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ESTABLISHMENT OF PRELIMINARY TEST ESTIMATORS AND PRELIMINARY TEST CONFIDENCE INTERVALS FOR MEASURES OF RELIABILITY OF AN EXPONENTIATED DISTRIBUTION BASED ON TYPE-II CENSORING

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1. Introduction

Life testing experiments are usually time consuming and expensive in nature. To reduce the cost and time of experimentation, various types of censoring schemes are used in the life testing experiments. This paper dwells on Type-II censoring scheme for developing preliminary test estimators (PTEs) and preliminary test confidence intervals (PTCIs) for the parameters and measures of reliability with respect to exponentiated distributions.

The reliability function $R(t)$ is defined as the probability of failure-free operation until time t. Thus, if the random variable X denotes the lifetime of an item or a system, then $R(t)=P(X>t)$. Another measure of reliability under stress strength setup is the probability $P=P(X>Y)$, which represents the reliability of an item or a system of random strength X subject to random stress Y. For details of work existing in literature, one may refer to Bartholomew (1957, 1963), Pugh (1963), Basu (1964), Tong (1974, 1975), Johnson (1975), Kelly *et al.* (1976), Sathe and Shah (1981), Chao (1982), Awad and Gharraf (1986), Tyagi and Bhattacharya (1989), Chaturvedi and Rani (1997, 1998), Chaturvedi and Surinder (1999), Chaturvedi and Tomer (2002, 2003), Chaturvedi and Singh (2006, 2008), Chaturvedi and Pathak (2012, 2013, 2014) and Chaturvedi and Malhotra (2016, 2017, 2018).

Quite often we come across cases in which there exists some prior information in respect of parameters, which may ultimately lead to improved inferential results. It is well known that the estimators with the prior information (called the restricted estimators) perform better than the estimators with no prior information (called the unrestricted estimators).

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However, when the prior information is doubtful (or not sure), one may combine the restricted and unrestricted estimator to obtain an estimator with better performance, which leads to the PTEs. The preliminary test approach was first discussed by Bancroft (1944) and further advancements were proposed by Sen and Saleh (1978), Saleh and Kibria (1993), Kibria (2004), Kibria and Saleh (1993, 2004, 2005, 2006, 2010), Saleh (2006) and Belaghi *et al.* (2014, 2015).

A lot of research work related with reliability estimation of different distributions has taken place. For a brief review, one may refer to Ljubo (1965), Tadikamalla (1980), Mudholkar and Srivastava (1993), Mudholkar *et al.* (1995), Mudholkar and Hutson (1996), Gupta *et al.* (1998), Gupta and Kundu (1999, 2001a,b, 2002, 2003a,b), Jiang and Murthy (1999), Gupta *et al.* (2002), Xie *et al.* (2002), Raqab (2002), Nassar and Eissa (2003, 2004), Lai *et al.* (2003), Kundu *et al.* (2005), Kundu and Gupta (2005), Kundu and Raqab (2005), Pal *et al.* (2006, 2007), Abdel-Hamid and AL-Hussaini (2009), Shawky and Abu-Zinadah (2009), AL-Hussaini (2010), AL-Hussaini and Hussein (2011), Abdul-Moniem and Abdel-Hameed (2012) and Chaturvedi and Vyas (2017).

In the present paper, we have dealt with an overview of exponentiated distributions in Section 2. The relevant results on the uniformly minimum variance unbiased estimators (UMVUEs) and the maximum likelihood estimators (MLEs) of parameter σ raised to certain power p, the measures of reliability functions, namely, $R(t)$ and P under Type-II censoring, as available in literature are reproduced in Section 3 for quick reference and use by us subsequently. In Section 4, we develop the PTEs for parameter σ raised to certain power p, $R(t)$ and P respectively based on their UMVUEs and MLEs. In Section 5, we derive the PTCIs for the parameter σ, $R(t)$ and P besides obtaining the expression of coverage probability of the PTCI for the parameter 'σ'. Finally, Section 6 depicts the supporting numerical results.

2. Exponentiated distributions

Let us consider a positive random variable X, with cumulative distribution function (cdf) $F(x)$. Then, for $\sigma > 0$,

$$G(x) = [F(x)]^{\sigma} \tag{1}$$

is also a cdf. Such distributions are referred to as exponentiated distributions. Denoting by $f(x)$, the probability density function corresponding to $F(x)$, we can write

$$g(x) = \sigma[F(x)]^{\sigma-1} f(x). \tag{2}$$

Let us suppose that n items are put on life testing and failure of only first r items are observed. Let us denote the r observed failure times by $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(r)}$, $(0 < r \leq n)$ which implies that $(n-r)$ items have survived until $X_{(r)}$.

