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BAYESIAN ESTIMATION OF LORENZ CURVE, GINI-INDEX AND VARIANCE OF LOGARITHMS IN A PARETO DISTRIBUTION

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1. INTRODUCTION

The use of the Pareto distribution as a model for various socio-economic phenomena dates back to the late nineteenth century when Pareto observed that the number of persons whose income exceed x can be approximated as $cx^{-a}, a > 0$. The distribution has played a very important role in the investigation of city population, occurrence of natural resources, insurance risk and business failures. Arnold and Press (1983) gave an extensive historical survey of its use in the context of income distribution. The Lorenz curve and Gini-index play a central role in the analysis of income distribution and the evaluation of welfare judgements. Also they have been extensively used in the study of inequality of distributions. The variance of logarithms is a widely used measure of dispersion, owing in part to its natural link with wage determination models and its special relationship with the lognormal distribution.

Given a distribution function $F(x)$ with finite mean μ, the Lorenz curve $L(p)$ (Gastwirth, 1972) is defined as

$$L(p) = \frac{1}{\mu}\int_0^p F^{-1}(x)dx, \; 0 < p < 1 \tag{1}$$

where

$$F^{-1}(x) = Sup\{y : F(y) \leq x\}.$$

If the distribution which is studied is the income of a certain population then $L(p)$ denotes the fraction of the total income received by the $100p\%$ of the population which has the lowest income. The Gini-index is twice the area between the Lorenz curve and the elegatarian line. The Gini-index G is defined as

$$G = 1 - 2\int_0^1 L(p)dp. \tag{2}$$

Also, the variance of logarithm (Foster and Ok, 1999) is defined as

$$V = Var(\log X) . \tag{3}$$

For the applications of Lorenz curve and Gini-index we refer to Chandra and Singpurwalla (1981), Moothathu (1991) and Bhattacharjee (1993). Most of the methods concerning estimators of the Lorenz curve and Gini-index are centered on classical procedures. For example, Kakwani and Podder (1973, 1976) proposed a prominent method for the estimation of Lorenz curve and inequality measures with grouped data. Further, Chotikapanich and Griffiths (2002) estimated the Lorenz curve using the Dirichlet distribution for the error terms. Moothathu (1990) discussed the problem of estimation of Lorenz curve and Gin-index for the Pareto distribution in the classical framework. In the present work, we estimate the Lorenz curve and Gini-index for the Pareto distribution, in the Bayesian framework with a conjugate prior. Also, we obtain the Bayesian and classical estimators of variance of logarithms for the Pareto distribution.

The present article is organized as follows. In section 2, we consider the Bayesian estimation of Lorenz curve and Gini-index in the two situations namely when θ is known and when θ is unknown. In section 3, the Bayes and the maximum likelihood estimators of variance of logarithm have been obtained, in the two situations mentioned above. The performance of the estimators are compared on the basis of bias and mean square errors, in the last section.

2. ESTIMATION OF LORENZ CURVE AND GINI-INDEX

Let $x_1, x_2, ..., x_n$ be a random sample from the Pareto distribution with density function

$$f(x, a, \theta) = a\theta^a x^{-(a+1)}, \quad x \geq \theta > 0, \quad a > 1. \tag{4}$$

For the model (4), the Lorenz curve and Gini-index simplifies to

$$L(p) = 1 - (1-p)^{1-a^{-1}}, \; 0 < p < 1 \tag{5}$$

and

$$G = (2a-1)^{-1} \tag{6}$$

2.1 *Estimation with known* θ

The likelihood function in this set-up can be easily written as

$$l(\underline{x} \mid a, \theta) = C_1 a^n \exp(-at) \tag{7}$$

where

$$t = \sum_{i=1}^{n} \log\left(\frac{x_i}{\theta}\right).$$

The inferential procedures for Pareto model in the Bayesian framework has been studied and discussed by Malik (1970), Zellner (1971), Arnold and Press (1983) and Jeevanand and Nair (1992). The likelihood function provides a conjugate prior namely

$$g(a) = C_2 a^{r-1} \exp(-\tau a), \quad r, \tau, a > 0. \tag{8}$$

The symbol C with various suffixes stands for the normalizing constants. Combining (7) and (8), the posterior density turns out to be

