DISTRIBUTION OF THE LIKELIHOOD RATIO STATISTIC FOR TESTING HOMOGENEITY OF COVARIANCE MATRICES OF COMPLETELY SYMMETRIC GAUSSIAN MODELS

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1. INTRODUCTION

Let $\mathbf{X}_i$ be a $p \times 1$ random vector, which has a multivariate normal distribution with mean vector $\boldsymbol{\mu}_i = \mu_i \mathbf{e}$ and covariance matrix $\Sigma_i = \sigma_i^2[(1-\rho_i)I_p + \rho_i J]$ where $\sigma_i^2 > 0, \mu_i$ and $\rho_i (-1/(p-1) < \rho < 1)$ are unknown scalars, $i = 1, \ldots, m$, $\mathbf{e} = (1,1,\ldots,1)'_{p\times 1}$, I_p is the identity matrix of order p and J is a $p \times p$ matrix with each entry unity. Under this model consider the problem of testing equality of covariance matrices, *i. e.*,

$$H_0 : \Sigma_1 = \ldots = \Sigma_m = \sigma^2[(1-\rho)I_p + \rho J]$$

against general alternatives.

Han (1975) derived the likelihood ratio criterion (LRC) for testing homogeneity of covariance matrices under compound symmetric Gaussian models and has shown that the test based on the modified LRC is better than the test derived by using Roy's union intersection procedure (see Srivastava, 1965; Krishnaiah and Pathak, 1967). Han (1975) and Gupta and Nagar (1986, 1987) have derived distributional results in the null as well as non-null case. Compound symmetry is often found in repeated measures designs which are not time dependent. If the order of the responses does not matter than the covariances are exchangeable. The term exchangeable is synonymous with compound symmetry. The completely symmetric Gaussian models arise in symmetries in animals and plants. These models have also been shown to be useful in areas like medical research and psychometrics.

The problem of testing H_0 was considered by Tayal, Gupta and Nagar (1988). They derived likelihood ratio criterion Λ, its null moments and several distributional results. Suppose that independent random samples $\mathbf{X}_{i1},\ldots,\mathbf{X}_{iN_i}$ are taken from the i-th population, $i=1,\ldots,m$. Let Q be an orthogonal matrix with first row $\mathbf{e}'/\sqrt{p}$. Then $Q\Sigma_i Q' = \mathrm{diag}(a_i, b_i, \ldots, b_i) = \Sigma_i^*$ and $\mathbf{v}_i = \mu_i Q\mathbf{e} = \sqrt{p}\mu_i \mathbf{e}_1$, say, where $a_i = \sigma_i^2[1+(p-1)\rho_i], b_i = \sigma_i^2(1-\rho_i)$, and $\mathbf{e}_1 = (1,0,\ldots,0)'_{p\times 1}$. The null hypothesis for testing equality of covariance matrices is equivalent to $H: a_1 = \cdots = a_m; b_1 = \cdots = b_m$. Let $\mathbf{Y}_{iu} = Q\mathbf{X}_{iu}$ and y_{iju} be the j-th component $(j=1,\ldots,p)$ of $\mathbf{Y}_{iu}, u=1,\ldots,N_i$, $i=1,\ldots,m$. Further, let $W_{1i} = \sum_{u=1}^{N_i}(y_{i1u} - y_{i1\cdot})^2$, $N_i y_{i1\cdot} = \sum_{u=1}^{N_i} y_{i1u}, W_{2i} = \sum_{j=2}^{p}\sum_{u=1}^{N_i} y_{i1u}^2$, and $N_0 = \sum_{i=1}^{m} N_i$. The likelihood ratio statistic Λ for testing H can be expressed as (Tayal, Gupta and Nagar, 1988),

$$\Lambda = \frac{N_0^{N_0 p/2}}{\prod_{i=1}^{m} N_i^{N_i p/2}} \cdot \frac{\prod_{i=1}^{m} W_{1i}^{N_i/2}}{\left[\sum_{i=1}^{m} W_{1i}\right]^{N_0/2}} \cdot \frac{\prod_{i=1}^{m} W_{2i}^{(p-1)N_i/2}}{\left[\sum_{i=1}^{m} W_{2i}\right]^{(p-1)N_0/2}}. \tag{1}$$