Using (2), the joint pdf of n order statistics $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ is given by

$$g^*(x_{(1)}, x_{(2)}, \ldots, x_{(n)}; \sigma) = n!\sigma^n \prod_{i=1}^{n} f(x_{(i)})[F(x_{(i)})]^{\sigma-1} \tag{3}$$

or

$$g^*(x_{(1)}, x_{(2)}, \ldots, x_{(n)}; \sigma) = n!\sigma^n \prod_{i=1}^{n} \exp\{-\sigma(-\log F(x_{(i)}))\} \left(\frac{f(x_{(i)})}{F(x_{(i)})} \right). \tag{4}$$

Considering the transformation, $y_{(i)} = -\log F(x_{(i)})$, the joint pdf of $Y_{(1)} \leq Y_{(2)} \leq \ldots \leq Y_{(n)}$, is given by

$$h^*(y_{(1)}, y_{(2)}, \ldots, y_{(n)}; \sigma) = n!\sigma^n \exp\left(-\sigma \sum_{i=1}^{n} y_{(i)} \right) \tag{5}$$

on integrating out $Y_{(r+1)} \leq Y_{(r+2)} \leq \ldots \leq Y_{(n)}$ from (5), the joint pdf of $Y_{(1)} \leq Y_{(2)} \leq \ldots \leq Y_{(r)}$ is as follows

$$h^{**}(y_{(1)}, y_{(2)}, \ldots, y_{(r)}; \sigma) = n(n-1)\ldots(n-r+1)\sigma^n \exp\left\{ -\sigma \left(\sum_{i=1}^{r} y_{(i)} + (n-r)y_{(r)} \right) \right\}.$$

Since $F(x_i)$, being cdf, follows the $U(0,1)$ distribution, $-\log F(x_{(i)})$ follows the exponential distribution with mean life $1/\sigma$.

Consider the transformation $Z_i = (n-i+1)(Y_{(i)} - Y_{(i-1)})$, $i = 1, 2, \ldots, r$

$$\Rightarrow \quad \sum_{i=1}^{r} z_i = \left(\sum_{i=1}^{r} y_{(i)} + (n-r)y_{(r)} \right) = S_r.$$

S_r, being the sum of exponential variates, follows the gamma distribution with pdf

$$t(s_r, \sigma) = \frac{\sigma^r}{\Gamma_r} s_r^{r-1} \exp(-\sigma s_r). \tag{6}$$

3. Available results on classical estimation

In the present paper, we intend to utilize appropriately certain results on UMVUE and MLE for σ^p, $R(t)$ and P, derived by Chaturvedi and Vyas (2017). These are consolidated and reproduced below for quick reference through Result 1 and Result 2.

RESULT 1. *The UMVUE and MLE of σ^p and $R(t)$ are as follows:*

(i) *For $(p \neq 0)$, the UMVUE of σ^p, i.e.*

$$\widehat{\sigma}_U^p = \frac{\Gamma_r}{\Gamma_{(r-p)}} S_r^{-p}, \quad (p < r).$$

(ii) *For* $(p \neq 0)$, *the MLE of* σ^p, *i.e.*

$$\widehat{\sigma}^p_{ML} = \left(\frac{r}{S_r}\right)^p .$$

(iii) *The UMVUE of* $R(t)$, *i.e.*

$$\widehat{R}_U(t) = 1 - \left[1 + \frac{\log F(t)}{S_r}\right]^{r-1}; \quad -\log F(t) < S_r .$$

(iv) *The MLE of* $R(t)$, *i.e.*

$$\widehat{R}_{ML}(t) = 1 - (F(t))^{\frac{r}{S_r}} .$$

Suppose X and Y are two independent random variables following the classes of distribution $g_1(x;\sigma_1)$ and $g_2(y;\sigma_2)$, respectively, where

$$g_1(x;\sigma_1) = \sigma_1 F_1(x)^{\sigma_1 - 1} f_1(x),$$
$$g_2(y;\sigma_2) = \sigma_2 F_2(y)^{\sigma_2 - 1} f_2(y).$$

Let us suppose that n and m items are put on test corresponding to X and Y, respectively. Further, the failure times of r and l units are observed from X and Y, respectively.

As done earlier, $S_r = \sum_{j=1}^{r} Y_{1(j)} + (n-r) Y_{1(r)}$ and $T_l = \sum_{j=1}^{l} Y_{2(j)} + (m-l) Y_{2(l)}$.