$$f(a \mid \underline{x}) = C_3 a^{N-1} \exp(-aT), \quad a \geq 0 \tag{9}$$

where

$$T = t + \tau, \quad N = n + r.$$

Using equation (5), the posterior distribution of L can be obtained as

$$f(L \mid \underline{x}) = \frac{[C_4(p,0)]^{-1}}{1-L} A_N(L,p) B(L,p) (1-p)^{-B(L,p)T}, \quad 0 < L < p \tag{10}$$

where

$$C_4(p,d) = \int_0^p \frac{L^d}{1-L} A_N(L,p) B(L,p) (1-p)^{-B(L,p)T} \, dL \tag{11}$$

with

$$A_k(L,p) = \left(\frac{\log(1-p)}{\log(1-p) - \log(1-L)} \right)^k$$

and

$$B(L,p) = \frac{1}{\log(1-p) - \log(1-L)}.$$

(11) can be easily evaluated by using suitable numerical methods. One can have estimators for L by specifying appropriate loss functions and using (10). Under squared error loss function the Bayes estimator for Lorenz curve simplifies to

$$\widehat{L}_1 = \frac{C_4(p,1)}{C_4(p,0)} \tag{12}$$

with Bayes risk

$$R(L,\widehat{L}_1) = \frac{C_4(p,2)}{C_4(p,0)} - (\widehat{L}_1)^2 \tag{13}$$

Now we derive a Bayesian estimator for the Gini-index. Using (6) and (9), we get the posterior distribution of G as

$$f(G \mid \underline{x}) = \frac{[C_5(0)]^{-1}}{G^2} D_{N-1}(G)\exp(-D_1(G)T), \quad 0 < G < 1 \tag{14}$$

where

$$C_5(d) = \int_0^1 G^{d-2} D_{N-1}(G)\exp(-D_1(G)T)dG, \quad 0 < G < 1 \tag{15}$$

with

$$D_k(G) = \left(\frac{G+1}{2G}\right)^k.$$

The Bayes estimate of Gini-index under squared error loss function is

$$\widehat{G}_1 = \frac{C_5(1)}{C_5(0)} \tag{16}$$

and the expected loss when (16) is used as estimate is

$$R(G,\widehat{G}_1) = \frac{C_5(2)}{C_5(0)} - (\widehat{G}_1)^2 \tag{17}$$

To evaluate (16) and (17) we seek numerical integration.

2.2 *Estimation with unknown* θ

The most general and perhaps a more realistic situation is when both the location and scale parameters are unknown. In this section we attend to the problem of estimation of Lorenz curve and Gini-index when θ and a are unknown. Here the likelihood can be written as

$$l(\underline{x} \mid a,\theta) = C_6 a^n \theta^{na} \exp(-az) \tag{18}$$

where

$$z = \sum_{i=1}^{n} \log x_i \ .$$

The kernel of the likelihood suggests the following conjugate prior for (a,θ), namely

$$g(a,\theta) = C_7 a^{\nu} \theta^{\kappa a} \exp(-a\delta), \ \ a > 1, \ \ 0 \le \theta \le \theta_0, \ \ \theta_0, \nu, \kappa, \delta > 0 \,. \tag{19}$$

The corresponding posterior distribution simplifies to

$$f(a,\theta \,|\, \underline{x}) = C_8 a^{M} \theta^{Na-1} \exp(-aZ), \ \ a > 1, \ \ 0 \le \theta \le \min(\theta_0, X_{(1)}) \tag{20}$$

where

$$M = n + \nu, \ \ N = n + \kappa, \ \ Z = z + \delta, \ \ X_{(1)} = \min(X_1, X_2, ..., X_n) \,.$$

Now using (5) and (20), we get the joint posterior distribution of (L,θ), and marginal posterior distribution of L is obtained by integrating the joint posterior distribution with respect to θ, we get the marginal posterior density of the Lorenz curve, after simplification, as

$$f(L \,|\, \underline{x}) = \frac{[C_9(p,0)]^{-1}}{1-L} A_N(L,p) B(L,p) (1-p)^{-B(L,p)\Omega}, \ \ 0 < L < p \tag{21}$$

where

$$C_9(p,d) = \int_0^p \frac{L^d}{1-L} A_N(L,p) B(L,p) (1-p)^{-B(L,p)\Omega} \, dL \tag{22}$$

with

$$\Omega = Z - N \log \theta_0 \,.$$

Under the squared error loss function the Bayes estimate and the corresponding risk of the Lorenz curve is given by

$$\widehat{L}_2 = \frac{C_9(p,1)}{C_9(p,0)} \tag{23}$$

and

$$R(L, \widehat{L}_2) = \frac{C_9(p,2)}{C_9(p,0)} - (\widehat{L}_2)^2 \tag{24}$$

Now we derive an estimator for the Gini-index. When both θ and a are unknown. Proceeding analogous to the estimation of Lorenz curve, we get the posterior distribution of (G,θ) as

$$f(G,\theta \mid \underline{x}) = \frac{[C_{11}(0)]^{-1}}{G^2} D_M(G)\theta^{(D_1(G))N-1}\exp(-D_1(G)Z),\ 0<G<1,\ 0<\theta<\theta_0 \tag{25}$$