The h-th null moment of Λ is available as

$$\begin{aligned} \mathrm{E}(\Lambda^h) = {} & \frac{N_0^{N_0 ph/2}}{\prod_{i=1}^{m} N_i^{N_i ph/2}} \frac{\Gamma[(N_0-m)/2]\Gamma[(p-1)N_0/2]}{\Gamma[\{N_0(1+h)-m\}/2]\Gamma[(p-1)N_0(1+h)/2]} \\ & \times \prod_{i=1}^{m} \frac{\Gamma[\{N_i(1+h)-1\}/2]\Gamma[(p-1)N_i(1+h)/2]}{\Gamma[(N_i-1)/2]\Gamma[(p-1)N_i/2]} \end{aligned} \tag{2}$$

where $\mathrm{Re}(N_i h) > -(N_i - 1)$, $i=1,\ldots,m$ and $\mathrm{Re}(\cdot)$ denotes the real part of $(\cdot)$.

Tayal, Gupta and Nagar (1988) used the above moment expression to derive exact and asymptotic null distributions of Λ. They obtained, for arbitrary p, asymptotic expansion of the distribution of a constant multiple of $-2\ln\Lambda$ in series involving chi-square distributions. They also derived exact distribution of $\Lambda^{2/N}$ when $p=2$ and $N_1 = \cdots = N_m = N$ using the residue theorem (see Nagar, Jain, and Gupta, 1988; Gupta and Nagar, 1999). As observed by them, the exact distribution of Λ for arbitrary p, using the residue theorem, does not seem possible to derive. We therefore, in this article, derive the distribution of $U = (\Lambda^{2/N_0})^{1/s}$, where s $(s>0)$ is an adjustable constant, in terms of beta series for arbitrary p.

The distribution of $U = (\Lambda^{2/N_0})^{1/s}$, in a series of beta distributions, is derived in Section 2. In Section 3, a comparison is made between the significance points obtained by using results derived in Section 2 and exct results obtained by Tayal, Gupta and Nagar (1988). It is found that the expansion in terms of beta is fairly good and can be used to compute percentage points for $p \geq 3$.

2. NULL DISTRIBUTION

Let $U=(\Lambda^{2/N_0})^{1/s}$. Substituting $2h/N_0 s$ for h, $\gamma_i=N_i/N_0$, $i=1,\ldots,m$ and then $N_0=n+2\delta$ in (2), we obtain

$$\mathrm{E}(U^h)=\frac{1}{\prod_{i=1}^{m}\gamma_i^{\gamma_i ph/s}}\cdot\frac{\Gamma[(n-m)/2+\delta]\Gamma[(p-1)(n/2+\delta)]}{\Gamma[(n-m)/2+h/s+\delta]\Gamma[(p-1)(n/2+h/s+\delta)]}$$
$$\times\prod_{i=1}^{m}\frac{\Gamma[\gamma_i(n/2+h/s+\delta)-1/2]\Gamma[\gamma_i(p-1)(n/2+h/s+\delta)]}{\Gamma[\gamma_i(n/2+\delta)-1/2]\Gamma[\gamma_i(p-1)(n/2+\delta)]}.$$

Now taking the inverse Mellin transform of the above expression, we get the density of U as

$$f(u)=K(2\pi\iota)^{-1}\int_{-\iota\infty}^{+\iota\infty}\frac{\prod_{i=1}^{m}\Gamma[\gamma_i(n/2+h/s+\delta)-1/2]}{\prod_{i=1}^{m}\gamma_i^{\gamma_i ph/s}\Gamma[(n-m)/2+h/s+\delta]}$$
$$\times\frac{\prod_{i=1}^{m}\Gamma[\gamma_i(p-1)(n/2+h/s+\delta)]}{\Gamma[(p-1)(n/2+h/s+\delta)]}u^{-h-1}dh, \tag{3}$$

where $0<u<1$, $\iota=\sqrt{-1}$ and

$$K=\frac{\Gamma[(n-m)/2+\delta]\Gamma[(p-1)(n/2+\delta)]}{\prod_{i=1}^{m}\Gamma[\gamma_i(n/2+\delta)-1/2]\prod_{i=1}^{m}\Gamma[\gamma_i(p-1)(n/2+\delta)]}. \tag{4}$$