RESULT 2. *The UMVUE and MLE of P are respectively as follows:*

(i) *The UMVUE, i.e.*

$$\widehat{P}_U = \begin{cases} (l-1)\displaystyle\int_0^{\frac{S_r}{T_l}} (1-z)^{l-2}\left[1 - \left\{1 - \frac{T_l}{S_r} z\right\}^{r-1}\right], & S_r < T_l , \\ (l-1)\displaystyle\int_0^{1} (1-z)^{l-2}\left[1 - \left\{1 - \frac{T_l}{S_r} z\right\}^{r-1}\right], & S_r \geqslant T_l . \end{cases}$$

(ii) *The MLE, i.e.*

$$\widehat{P}_{ML} = \frac{r T_l}{r T_l + l S_r} .$$

4. Proposed preliminary test estimators

Let us suppose that the prior information of the parameter can be expressed in the form of following hypothesis

$$H_0 : \sigma = \sigma_0 \text{ against } H_1 : \sigma \neq \sigma_0.$$

From (6), we know that

$$2\sigma S_r \sim \chi^2_{2r}. \tag{7}$$

Therefore, the critical region is given by

$$(0 < S_r < k_0) \cup (k_1 < S_r < \infty),$$

where $k_0 = \frac{1}{2\sigma_0}\chi^2_{2r}\left(\frac{\alpha}{2}\right)$, $k_1 = \frac{1}{2\sigma_0}\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)$ and α is the level of significance. Let us suppose

$$\chi^2_{2r}\left(\frac{\alpha}{2}\right) = C_2 \text{ and } \chi^2_{2r}\left(1-\frac{\alpha}{2}\right) = C_1 \tag{8}$$

and $I(A)$ be the indicator function of the following set

$$A = \{\chi^2_{2r}; C_2 \leq \chi^2_{2r} \leq C_1\}.$$

The PTEs of σ^p based on UMVUE and MLE are then given respectively by

$$\widehat{\sigma}^p_{PT\_U} = \widehat{\sigma}^p_U - (\widehat{\sigma}^p_U - \sigma^p_0)I(A) \tag{9}$$

and

$$\widehat{\sigma}^p_{PT\_ML} = \widehat{\sigma}^p_{ML} - (\widehat{\sigma}^p_{ML} - \sigma^p_0)I(A), \tag{10}$$

where $\widehat{\sigma}^p_U$ and $\widehat{\sigma}^p_{ML}$ are as defined in Result 1(i) and Result 1(ii), respectively.

Next, we find the PTEs of $R(t)$ and P based on UMVUEs and MLEs. Using Result 1(iii) and Result 1(iv) on the UMVUE and MLE of $R(t)$, the PTEs of $R(t)$ based on UMVUE and MLE are given respectively by

$$\widehat{R}_{PT\_U}(t) = \widehat{R}_U(t) - (\widehat{R}_U(t) - R_0(t))I(A), \tag{11}$$

where $R_0(t) = 1 - F(t)^{\sigma_0}$, under H_0 and

$$\widehat{R}_{PT\_ML}(t) = \widehat{R}_{ML}(t) - (\widehat{R}_{ML}(t) - R_0(t))I(A). \tag{12}$$

Let us now derive the PTEs of P based on UMVUE and MLE. We know that

$$P = \frac{\sigma_1}{\sigma_1 + \sigma_2}.$$

Suppose, we want to test

$$H_0 : P = P_0 \text{ against } H_1 : P \neq P_0$$
$$P = P_0 \text{ gives } \sigma_1 = k\sigma_2 \text{ where } k = \frac{P_0}{1-P_0}.$$

Therefore, H_0 is equivalent to

$$H_0 : \sigma_1 = k\sigma_2 \text{ against } H_1 : \sigma_1 \neq k\sigma_2.$$

We know that

$$2\sigma_1 S_r \sim \chi^2_{2r} \text{ and } 2\sigma_2 T_l \sim \chi^2_{2l}.$$

Therefore,

$$\frac{\sigma_1 S_r l}{\sigma_2 T_l r} \sim F_{2r,2l}. \tag{13}$$

The critical region for testing $H_0 : P = P_0$ is given by

$$\left(\frac{S_r}{T_l} < k_2\right) \cup \left(k_3 < \frac{S_r}{T_l}\right),$$

where

$$k_2 = \frac{r}{kl} F_{2r,2l}\left(\frac{\alpha}{2}\right) \text{ and } k_3 = \frac{r}{kl} F_{2r,2l}\left(1 - \frac{\alpha}{2}\right). \tag{14}$$

Let $I(B)$ be indicator function of the set

$$B = \{F_{2r,2l}; C_4 \leq F_{2r,2l} \leq C_3\},$$

where $C_3 = F_{2r,2l}\left(1 - \frac{\alpha}{2}\right)$; $C_4 = F_{2r,2l}\left(\frac{\alpha}{2}\right)$.