Integrating out θ, from (25), we get the marginal posterior distribution of G as

$$f(G \mid \underline{x}) = \frac{[C_{11}(0)]^{-1}}{G^2} D_{M-1}(G)\exp(-D_1(G)\Omega),\quad 0<G<1 \tag{26}$$

where

$$C_{11}(d) = \int_0^1 G^{d-2} D_{M-1}(G)\exp(-D_1(G)\Omega)dG \tag{27}$$

From (27), we obtain the Bayes estimate of the Gini-index as

$$\widehat{G}_2 = \frac{C_{11}(1)}{C_{11}(0)} \tag{28}$$

and the expected loss when (28) is used as estimate is

$$R(G,\widehat{G}_2) = \frac{C_{11}(2)}{C_{11}(0)} - (\widehat{G}_2)^2 \tag{29}$$

3. ESTIMATION OF VARIANCE OF LOGARITHMS

In this section we find the MLE and the Bayes estimators for the variance of logarithms under the two alternatives namely (i) the scale parameter θ is known and (ii) θ is unknown. The variance of logarithm for model (4) simplifies to

$$V = \frac{1}{a^2} \tag{30}$$

3.1 *Maximum likelihood estimator when θ is known*

The likelihood function (7) can be written as, (using (30))

$$l(\underline{x} \mid V) = C_{12} V^{-\frac{n}{2}} \exp\left(-tV^{-\frac{1}{2}}\right) \tag{31}$$

The normal equation for estimating V takes the form

$$\frac{d\log l}{dV}=\frac{t}{2V^{3/2}}-\frac{n}{2V}=0 \tag{32}$$

Solving (32), we get the MLE of V, $\hat{\eta}_1$, as

$$\hat{\eta}_1=\left(\frac{t}{n}\right)^2 \tag{33}$$

The expected information is

$$I(\hat{\eta}_1)=-E\left(\frac{\partial^2\log l}{\partial V^2}\right)=\frac{-n}{2V^2}+\frac{3}{4V^{5/2}}E(t)$$

Since $t=\sum_{i=1}^{n}\log\left(\frac{x_i}{\theta}\right)$ follow the Gamma distribution $G(a,n)$, we get

$$I(\hat{\eta}_1)=\frac{n}{4V^2} \tag{34}$$

Using (34), the variance of the estimator can be expressed as

$$Var(\hat{\eta}_1)=\frac{1}{I(\hat{\eta}_1)}=\frac{4V^2}{n} \tag{35}$$

3.2 *Maximum likelihood estimator when* θ *is unknown*

The likelihood function in this set-up can be written as

$$l(\underline{x}\,|\,V)=C_{13}V^{-\frac{n}{2}}\theta^{nV^{-\frac{1}{2}}}\exp\left(-zV^{-\frac{1}{2}}\right) \tag{36}$$

Differentiating logarithm of (40) with respect to V and equating to zero, we get

$$\frac{d\log l}{dV}=\frac{n}{2V}+\frac{z-n\log\theta}{2V^{\frac{3}{2}}}=0 \tag{37}$$

Solving (37), we get the MLE of V, denoted by $\hat{\eta}_2$, as

$$\hat{\eta}_2=\frac{z^2-2nz\log(X_{(1)})+n^2(\log(X_{(1)}))^2}{n^2} \tag{38}$$

The expected information, using the distribution of $t=\sum_{i=1}^{n}\log\left(\frac{x_i}{x_{(1)}}\right)$ $(G(a,n-1))$,

$$I(\hat{\eta}_2)=\frac{n}{2V^2}-\frac{3}{4V^{5/2}}E(t)=\frac{n-3}{4V^2} \text{ and} \tag{39}$$

the variance of the estimator can be expressed as

$$Var(\hat{\eta}_2)=\frac{4V^2}{n-3}, n>3. \tag{40}$$

3.3 *Bayes estimator when θ is known*

Using (9) and (30), we get the posterior distribution of V as

$$f(V\mid \underset{\sim}{x})=[C_{14}(0)]^{-1}V^{-\left(\frac{N}{2}+1\right)}\exp\left(\frac{-T}{\sqrt{V}}\right), 0<V<1 \tag{41}$$

where

$$C_{14}(d)=\int_0^1 V^{\left(\frac{2d-N}{2}-1\right)}\exp\left(\frac{-T}{\sqrt{V}}\right)dV \tag{42}$$