Now substituting $n/2+h/s=t/s$, we have

$$f(u)=K(2\pi\iota)^{-1}\prod_{i=1}^{m}\gamma_i^{\gamma_i pn/2}u^{ns/2-1}\int_{c-\iota\infty}^{c+\iota\infty}\varphi(t)u^{-t}dt, 0<u<1, \tag{5}$$

where $c=ns/2$ and

$$\varphi(t)=\frac{\prod_{i=1}^{m}\Gamma[\gamma_i(t/s+\delta)-1/2]\prod_{i=1}^{m}[\gamma_i(p-1)(t/s+\delta)]}{\prod_{i=1}^{m}\gamma_i^{\gamma_i pt/s}\Gamma(t/s-m/2+\delta)\Gamma[(p-1)(t/s+\delta)]}. \tag{6}$$

Expandig the logarithm of $\varphi(t)$ using Barnes' expansion and converting back, one obtains

$$\phi(t)=(2\pi)^{m-1}(p-1)^{-(m-1)/2}\prod_{i=1}^{m}\gamma_i^{p\delta\gamma_i-3/2}\left(\frac{t}{s}\right)^{-\nu}\left[1+\sum_{\alpha=1}^{\infty}\frac{Q_\alpha}{t^\alpha}\right], \tag{7}$$

where $\nu = m-1$,

$$\mathcal{Q}_\alpha = \frac{1}{\alpha}\sum_{r=1}^{\alpha} r A_r \mathcal{Q}_{\alpha-r}, \alpha = 1,2,3,\ldots, \mathcal{Q}_0 = 1, \tag{8}$$

$$A_r = \frac{(-1)^{r+1} s^r}{r(r+1)}\left[\sum_{r=1}^{\alpha}\frac{B_{r+1}(\gamma_i\delta - 1/2)}{\gamma_i^r} + \sum_{i=1}^{m}\frac{B_{r+1}(\gamma_i(p-1)\delta)}{(p-1)^r\gamma_i^r} - \frac{B_{r+1}((p-1)\delta)}{(p-1)^r} - B_{r+1}\left(\delta - \frac{m}{2}\right)\right] \tag{9}$$

and $B_r(\cdot)$ is the Bernoulli polynomial of degree r and order one. Substituting (7) in (5) we get

$$f(u) = K(2\pi)^{m-1}(p-1)^{-(m-1)/2}\prod_{i=1}^{m}\gamma_i^{p\gamma_i(n/2+\delta)-3/2} s^\nu u^{ns/2-1} \times(2\pi\iota)^{-1}\int_{c-\iota\infty}^{c+\iota\infty}\left[1+\sum_{\alpha=1}^{\infty}\frac{\mathcal{Q}_\alpha}{t^\alpha}\right] t^{-\nu}u^{-t}dt,\ 0<u<1 \tag{10}$$

Also, since $t^{-\nu}[1+\sum_{r=1}^{\infty}\mathcal{Q}_r t^{-r}] = O(t^{-\nu})$, using Nair (1940), one can expand $t^{-\nu}[1+\sum_{r=1}^{\infty}\mathcal{Q}_r t^{-r}]$ in factorial series as

$$t^{-\nu}\left[1+\sum_{r=1}^{\infty}\mathcal{Q}_r t^{-r}\right] = \sum_{\alpha=0}^{\infty} \mathrm{R}_\alpha \frac{\Gamma(t+a)}{\Gamma(t+a+\nu+\alpha)} \tag{11}$$

where a is a constant to be determined later and the coefficients R_α's can be determined explicitly as done below. Expanding logarithm of $\Gamma(t+a)/\Gamma(t+a+\nu+\alpha)$ using Barnes' expansion and converting back, one obtains

$$\frac{\Gamma(t+a)}{\Gamma(t+a+\nu+\alpha)} = t^{-\nu-\alpha}\left[1+\sum_{j=1}^{\infty}C_{\alpha j}t^{-j}\right] \tag{12}$$

where

$$C_{\alpha j} = \frac{1}{j}\sum_{\ell=1}^{j}\ell A_{\alpha\ell}C_{\alpha,j-\ell}, j = 1,2,\ldots,\ldots, C_{\alpha 0} = 1, \tag{13}$$

and

$$A_{\alpha\ell} = \frac{(-1)^{\ell}}{\ell(\ell+1)}[B_{\ell+1}(a+\nu+\alpha) - B_{\ell+1}(a)]\,. \tag{14}$$