As seen earlier in Result 2(i) and Result 2(ii), the UMVUE and MLE of P when X, Y belong to same family of distribution are respectively given by

$$\widehat{P}_U = \begin{cases} (l-1)\displaystyle\int_0^{\frac{S_r}{T_l}} (1-z)^{l-2}\left[1 - \left\{1 - \frac{T_l}{S_r} z\right\}^{r-1}\right], & S_r < T_l, \\ (l-1)\displaystyle\int_0^{1} (1-z)^{l-2}\left[1 - \left\{1 - \frac{T_l}{S_r} z\right\}^{r-1}\right], & S_r \geqslant T_l \end{cases}$$

and

$$\widehat{P}_{ML} = \frac{rT_l}{rT_l + lS_r}.$$

The PTEs of P based on UMVUE and MLE are then obtained respectively by

$$\widehat{P}_{PT\_U} = \widehat{P}_U - (\widehat{P}_U - P_0)I(B) \tag{15}$$

and

$$\widehat{P}_{PT\_ML} = \widehat{P}_{ML} - (\widehat{P}_{ML} - P_0)I(B), \tag{16}$$

where P_0 is the assumed value of P under H_0.

5. Preliminary test confidence intervals

In this section, we derive the PTCIs for σ, $R(t)$ and P based on their UMVUEs and MLEs.

From (7), we know that, $2\sigma S_r \sim \chi^2_{2r}$

$$\Rightarrow \quad P\left\{ \frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2S_r} < \sigma < \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2S_r} \right\} = 1-\alpha, \tag{17}$$

where α is the significance level.

From Result 1(i) and Result 1(ii), we know that $\widehat{\sigma}_U = \frac{r-1}{S_r}$ and $\widehat{\sigma}_{ML} = \frac{r}{S_r}$, therefore, $100(1-\alpha)\%$ equal tail confidence interval (CI) for σ based on UMVUE and MLE may be written as

$$I_{ET\_\sigma\_U} = \left[\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2(r-1)} \widehat{\sigma}_U, \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2(r-1)} \widehat{\sigma}_U \right] \tag{18}$$

and

$$I_{ET\_\sigma\_ML} = \left[\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2r} \widehat{\sigma}_{ML}, \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2r} \widehat{\sigma}_{ML} \right]. \tag{19}$$

The proposed PTCI of σ based on UMVUE and MLE are as follows:

$$I_{PT\_\sigma\_U} = \left[\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2(r-1)} \widehat{\sigma}_{PT\_U}, \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2(r-1)} \widehat{\sigma}_{PT\_U} \right] \tag{20}$$

$$I_{PT\_\sigma\_ML} = \left[\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2r} \widehat{\sigma}_{PT\_ML}, \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2r} \widehat{\sigma}_{PT\_ML} \right], \tag{21}$$

where $\widehat{\sigma}_{PT\_U}$ and $\widehat{\sigma}_{PT\_ML}$ are as defined in (9) and (10), respectively.

Next we derive the PTCI for $R(t)$. Using (1), we know that

$$R(t) = 1 - \exp\{\sigma \log F(t)\}. \tag{22}$$

Using (17) and (22), we can write

$$P\left\{1-\exp\left(\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2S_r}\log F(t)\right)<R(t)<1-\exp\left(\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2S_r}\log F(t)\right)\right\}=1-\alpha.$$

Therefore, $100(1-\alpha)\%$ equal tail CI for $R(t)$ may be written as

$$I_{ET\_R}=\left[1-\exp\left(\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2S_r}\log F(t)\right),1-\exp\left(\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2S_r}\log F(t)\right)\right]. \tag{23}$$

From Result 1(iii), we can write

$$\frac{\log F(t)}{S_r}=(1-\widehat{R}_U(t))^{\frac{1}{r-1}}-1. \tag{24}$$

Using (24) in (23), $100(1-\alpha)\%$ equal tail CI for $R(t)$ based on UMVUE may be written as

$$I_{ET\_R\_U}=\left[1-\exp\left\{\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2}((1-\widehat{R}_U(t))^{\frac{1}{r-1}}-1)\right\},\right.$$
$$\left.1-\exp\left\{\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2}((1-\widehat{R}_U(t))^{\frac{1}{r-1}}-1)\right\}\right]$$

From Result 1(iv), we can write

$$\log F(t)=\frac{S_r}{r}\log(1-\widehat{R}_{ML}(t)). \tag{25}$$

Using (25) in (23), $100(1-\alpha)\%$ equal tail CI for $R(t)$ based on MLE may be written as

$$I_{ET\_R\_ML}=\left[1-\exp\left\{\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2r}\log(1-\widehat{R}_{ML}(t))\right\},\right.$$
$$\left.1-\exp\left\{\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2r}\log(1-\widehat{R}_{ML}(t))\right\}\right].$$

