A MODIFICATION OF SILBER'S ALGORITHM TO DERIVE BOUNDS ON GINI'S CONCENTRATION RATIO FROM GROUPED OBSERVATIONS

T. Ogwang, B. Wang

1. INTRODUCTION

The problem of determining the bounds of Gini's concentration ratio (or the Gini index) from grouped data has been considered by Gastwirth (1972), Mehran (1975), Fuller (1979), Murray (1978), Giorgi and Pallini (1987), Silber (1990), Cowell (1991), and Ogwang (2003), among others. The need for determining the bounds of the Gini index instead of its point estimates arises because of the sensitivity of the latter to the specification of the underlying Lorenz Curve (LC), as demonstrated by Ogwang and Rao (1996) and Ryu and Slottje (1999), among others. The lower bound assumes that all the income-receiving units (individuals or households) in a given income bracket receive the average income for that bracket. The upper bound incorporates a grouping correction by assuming maximum inequality in each income bracket.

The bounds of the Gini index derived by Gastwirth, Fuller, Silber, and Ogwang are based on similar geometric approaches. Specifically, the lower bound is obtained by drawing a series of line segments joining the observed points on the LC, and the Gini index is given by one minus twice the area below the piecewise linear LC. The corresponding upper bound is obtained by constructing tangents to the LC at the observed points, and one minus twice the area below these tangents gives the Gini index. Ogwang established the equivalence between Gastwirth's, Fuller's, and Ogwang's upper bounds, all of which require information on the limits of the income brackets and group mean incomes or the overall mean income.

Silber has derived the coordinates of the points of intersection of the tangents to the LC at the observed points that facilitate the computation of the upper bound of the Gini index without using information on the limits of the income brackets, the group mean incomes, or the overall mean income. His algorithm for determining the bounds of the Gini index applies the **G**-matrix operator in conjunction with the geometric method. Although Silber's and Mehran's upper bounds have the same information requirements, the former seems simpler, which renders it potentially attractive for empirical work. Since Silber's upper bound is also based on a geometric approach, the resulting estimate should be identical to Gastwirth's, Fuller's, and Ogwang's upper bounds if the same information is used. However, our experience with several data sets indicates that this is not the case. This discrepancy arises because Silber's coordinates of the points of intersection of the tangents to the LC at the observed points are based purely on population shares and income shares whereas the upper bounds derived by Gastwirth, Fuller, and Ogwang also utilize additional information pertaining to the income brackets.

We believe that if there is additional information pertaining to the income brackets, such as the limits of the income brackets, the group mean incomes, or the overall mean income, such information should also be used to estimate the bounds of the Gini index (and other inequality measures). It is therefore necessary to modify Silber's coordinates in cases where such information is provided. For a theoretical discussion on the importance of reporting group means, see, for example, Krieger (1983).

The purpose of this paper is to derive modified coordinates of the points of intersection of the tangents to the LC at the observed points assuming that there is information on the limits of the income brackets and full or sparse information on mean incomes. We also show that if the modified coordinates are incorporated into Silber's algorithm, the resulting estimate of the upper bound is identical to estimates of the upper bound based on Gastwirth's, Fuller's, and Ogwang's formulas. However, by applying the **G**-matrix operator, the empirical implementation of the proposed methodology is undoubtedly simpler than the empirical implementation of Gastwirth's, Fuller's, and Ogwang's formulas. Thus, the proposed methodology is computationally simpler than Gastwirth's, Fuller's, and Ogwang's formulas, yet the resulting estimates of the bounds are identical.

The format of the rest of the paper is as follows: In section 2, we present modified coordinates of the points of intersection of the tangents to the LC at the observed points, taking into account information on the limits of the income brackets and the overall mean income or at least one of the group mean incomes. The connection between the modified coordinates and Silber's coordinates is also established and the associated bounds are presented. An illustrative example is presented in section 3 and concluding remarks are made in section 4.

2. THE MODIFIED COORDINATES AND THE BOUNDS

As is standard practice with the geometric approach, the observed points on the LC, derived from the grouped data, are taken to be fixed. Let us assume that the data are divided into $k+1$ income brackets with $a_0, a_1, ..., a_{k+1}$ as the interval endpoints $(0 \leq a_0 < a_1 < ... < a_{k+1} \leq \infty)$, which are assumed to be provided. To ensure the validity of the upper bound, it is assumed that the sample is drawn from a continuous distribution.

Let μ_i, μ, p_i, and $L(p_i)$ denote the mean income in income bracket (a_{i-1}, a_i), the overall mean income, the cumulative fraction of income receiving units whose incomes are less than a_i, and the corresponding cumulative fraction of income, respectively. The corresponding LC is defined by a set of ordered points $(p_i, L(p_i))$, $i = 0,1,...,k+1$ as illustrated by the dashed convex curve in Figure 1, assuming that the incomes are classified into three brackets (i.e., $k+1=3$). By definition, $(p_0, L(p_0)) = (0,0)$ and $(p_{k+1}, L(p_{k+1})) = (1,1)$. The line segment joining $(0,0)$ and $(1,1)$ is the perfect equality (egalitarian) line.