Sustituting (12) in (11) and comparing the coefficients of like powers of t on both sides, one gets

$$Q_\alpha = \sum_{j=0}^{\alpha} R_{\alpha-j} C_{\alpha-j,j}, \alpha = 1,2,\ldots,\ldots, R_0 = 1. \tag{15}$$

Now using (11) in (10) and integrating term by term which is valid, the density of U is obtained as

$$f(u) = K(2\pi)^{m-1}(p-1)^{-(m-1)/2}\prod_{i=1}^{m} \gamma_i^{p\gamma_i(n/2+\delta)-3/2} s^{\nu} \\ \times \sum_{\alpha=0}^{\infty} R_\alpha \frac{u^{ns/2+a-1}(1-u)^{\nu+\alpha-1}}{\Gamma(\nu+\alpha)}, 0 < u < 1. \tag{16}$$

Since the series in (16) is uniformly convergent for $0 < u < 1$, the distribution function can be derived by integrating term by term, and this procedure would then lead to a series in incomplete beta function given by

$$F(u) = K(2\pi)^{m-1}(p-1)^{-(m-1)/2}\prod_{i=1}^{m} \gamma_i^{p\gamma_i(n/2+\delta)-3/2} s^{\nu} \\ \times \sum_{\alpha=0}^{\infty} R_\alpha I_u(ns/2+a, \nu+\alpha)\frac{\Gamma(ns/2+a)}{\Gamma(ns/2+a+\nu+\alpha)}, \tag{17}$$

where $I_u(\cdot,\cdot)$ is the incomplete beta function. Also expanding K and $\Gamma(ns/2+a)/\Gamma(ns/2+a+\nu+\alpha)$ and using Barnes' expansion, one has

$$K = (2\pi)^{-(m-1)}(p-1)^{(m-1)/2}\prod_{i=1}^{m} \gamma_i^{-p\gamma_i(n/2+\delta)+3/2}\left(\frac{n}{2}\right)^{\nu}\left[1 + \sum_{\alpha=1}^{\infty}\frac{Q_\alpha^*}{(n/2)^\alpha}\right], \tag{18}$$

where

$$Q_\alpha^* = \frac{1}{\alpha}\sum_{r=1}^{\alpha} r A_r^* Q_{\alpha-r}^*, \; Q_0^* = 1, \tag{19}$$

$$A_r^* = -\frac{A_r}{s^r}, \tag{20}$$

$$\frac{\Gamma(ns/2+a)}{\Gamma(ns/2+a+\nu+\alpha)}=\left(\frac{1}{2}ns\right)^{-\nu-\alpha}\left[1+\sum_{j=1}^{\infty}C_{\alpha j}\left(\frac{1}{2}ns\right)^{-j}\right] \tag{21}$$

and $C_{\alpha j}$'s are given by (13). Substituting (18) and (21) in (17) and simplifying we get the following result:

$$F(u)=I_u\left(\frac{1}{2}ns+a,\nu\right)+\sum_{\alpha=1}^{\infty}\frac{1}{(n/2)^{\alpha}}G_{\alpha} \tag{22}$$

where

$$G_{\alpha}=\sum_{j=0}^{\alpha}R_{\alpha-j}I_u\left(\frac{1}{2}ns+a,\nu+\alpha-j\right)\sum_{\ell=0}^{j}\frac{Q_{\ell}^{*}C_{\alpha-j,j-\ell}}{s^{\alpha-\ell}}. \tag{23}$$