The proposed PTCIs of $R(t)$ based on its UMVUE and MLE are as follows:

$$I_{PT\_R\_U} = \left[1-\exp\left\{\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2}(1-\widehat{R}_{PT\_U}(t))^{\frac{1}{r-1}}-1\right\},\right.$$
$$\left.1-\exp\left\{\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2}((1-\widehat{R}_{PT\_U}(t))^{\frac{1}{r-1}}-1)\right\}\right]$$
$$I_{PT\_R\_ML} = \left[1-\exp\left\{\frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2r}\log(1-\widehat{R}_{PT\_ML}(t))\right\},\right.$$
$$\left.1-\exp\left\{\frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2r}\log(1-\widehat{R}_{PT\_ML}(t))\right\}\right].$$

Now, we derive the CI for P. We know that $P = \dfrac{\sigma_1}{\sigma_1+\sigma_2} = \dfrac{1}{1+\frac{\sigma_2}{\sigma_1}}$ and from (13), $\dfrac{\sigma_2 T_l r}{\sigma_1 S_r l} \sim F_{2l,2r}$ using the same approach as above, $100(1-\alpha)\%$ equal tail CI for P may be written as

$$I_{ET\_P} = \left[\frac{1}{1+\frac{lS_r}{rT_l}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \frac{1}{1+\frac{lS_r}{rT_l}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right]. \tag{26}$$

From Result 2(i), we can write

$$\frac{S_r}{T_l} = \begin{cases} \dfrac{\widehat{P}_U}{d1-d2+d3-d4}, & S_r < T_l, \\ \dfrac{\widehat{P}_U}{1-d5}, & T_l \leqslant S_r, \end{cases} \tag{27}$$

where

$$d1 = \frac{T_l}{S_r},$$
$$d2 = \sum_{i=0}^{l-1}(-1)^i \frac{(l-1)!}{i!(l-i-1)!}\left(\frac{S_r}{T_l}\right)^{i-1},$$
$$d3 = \sum_{i=0}^{l-2}(-1)^i \frac{(l-1)!}{(i+1)!(l-i-2)!}\left(\frac{S_r}{T_l}\right)^{i},$$
$$d4 = \sum_{i=0}^{l-2}(-1)^i \frac{(l-1)!(r-1)!}{(i+r)!(l-i-2)!}\left(\frac{S_r}{T_l}\right)^{i},$$
$$d5 = \sum_{i=0}^{r-1}(-1)^i \frac{(l-1)!(r-1)!}{(r-1-i)!(l+i-1)!}\left(\frac{T_l}{S_r}\right)^{i+1}.$$

Using (27) in (26), we can write $100(1-\alpha)\%$ equal tail CI for P based on UMVUE as

$$I_{ET\_P\_U} = \begin{cases} \left[\dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_U}{(d1-d2+d3-d4)}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_U}{(d1-d2+d3-d4)}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right], & S_r < T_l \\ \left[\dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_U}{(1-d5)}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_U}{(1-d5)}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right], & S_r \geqslant T_l \end{cases}$$

From Result 2(ii), we can write

$$\frac{lS_r}{rT_l} = \frac{1-\widehat{P}_{ML}}{\widehat{P}_{ML}}. \tag{28}$$

Using (28) in (26), we can write $100(1-\alpha)\%$ equal tail CI for P based on MLE as

$$I_{ET\_P\_ML} = \left[\frac{1}{1+\left\{\frac{1-\widehat{P}_{ML}}{\widehat{P}_{ML}}\right\}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \frac{1}{1+\left\{\frac{1-\widehat{P}_{ML}}{\widehat{P}_{ML}}F_{2l,2r}\left(\frac{\alpha}{2}\right)\right\}}\right].$$

The proposed PTCIs of P based on its UMVUE and MLE are then as follows:

$$I_{PT\_P\_U} = \begin{cases} \left[\dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_{PT\_U}}{(d1-d2+d3-d4)}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_{PT\_U}}{(d1-d2+d3-d4)}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right], & S_r < T_l \\ \left[\dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_{PT\_U}}{(1-d5)}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \dfrac{1}{1+\frac{l}{r}\frac{\widehat{P}_{PT\_U}}{(1-d5)}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right], & S_r \geqslant T_l \end{cases}$$

$$I_{PT\_P\_ML} = \left[\frac{1}{1+\left\{\frac{1-\widehat{P}_{PT\_ML}}{\widehat{P}_{PT\_ML}}\right\}F_{2l,2r}\left(1-\frac{\alpha}{2}\right)}, \frac{1}{1+\left\{\frac{1-\widehat{P}_{PT\_ML}}{\widehat{P}_{PT\_ML}}\right\}F_{2l,2r}\left(\frac{\alpha}{2}\right)}\right]$$

