questioning method has been shown in Figure 1 and Figure 2 respectively.

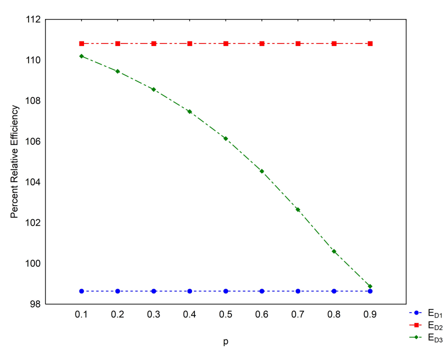

Figure 1 – Percent relative efficiencies with respect to direct method under three scrambled response models in two occasions successive sampling.

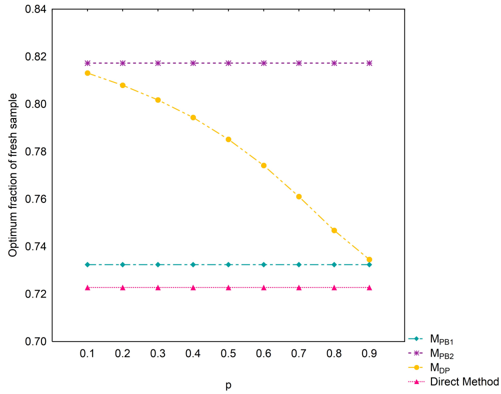

Figure 2 – Optimum value of fraction of sample drawn afresh for proposed improved class of estimator under three scrambled response models and under direct method.

10. INTERPRETATION OF RESULTS AND EPILOGUE

The following interpretations can be drawn from the empirical and simulation studies.

1. Observations from Table 3:
   (a) It is observed that the proposed improved class of estimators turns out to be more efficient than the modified estimators by Singh and Pal (2015, 2017) under all the three considered scrambled response models.
   (b) The optimum value of fraction of fresh sample to be drawn exists for all the estimators under all considered models.
   (c) The improved class of estimator $\hat{\hat{Y}}$ proves to be more efficient for higher values of $\phi_x(\phi_y)$ in comparison to lower values when compared with the estimator $\hat{\hat{Y}}_{s1}$ and for the estimator $\hat{\hat{Y}}_{s2}$, proves to be more efficient for lower values of $\phi_x(\phi_y)$ in comparison to higher values.
   (d) The model M_{PB1} yields more efficiency than M_{PB2} and M_{DP} for the estimator $\hat{\hat{Y}}_{s1}$. However, for the estimator $\hat{\hat{Y}}_{s2}$, the model M_{PB2} yields more efficiency than M_{PB1} and M_{DP}.
2. Observations from Table 4:
   (a) The proposed improved class of estimator is performing more efficiently than $\hat{\hat{Y}}_{s1}$ and $\hat{\hat{Y}}_{s2}$ respectively in terms of percent relative efficiency.
   (b) For Set-I and Set-II, $E_{s1j} \geq E_{s2j}\ \forall\ j = 1, 2, 3$.

(c) In simulation, similar behaviour of proposed improved class of estimator is observed as in empirical studies.

3. From Figure 1, it is clear that the proposed improved class of estimators under scrambled response model M_{DP} is performing better than M_{PB1} when compared with direct method for varying model parameter, the model M_{DP} efficiency decreases for higher values of model parameter which is in accordance with Diana and Perri (2010). However, more efficiency have been observed in M_{PB2} when compared with direct method.

4. Figure 2 reflects that the optimum fraction of sample to be drawn afresh for improved class of estimator under M_{DP} decreases with increasing value of model parameter.

Therefore, the proposed improved class of estimators provides several advantages with the scrambled response models considered. The numerical applications through simplistic simulations reflects how the proposed improved class of estimator is fare in practice. Hence, it may be recommended for its practical use.

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SUMMARY

Surveys related to sensitive issues are accompanied with social desirability response bias which flaw the validity of analysis. This problem became serious when sensitive issues are estimated on successive occasions. The scrambled response technique is an alternative solution as it preserve respondents anonymity. Therefore, the present article endeavours to propose an improved class of estimators for estimating sensitive population mean at current occasion using an innocuous variable in two occasion successive sampling. Detailed properties of the estimators are analysed. Optimum allocation to fresh and matched samples are obtained. Many existing estimators in successive sampling have been modified to work for sensitive population mean estimation under scrambled response technique. The proposed estimators has been compared with recent modified estimators. Theoretical considerations are integrated with empirical and simulation studies to ascertain the efficiency gain derived from the proposed improved class of estimators.

Keywords: Sensitive variable; Successive occasions; Scrambled response model; Population mean; Optimum matching fraction.