Under squared error loss function the Bayes estimator for V simplifies to

$$\widehat{V}_1=\frac{C_{14}(1)}{C_{14}(0)} \tag{43}$$

with Bayes risk

$$R(V,\widehat{V}_1)=\frac{C_{14}(2)}{C_{14}(0)}-(\widehat{V}_1)^2. \tag{44}$$

3.4 *Bayes estimator when θ is unknown*

Finally when both θ and a are unknown, w get the posterior distribution of V as

$$f(V\mid \underset{\sim}{x})=[C_{15}(0)]^{-1}V^{-\left(\frac{N}{2}+1\right)}\exp\left(\frac{-\Omega}{\sqrt{V}}\right), 0<V<1 \tag{45}$$

where

$$C_{15}(d) = \int_0^1 V^{\left(\frac{2d-N}{2}-1\right)} \exp\left(\frac{-\Omega}{\sqrt{V}}\right) dV \tag{46}$$

Under squared error loss function the Bayes estimator for V is defined as

$$\widehat{V}_2 = \frac{C_{15}(1)}{C_{15}(0)} \tag{47}$$

with Bayes risk

$$R(V, \widehat{V}_2) = \frac{C_{15}(2)}{C_{15}(0)} - (\widehat{V}_2)^2 \tag{48}$$

To evaluate (46), we seek numerical integration.

4. COMPARISON

In this section we compare the Bayes estimators of the Lorenz curve and Gini-index with the corresponding classical estimators (Moothathu, 1990) using a simulation study. We also make comparisons of the Bayes estimators and the maximum likelihood estimators of the variance of logarithms proposed in this article. We present the simulation results concerning the average, bias and mean square errors of all these estimators. In all the simulation results presented here, the bias of an estimator can be determined as the average value of the estimate report in the table - parameter value set. The variance of an estimator was determined as the sample variance obtained from all the simulations carried out. Finally, the mean square error of estimator is (variance of the estimator + (bias) ^{2}).

We simulated the average, bias and mean square error values of these estimators for sample sizes 25, 50, 100 and 150 using 2000 Monte Carlo runs. The values of the Bayes and maximum likelihood estimators of the Lorenz curve, Gini-index and variance of logarithms, are presented in the tables listed below, for different choices of the parameters a and θ. Bayes estimator was evaluated for the prior hyper-parameters $m, \tau, \upsilon = 0, 1$, *2* and their corresponding values are shown in various tables. From the tables listed below, it is revealed that the Bayes estimator is not seems very sensitive with variation of the prior parameters. It is to be noted that the bias and mse of the various Bayes estimators becomes smaller as the sample size increases.

It is clear from the tables that the proposed Bayes estimators for the Lorenz curve, Gini-index and variance of logarithms are on the whole superior in all the cases considered as compared to the MLEs in terms of bias and mse.

In figure 1, we plot the original Lorenz curve and estimates of Lorenz curves using Bayesian and MLE, for various values of p. From the figure it can be seen that the Lorenz curve estimated using the proposed estimator is very close to the original Lorenz curve than the Lorenz curve estimated using MLE, uniformly in p.

TABLE 1

Estimators of Lorenz curve when θ known ($p = 0.2$)

Parameter	Estimator	Average	Bias	MSE
$n = 25,\ a = 2.2$	$\widehat{L_1}$	0.11472	0.00012	0.00023
$m = \tau = 0,\ \theta = 3.5$	$\widehat{\beta_1}$	0.11251	-0.00208	0.00031
$n = 50,\ a = 2.5$	$\widehat{L_1}$	0.12315	-0.00216	0.00008
$m = \tau = 1,\ \theta = 3.6$	$\widehat{\beta_1}$	0.12642	0.00112	0.00011
$n = 100,\ a = 2.8$	$\widehat{L_1}$	0.13396	0.00032	0.00004
$m = \tau = 2,\ \theta = 3.7$	$\widehat{\beta_1}$	0.13330	-0.00033	0.00005
$n = 150,\ a = 2.8$	$\widehat{L_1}$	0.13396	0.00032	0.00004
$m = 1,\ \tau = 2,\ \theta = 3.7$	$\widehat{\beta_1}$	0.13330	-0.00033	0.00005