Now we obtain the coverage probability of PTCI of σ based on its UMVUE

We know that $T = 2\sigma S_r \sim \chi^2_{2r}$

$$
\begin{aligned}
P(\sigma \in I_{PT\_\sigma\_U}) = & P\left\{\sigma \in (a_1\sigma_0, a_2\sigma_0), \chi^2_{2r}\left(\frac{\alpha}{2}\right) < 2\sigma_0 S_r < \chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right\} \\
& + P\left\{\sigma \in (a_1\widehat{\sigma}_U, a_2\widehat{\sigma}_U), 2\sigma_0 S_r < \chi^2_{2r}\left(\frac{\alpha}{2}\right)\right\} \\
& + P\left\{\sigma \in (a_1\widehat{\sigma}_U, a_2\widehat{\sigma}_U), 2\sigma_0 S_r > \chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right\},
\end{aligned}
$$

where $a_1 = \frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{2(r-1)}$ and $a_2 = \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{2(r-1)}$.
Denoting $\delta = \frac{\sigma}{\sigma_0}$, we get

$$
\begin{aligned}
P(\sigma \in I_{PT\_\sigma\_U}) = & P\left\{a_1 < \delta < a_2; \delta\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \delta\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right\} \\
& + P\left\{\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \chi^2_{2r}\left(1-\frac{\alpha}{2}\right); T < \delta\chi^2_{2r}\left(\frac{\alpha}{2}\right)\right\} \\
& + P\left\{\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \chi^2_{2r}\left(1-\frac{\alpha}{2}\right); T > \delta\chi^2_{2m}\left(1-\frac{\alpha}{2}\right)\right\}
\end{aligned}
$$

or

$$
\begin{aligned}
P(\sigma \in I_{PT\_\sigma\_U}) = & P\left\{\delta\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \delta\chi^2_{2r}\left(1-\frac{\alpha}{2}\right) I_{a_1 a_2}(\delta)\right. \\
& + P\left\{\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \min\left(\chi^2_{2r}\left(1-\frac{\alpha}{2}\right), \delta\chi^2_{2r}\left(\frac{\alpha}{2}\right)\right)\right\} \\
& + P\left\{\max\left(\chi^2_{2r}\left(\frac{\alpha}{2}\right), \delta\chi^2_{2r}\left(1-\frac{\alpha}{2}\right) < T < \chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right)\right\}.
\end{aligned}
$$

Let us denote $P\left\{\delta\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \delta\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right\} I_{a_1 a_2}(\delta)$ by A. Considering all possible cases of δ, we may write

$$
P(\sigma \in I_{PT\_\sigma\_U}) = \begin{cases}
A + 1 - \alpha, & 0 < \delta \le \frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)} \\
A + 1 - \alpha, & \delta > \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{\chi^2_{2r}\left(\frac{\alpha}{2}\right)} \\
A + P\left\{\chi^2_{2r}\left(\frac{\alpha}{2}\right) < T < \delta\chi^2_{2r}\left(\frac{\alpha}{2}\right)\right\}, & 1 < \delta \le \frac{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)}{\chi^2_{2r}\left(\frac{\alpha}{2}\right)} \\
A + P\left\{\delta\chi^2_{2r}\left(1-\frac{\alpha}{2}\right) < T < \chi^2_{2r}\left(1-\frac{\alpha}{2}\right)\right\}, & \frac{\chi^2_{2r}\left(\frac{\alpha}{2}\right)}{\chi^2_{2r}\left(1-\frac{\alpha}{2}\right)} < \delta \le 1
\end{cases}
$$

Similarly, coverage probability for other PTCI may be obtained.

6. Numerical findings

By using the method of inverse cumulative density, we have generated 100 observations by making use of the transformation $y_i = -\left(\frac{1}{\sigma}\right)\log G(x_i, \sigma)$ with $\sigma = 1.5$. It may be noted here that $G(x)$, being the cdf, follows the uniform distribution. Further suppose that X_i's, the failure times of experimental units, follow the Weibull distribution with shape parameter $\gamma = 1.25$ and scale parameter $\lambda = 10$.

Estimates of σ

For each combination of r and p, the UMVUEs and MLEs of σ are obtained. PTEs of σ based on UMVUEs and MLEs are further obtained. We replicated this process 1500 times in order to obtain 1500 estimates. Thereafter mean value of estimates and corresponding mean square error (MSE) are obtained using these 1500 estimates. The results are shown in Table 1. From Table 1 it may be observed that PTEs are more close to the actual value of parameter and their MSE is less than the MSE of traditional UMVUEs and MLEs, thereby establishing superiority of PTEs. Also, it is observed that as r increases, PTEs tend to get closer to the true value.