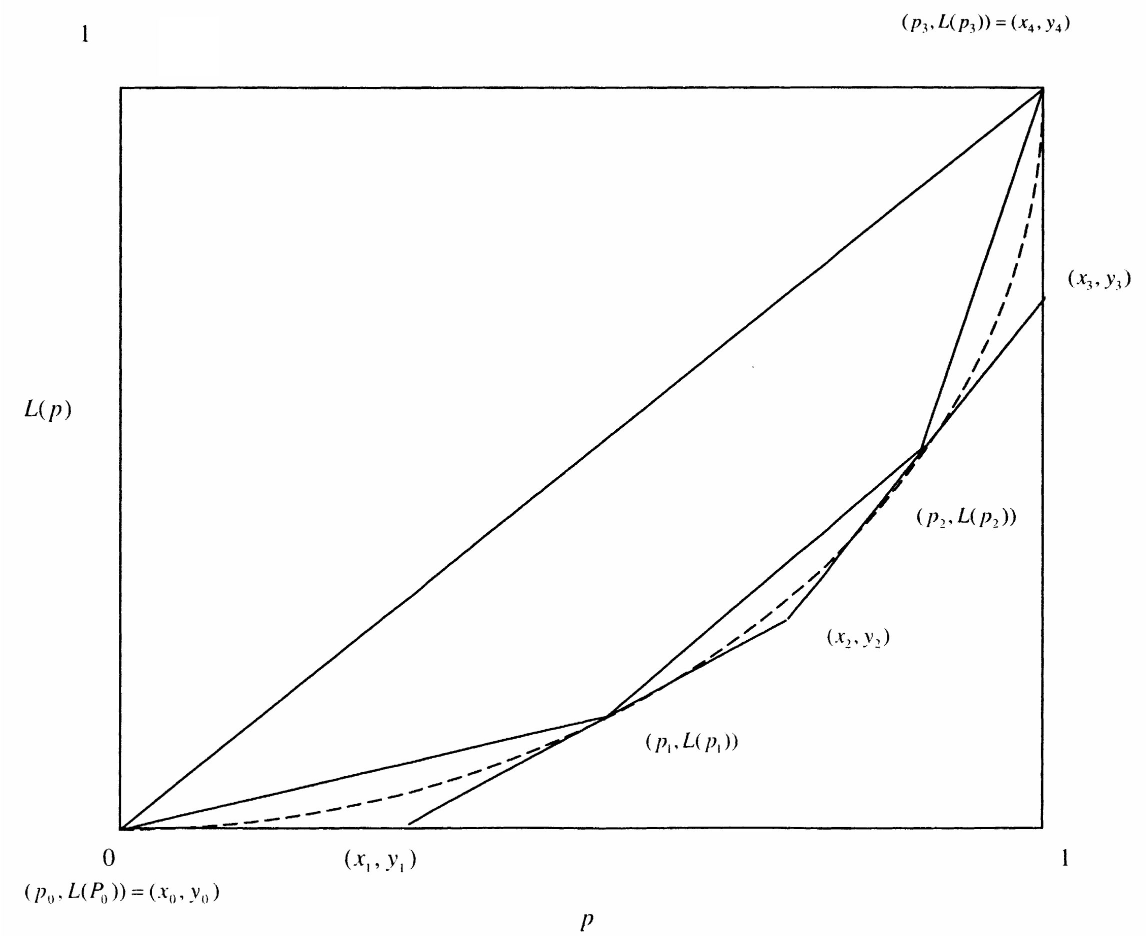

Figure 1 – Lorenz curve coordinates assuming three income brackets.

Let (x_i, y_i) be the coordinates of the point of intersection of the tangents to the LC at $(p_{i-1}, L(p_{i-1}))$ and $(p_i, L(p_i))$, $i = 1,2,...,k+1$. To simplify the exposition of Silber's **G**-matrix approach assuming that the incomes are classified into $(k+1)$ brackets, it is convenient to define $(x_0, y_0) = (0,0)$ and $(x_{k+2}, y_{k+2}) = (1,1)$.

Let f_i denote the share of income receiving units that belong to income bracket (a_{i-1}, a_i), $i = 1,2,...,k+1$ and s_i, the corresponding share of income. Thus, $f_i = (p_i - p_{i-1})$ or, equivalently, $p_i = \sum_{j=1}^{i} f_j$. Also, $s_i = (L(p_i) - L(p_{i-1}))$ or, equivalently, $L(p_i) = \sum_{j=1}^{i} s_j$.