TABLE 2

Estimators of Lorenz curve when θ unknown ($p = 0.2$)

Parameter	Estimator	Average	Bias	MSE
$n = 25,\ a = 2.2$	$\widehat{L_2}$	0.11381	-0.00078	0.00028
$\nu = \kappa = \delta = 0,\ \theta = 3.5$	$\widehat{\beta_2}$	0.09260	-0.02199	0.00082
$n = 50,\ a = 2.5$	$\widehat{L_2}$	0.12581	0.00050	0.00013
$\nu = \kappa = \delta = 1,\ \theta = 3.6$	$\widehat{\beta_2}$	0.08752	-0.03778	0.00158
$n = 100,\ a = 2.8$	$\widehat{L_1}$	0.13468	0.00104	0.00007
$\nu = \kappa = \delta = 2,\ \theta = 3.7$	$\widehat{\beta_2}$	0.07661	-0.05702	0.00331
$n = 150,\ a = 2.8$	$\widehat{L_2}$	0.13574	-0.00027	0.00002
$\nu = \kappa = 1,\ \delta = 2,\ \theta = 3.7$	$\widehat{\beta_2}$	0.07432	-0.06170	0.00383

TABLE 3

Estimators of Gini-index when θ known

Parameter	Estimator	Average	Bias	MSE
$n = 25,\ a = 2.2$	$\widehat{G_1}$	0.31196	0.01784	0.00609
$m = \tau = 0,\ \theta = 3.5$	$\widehat{\lambda_1}$	0.30691	0.01279	0.00678
$n = 50,\ a = 2.5$	$\widehat{G_1}$	0.25264	0.00264	0.00162
$m = \tau = 1,\ \theta = 3.6$	$\widehat{\lambda_1}$	0.23410	-0.01589	0.00171
$n = 100,\ a = 2.8$	$\widehat{G_1}$	0.22912	0.01173	0.00079
$m = \tau = 2,\ \theta = 3.7$	$\widehat{\lambda_1}$	0.21345	-0.00394	0.00083
$n = 150,\ a = 2.9$	$\widehat{G_1}$	0.21994	0.01161	0.00044
$m = 1,\ \tau = 2,\ \theta = 3.8$	$\widehat{\lambda_1}$	0.20876	0.00043	0.00052

TABLE 4

Estimators of Gini-index when θ unknown

Parameter	Estimator	Average	Bias	MSE
$n=25,\ a=2.2$	$\widehat{G_2}$	0.32947	0.03535	0.01034
$\nu=\kappa=\delta=0,\ \theta=3.5$	$\widehat{\lambda_2}$	0.32072	0.02661	0.01073
$n=50,\ a=2.5$	$\widehat{G_2}$	0.29167	0.04167	0.00547
$\nu=\kappa=\delta=1,\ \theta=3.6$	$\widehat{\lambda_2}$	0.29463	0.04463	0.00569
$n=100,\ a=2.8$	$\widehat{G_2}$	0.24529	0.02790	0.00174
$\nu=\kappa=\delta=2,\ \theta=3.7$	$\widehat{\lambda_2}$	0.24992	0.03253	0.00205
$n=150,\ a=2.9$	$\widehat{G_2}$	0.21405	0.00571	0.00035
$\nu=\kappa=1,\ \delta=2,\ \theta=3.8$	$\widehat{\lambda_2}$	0.21678	0.00845	0.00040

TABLE 5

Estimators of variance of logarithms when θ known

Parameter	Estimator	Average	Bias	MSE
$n=25,\ a=2.2$	$\widehat{V_1}$	0.22673	0.02012	0.00910
$m=\tau=0,\ \theta=3.5$	$\widehat{\eta_1}$	0.23064	0.02403	0.01128
$n=50,\ a=2.5$	$\widehat{V_1}$	0.18368	0.02367	0.00322
$m=\tau=1,\ \theta=3.6$	$\widehat{\eta_1}$	0.18076	0.00208	0.00388
$n=100,\ a=2.8$	$\widehat{V_1}$	0.13265	0.00510	0.00059
$m=\tau=2,\ \theta=3.7$	$\widehat{\eta_1}$	0.12460	-0.00295	0.00063
$n=150,\ a=2.8$	$\widehat{V_1}$	0.11820	-0.00070	0.00047
$m=1,\ \tau=2,\ \theta=3.7$	$\widehat{\eta_1}$	0.12504	0.00614	0.00058