Estimates of $R(t)$

For each combination of r and t, the UMVUEs, MLEs and PTEs of $R(t)$ based on UMVUE, MLE are obtained using the similar approach as mentioned above. The MSE for each estimator is further obtained. The results may be seen in Table 2 and it may be observed that PTEs are more close to the actual value of $R(t)$ and their MSE is less than the MSE of traditional UMVUEs and MLEs of $R(t)$.

Estimates of P

For each combination of (r, l) and p, the UMVUEs, MLEs and PTEs of P based on UMVUE, MLE are obtained besides the MSE for each estimator using the similar approach as mentioned above. The results may be seen in Table 3. Similar observations as above may be drawn from Table 3 as well. Therefore, we may conclude that the PTEs perform better than the classical estimators as the MSE of the PTEs is observed to be less than the MSE of the classical estimators under the simulated data set.

Finally, the coverage probability (CP) of PTCI of σ is plotted against δ, which may be seen in Figure 1. It may be seen that for fixed values of n, r and $\alpha = 0.15$, the CP, as a function of δ, decreases monotonically. It then increases and crosses the line $1-\alpha$, and deceases again. It increases and decrease again until reaching the minimum value. Finally, it increases and tends to line $1-\alpha$ when δ becomes large. Further, it is observed that for small values of r, the domination interval is wider than for large values of r. Therefore, we may conclude that for some δ in specific interval, the coverage probability of PTCI of σ is more than that of equal tail confidence interval.

TABLE 1

Estimates and corresponding mean square error of the parameter σ. The values are truncated to 4 decimal points, therefore few values in the column of mean square error being very small are mentioned as 0.0000.

r	p	Estimates				Mean square error			
		$\widehat{\sigma}_U$	$\widehat{\sigma}_{PT\_U}$	$\widehat{\sigma}_{ML}$	$\widehat{\sigma}_{PT\_ML}$	$MSE(\widehat{\sigma}_U)$	$MSE(\widehat{\sigma}_{PT\_U})$	$MSE(\widehat{\sigma}_{ML})$	$MSE(\widehat{\sigma}_{PT\_ML})$
	2	0.5052	1.4549	0.5340	1.4562	0.0696	0.0020	0.0778	0.0019
55	4	0.2414	1.4361	0.2911	1.4384	0.0140	0.0041	0.0203	0.0038
	5	0.1670	1.4427	0.2214	1.4448	0.0096	0.0033	0.0169	0.0030
	2	0.6380	1.4641	0.6712	1.4654	0.0107	0.0013	0.0118	0.0011
60	4	0.3926	1.4433	0.4658	1.4471	0.0582	0.0032	0.0819	0.0028
	5	0.3118	1.4524	0.4036	1.4558	0.0196	0.0023	0.0328	0.0020
	2	1.1625	1.4862	1.2105	1.4883	0.0770	0.0002	0.0835	0.0001
75	4	1.3126	1.4873	1.5039	1.4973	0.0530	0.0002	0.0696	7.5217
	5	1.4376	1.4947	1.7648	1.5124	0.3609	0.0000	0.5439	0.0002

TABLE 2

Estimates and corresponding mean square error of $R(t)$. The values are truncated to 4 decimal points, therefore few values in the column of mean square error being very small are mentioned as 0.0000.

r	t	$R(t)$	Estimates				Mean Square Error			
			$\widehat{R}_U(t)$	$\widehat{R}_{PT\_U}(t)$	$\widehat{R}_{ML}(t)$	$\widehat{R}_{PT\_ML}(t)$	$MSE(\widehat{R}_U(t))$	$MSE(\widehat{R}_{PT\_U}(t))$	$MSE(\widehat{R}_{ML}(t))$	$MSE(\widehat{R}_{PT\_ML}(t))$
	1.5	0.9092	0.8210	0.9044	0.8218	0.9045	0.0008	0.0000	0.0007	0.0000
55	2	0.8729	0.7722	0.8686	0.7737	0.8687	0.0009	0.0000	0.0009	0.0000
	3	0.7987	0.6814	0.7924	0.6842	0.7925	0.0069	0.0000	0.0067	0.0000
	1.5	0.9092	0.8555	0.9068	0.8557	0.9068	0.0003	0.0000	0.0003	0.0000
60	2	0.8729	0.8108	0.8694	0.8116	0.8694	0.0013	0.0000	0.0013	0.0000
	3	0.7987	0.7256	0.7954	0.7277	0.7955	0.0001	0.0000	0.0001	0.0000
	1.5	0.9092	0.9259	0.9102	0.9251	0.9101	0.0001	0.0000	0.0001	0.0000
75	2	0.8729	0.8933	0.8740	0.8929	0.8740	0.0000	0.0000	0.0000	0.0000
	3	0.7987	0.8239	0.8005	0.8244	0.8005	0.0033	0.0000	0.0032	0.0000

TABLE 3

Estimates and corresponding mean square error of P. The values are truncated to 4 decimal points, therefore few values in the column of mean square error being very small are mentioned as 0.0000.