Substituting $\alpha=1,2$ in (2.21) we get

$$G_1=R_1I_u\left(\frac{ns}{2}+a,\nu+1\right)\frac{Q_0^{*}C_{10}}{s}+R_0I_u\left(\frac{ns}{2}+a,\nu\right)\left\{\frac{Q_0^{*}C_{01}}{s}+\frac{Q_1^{*}C_{00}}{s^0}\right\} \tag{24}$$

and

$$G_2=R_2I_u\left(\frac{ns}{2}+a,\nu+2\right)\frac{Q_0^{*}C_{20}}{s^2}+R_1I_u\left(\frac{ns}{2}+a,\nu+1\right)\left\{\frac{Q_0^{*}C_{11}}{s^2}+\frac{Q_1^{*}C_{10}}{s^1}\right\}$$

$$+R_0I_u\left(\frac{ns}{2}+a,\nu\right)\left\{\frac{Q_0^{*}C_{02}}{s^2}+\frac{Q_1^{*}C_{01}}{s^1}+\frac{Q_1^{*}C_{00}}{s^0}\right\}. \tag{25}$$

From (15), (8), (13), (10) and (20) we have $R_0=1$, $Q_1=R_1C_{10}+R_0C_{01}$, $Q_2=R_2C_{20}+R_1C_{11}+R_0C_{02}$, $Q_0=1, Q_1=A_1$, $Q_2=A_1^2/2+A_2$, $C_{00}=C_{10}=C_{20}=1$, $C_{11}=A_{11}, C_{01}=A_{01}$, $C_{02}=A_{01}^2/2+A_{02}$, $Q_0^{*}=1, Q_1^{*}=A_1^{*}$, $Q_2^{*}=A_1^{*2}/2+A_2^{*}$, $A_1^{*}=-A_1/s, A_2^{*}=-A_2/s^2$ and G_1 simplifies to

$$G_1=\frac{1}{s}(A_1-A_{01})\left\{I_u\left(\frac{1}{2}ns+a,\nu+1\right)-I_u\left(\frac{1}{2}ns+a,\nu\right)\right\}$$

where from (9) and (14) we get

$$A_1=\frac{s}{24}\left[\frac{11p-9}{p-1}\left(\sum_{i=1}^{m}\frac{1}{\gamma_i}-1\right)-24(m-1)\delta-3(m-1)(m+3)\right]$$

and $A_{01} = -\nu(2a+\nu-1)/2$. Now if we choose $a = -(\nu-1)/2$ and

$$\delta = \frac{(11p-9)\left[\sum_{i=1}^{m}(1/\gamma_i)-1\right]-3(p-1)(m-1)(m+3)}{24(p-1)(m-1)}$$

then $G_1 = 0$ and G_2 in (25) reduces to

$$G_2 = \left(-A_2^* - \frac{A_{02}}{s^2}\right)\left\{I_u\left(\frac{1}{2}ns+a,\nu+2\right)-I_u\left(\frac{1}{2}ns+a,\nu\right)\right\}$$

where from (9), (20) and (14), we have

$$A_2^* = -\frac{1}{8}\left[\sum_{i=1}^{m}\frac{1}{\gamma_i^2}-\frac{1}{6}m(m+1)(m+2)-4(m-1)\delta^2\right]$$

and $A_{02} = -\nu(1\text{-}\nu^2)/24$. In order to make $G_2 = 0$ we choose s^2 such that $-A_2^* - A_{02}/s^2 = 0$. Thus we obtain $s^2 = \nu(1-\nu^2)/24A_2^*$.

It may be noted here that the choice of s is possible only if $\nu > 1$. For $m = 2$, we have $\nu = 1$ and in this case we choose $s = 1$. Finally, simplifying G_3 in (14), from (22) we get the following asymptotic expansion of the distribution of U:

$$F(u) = I_u\left(\frac{ns}{2}+a,\nu\right)+\frac{8R_3}{n^3}\left\{I_u\left(\frac{ns}{2}+a,\nu+3\right)-I_u\left(\frac{ns}{2}+a,\nu\right)\right\}+O(n^{-4}). \qquad (26)$$

3. COMPARISON OF PERCENTAGE POINTS

For $N_1 = \cdots = N_m = N$ and $p = 2, m = 3, m = 4$, Tayal, Gupta and Nagar (1988) have given the following exact results of the distribution of $V = \Lambda^{1/N}$ in series involving Psi function (Abramowitz and Stegun, 1965; Luke, 1969):