TABLE 6

Estimators of variance of logarithms when θ unknown

Parameter	Estimator	Average	Bias	MSE
$n=25,\ a=2.2$	$\widehat{V_2}$	0.18528	-0.02134	0.00569
$\nu=\kappa=\delta=0,\ \theta=3.5$	$\widehat{\eta_2}$	0.16425	-0.04236	0.00593
$n=50,\ a=2.5$	$\widehat{V_2}$	0.16219	0.00219	0.00306
$\nu=\kappa=\delta=1,\ \theta=3.6$	$\widehat{\eta_2}$	0.17976	0.01976	0.00369
$n=100,\ a=2.8$	$\widehat{V_2}$	0.12091	-0.00664	0.00050
$\nu=\kappa=\delta=2,\ \theta=3.7$	$\widehat{\eta_2}$	0.14097	0.01342	0.00072
$n=150,\ a=2.9$	$\widehat{V_2}$	0.11785	-0.00105	0.00035
$\nu=\kappa=1,\ \delta=2,\ \theta=3.8$	$\widehat{\eta_2}$	0.12321	0.00430	0.00038

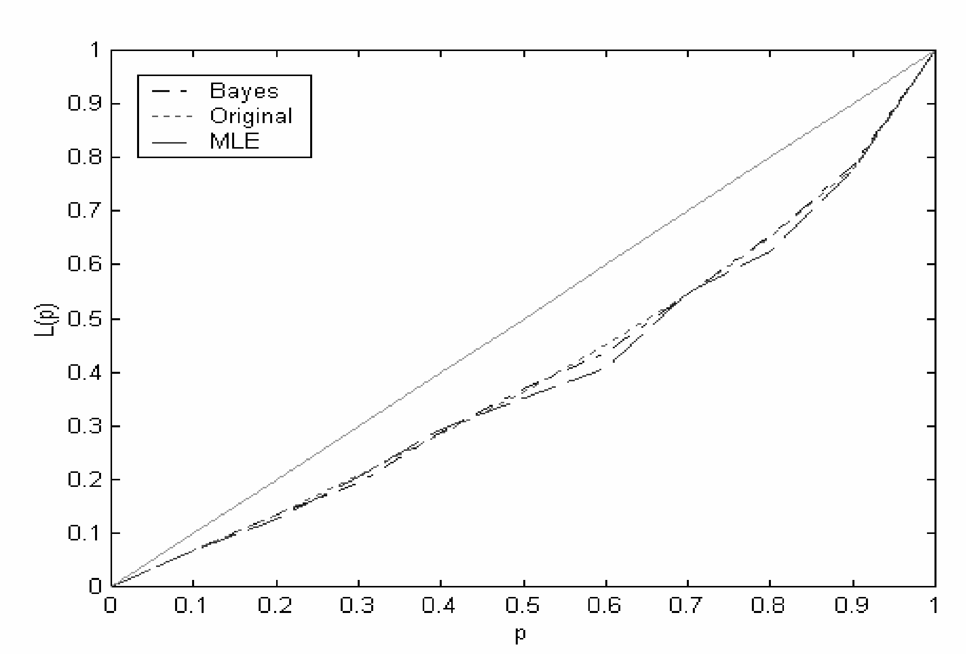

Figure 1 – Estimates of Lorenz curve (simulated data).

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RIASSUNTO

Stima bayesiana della curva di Lorenz, dell'indice di Gini e della varianza dei logaritmi in una distribuzione di Pareto

In questo articolo stimiamo la curva di Lorenz, l'indice di Gini e la varianza dei logaritmi per la distribuzione di Pareto adoperando l'impostazione bayesiana con distribuzione a priori coniugata. Il metodo Monte Carlo è stato usato per confrontare gli stimatori da noi proposti con lo stimatore di massima verosimiglianza suggerito da Moothathu (1990): gli stimatori proposti risultano molto efficienti.

SUMMARY

Bayesian estimation of Lorenz curve, Gini-index and variance of logarithms in a Pareto distribution

In this article, we estimate Lorenz curve, Gini-index and variance of logarithms for Pareto distribution using Bayesian framework with a conjugate prior. Our proposed Bayesian estimators are compared using a Monte Carlo study, to the MLE estimator proposed by Moothathu (1990) in terms of variance. It is found that the proposed estimators are highly efficient.