(r,l)	p	P	Estimates				Mean Square Error			
			$\widehat{P}_U$	$\widehat{P}_{PT\_U}$	$\widehat{P}_{ML}$	$\widehat{P}_{PT\_ML}$	$MSE(\widehat{P}_U)$	$MSE(\widehat{P}_{PT\_U})$	$MSE(\widehat{P}_{ML})$	$MSE(\widehat{P}_{PT\_ML})$
	2	0.5	0.4234	0.4963	0.4246	0.4964	0.0000	0.0000	0.0000	0.0000
(55,75)	4	0.5	0.4225	0.4952	0.4237	0.4953	0.0013	0.0000	0.0013	0.0000
	5	0.5	0.4225	0.4958	0.4237	0.4959	0.0000	0.0000	0.0000	0.0000
	2	0.5	0.5210	0.5011	0.5209	0.5010	0.0004	0.0000	0.0004	0.0000
60,60)	4	0.5	0.5226	0.5010	0.5224	0.5010	0.0027	0.0000	0.0026	0.0000
	5	0.5	0.5227	0.5013	0.5225	0.5012	0.0008	0.0000	0.0008	0.0000
	2	0.5	0.6037	0.5047	0.6023	0.5046	0.0007	0.0000	0.0007	0.0000
(75,55)	4	0.5	0.6025	0.5049	0.6011	0.5049	0.0003	0.0000	0.0003	0.0000
	5	0.5	0.6036	0.5044	0.6022	0.5044	0.0003	0.0000	0.0003	0.0000

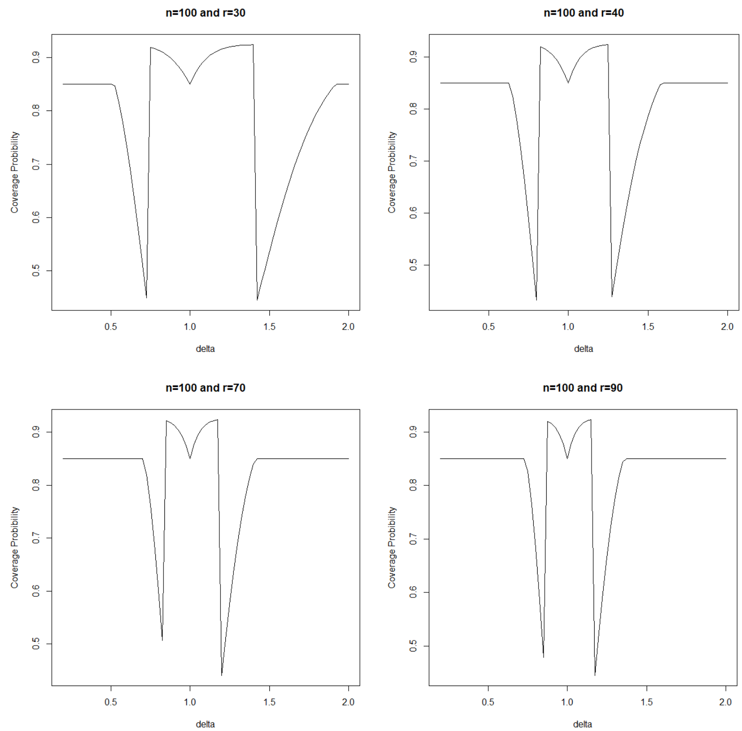

Figure 1 – Coverage probability of PTCI of σ plotted as a function of delta.

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SUMMARY

The present paper has developed the preliminary test estimators (PTEs) of the model parameter raised to certain power, σ^p, and the two measures of reliability, namely, the reliability function, $R(t)$ and the reliability of an item or a system, P of an exponentiated distribution, under Type-II censoring, based on their uniformly minimum variance unbiased estimators (UMVUEs) and maximum likelihood estimators (MLEs). The preliminary test confidence intervals (PTCIs) are also developed for σ, $R(t)$ and P based on their UMVUEs and MLEs. Further, the paper has derived expression for coverage probability of the PTCI of the model parameter, σ. Merits of the proposed PTEs are also established through analysis of simulated numerical data.

Keywords: Exponentiated distributions; Preliminary test estimator; Type-II censoring; Uniformly minimum variance unbiased estimator; Maximum likelihood estimator.