$p = 2, m = 3$

$$g(v) = \frac{\Gamma(N-1/3)\Gamma(N+1/3)}{\Gamma^2(N-1)}v^{N-2}\sum_{i=0}^{\infty}\left\{-\ln v + 2\psi(i+1)-\psi\left(i+\frac{1}{3}\right)\right.$$
$$\left.-\psi\left(i-\frac{1}{3}\right)\right\}\frac{(-1)\Gamma(i-1/3)\Gamma(i+1/3)}{(i!)^2\Gamma^2(1/3)\Gamma^2(2/3)}, 0 < v < 1.$$

$p = 2, m = 4$

$$g(v) = \frac{(N-1)\Gamma(N-1/2)\Gamma(N+1/2)}{\pi\Gamma^2(N-1)} v^{N-2}\Big[(-\ln v)^2 + 4(\ln 4 - 1)(-\ln v)$$

$$+4(\ln 4 - 1)^2 + 4 - \frac{2\pi^2}{3} + \frac{1}{\pi}\sum_{i=0}^{\infty} v^i \Big\{(-\ln v) + (i)^{-1} + 2\psi(i+1)$$

$$-\psi\left(i+\frac{1}{2}\right) - \psi\left(i-\frac{1}{2}\right)\Big\}\frac{\Gamma(i-1/2)\Gamma(i+1/2)}{i(i!)^2}\Big], \; 0 < v < 1.$$

The computation of the exact percentaje points has been carried out by using $F(x) = \int_0^x g(t)\,dt$ where $g(t)$ is the density of V. The computation is carried out by using series repreentation given above. First $F(x)$ is computed for various values of x. It is checked for monotonicity and for conditions $F(x) \to 0$ as $x \to 0$ and $F(x) \to 1$ as $x \to 1$. Then x is computed for various values of $F(x)$. The approximate percentage points have been obtained by using first term in (26). A comparison of the exact and approximate percentage points is given in Table 1 and table 2. It is observed that beta approximation performs fairly good.

TABLE 1

Comparison of the Exact significance points of V with Beta approximation, m=3, p=2

	$\alpha = 0.05$		$\alpha = 0.01$	
N	Exact	Beta	Exact	Beta
5	0.193203	0.192961	0.309137	0.308957
10	0.480094	0.48007	0.591986	0.591972
15	0.623450	0.623445	0.713466	0.713464
20	0.705784	0.705782	0.779584	0.779583
25	0.758814	0.758813	0.821004	0.821004
30	0.795732	0.795731	0.849352	0.849352
35	0.822883	0.822883	0.869962	0.869962
40	0.843681	0.84368	0.885618	0.885618

TABLE 2

Comparison of the Exact significance points of V with Beta approximation, m=4, p=2

	$\alpha = 0.05$		$\alpha = 0.01$	
N	Exact	Beta	Exact	Beta
5	0.12604	0.125765	0.212278	0.212038
10	0.395871	0.395836	0.499626	0.499603
15	0.550343	0.550335	0.639382	0.639377
20	0.643634	0.643631	0.718926	0.718924
25	0.705319	0.705318	0.769926	0.769926
30	0.74896	0.74896	0.805333	0.805333
35	0.781411	0.781411	0.831326	0.831326
40	0.806466	0.806465	0.85121	0.85121

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RIASSUNTO

Distribuzione del rapporto di verosimiglianza per verificare l'ipotesi di omogeneità delle matrici di covarianza di modelli gaussiani simmetrici completi

In questo lavoro viene derivata la distribuzione del rapporto di verosimiglianza per saggiare l'ipotesi di uguaglianza delle matrici di covarianza di modelli gaussiani simmetrici completi nel caso di serie beta. La soluzione trovata è confrontata con i risultati esatti.

SUMMARY

Distribution of the likelihood ratio statistic for testing homogeneity of covariance matrices of completely symmetric Gaussian models

Iin this article the distribution of the likelihood ratio test statistic for testing equality of covariance matrices of completely symmetric Gaussian models has been derived in beta series. Comparison of results with the exact results has also been done.