Following Silber (1990, p. 217), the lower bound of the Gini index is given by

$$G_L = e'Gs \tag{1}$$

where $\boldsymbol{e} = [f_{k+1}, f_k, ..., f_2, f_1]'$ is a $(k+1)$ x1 vector of shares of income-receiving units in each of the $(k+1)$ income brackets; $\boldsymbol{G}$ is a $(k+1)$ x$(k+1)$ matrix (the **G**-matrix) with elements g_{ij} such that $g_{ij} = -1$ if $i < j$, $g_{ij} = 0$ if $i = j$, and $g_{ij} = 1$ if $i > j$; $\boldsymbol{s} = [s_{k+1}, s_k, ..., s_2, s_1]'$ is a $(k+1)$ x1 vector of each of the $(k+1)$ income shares.

The corresponding upper bound is given by

$$G_U = d'Gw \tag{2}$$

where $d = [d_{k+2}, d_{k+1}, d_k, ..., d_2, d_1]'$ is a $(k+2)$ x1 vector with elements $d_i = (x_i - x_{i-1})$, $i = 1,2,...,k+2$, where the x_i's are defined above (note that $x_0 = 0$ and $x_{k+1} = x_{k+2} = 1$); G is a matrix (the **G**-matrix) with elements g_{ij} such that $g_{ij} = -1$ if $i < j$, $g_{ij} = 0$ if $i = j$, and $g_{ij} = 1$ if $i > j$; $\boldsymbol{w}$ = $[w_{k+2}, w_{k+1}, ..., w_2, w_1]'$ is a $(k+2)$ x1 vector with elements $w_i = (y_i - y_{i-1})$, $i = 1,2,...,k+2$, where the y_i's are defined above (note that $y_0 = 0$ and $y_{k+2} = 1$).

Since all the elements of the **G**-matrix in equation (1) are known and those of column vectors *e* and *s* are obtained directly from the observed points on the LC, no complications arise in the determination of the lower bound. However, as pointed out by Silber (1990, p. 217), determining the upper bound, using equation (2), necessitates determining the coordinates of the points of intersection of the tangents to the LC at the observed points. We shall first present Silber's coordinates followed by its modification, taking into account information on the limits of the income brackets as well as mean income.

Assuming that the incomes are classified into $(k+1)$ brackets, the following coordinates can be deduced from Silber's paper:

$$x_0 = y_0 = y_1 = 0,$$

$$x_1 = f_1 - \left(\frac{f_1 + f_2}{s_1 + s_2}\right) s_1, \tag{3}$$

$$x_i = \frac{\left(\dfrac{s_i + s_{i+1}}{f_i + f_{i+1}}\right)(f_1 + ... + f_i) - \left(\dfrac{s_{i-1} + s_i}{f_{i-1} + f_i}\right)(f_1 + ... + f_{i-1}) - s_i}{\left(\dfrac{s_i + s_{i+1}}{f_i + f_{i+1}}\right) - \left(\dfrac{s_{i-1} + s_i}{f_{i-1} + f_i}\right)}, \; i = 2,...,k \quad (4)$$

$$y_i = \frac{\left(\dfrac{s_{i-1} + s_i}{f_{i-1} + f_i}\right)\left(\dfrac{s_i + s_{i+1}}{f_i + f_{i+1}}\right) f_i - \left(\dfrac{s_{i-1} + s_i}{f_{i-1} + f_i}\right)(s_1 + ... + s_i) + \left(\dfrac{s_i + s_{i+1}}{f_i + f_{i+1}}\right)(s_1 + ... + s_{i-1})}{\left(\dfrac{s_i + s_{i+1}}{f_i + f_{i+1}}\right) - \left(\dfrac{s_{i-1} + s_i}{f_{i-1} + f_i}\right)}, \; i = 2,...,k \quad (5)$$

$$y_{k+1} = \left(\frac{s_k + s_{k+1}}{f_k + f_{k+1}}\right) - \left(\frac{s_k + s_{k+1}}{f_k + f_{k+1}}\right)(f_1 + ... + f_k) + (s_1 + ... + s_k), \quad (6)$$

$$x_{k+1} = x_{k+2} = y_{k+2} = 1.$$

Note that the last expression in the numerator of equation (5) incorporates a correction to a typographical error in the last equation in Silber (1990, p. 218), by replacing $(s_1 + ... + s_i)$ by $(s_1 + ... + s_{i-1})$.

The modified coordinates utilize some of the results derived in an earlier paper by Ogwang (2003). Let β_i denote the slope of the line segment joining $(p_{i-1}, L(p_{i-1}))$ and $(p_i, L(p_i))$, and β_i^*, the slope of the tangent to the LC at $(p_i, L(p_i))$, i.e. $\beta_i = \dfrac{(L(p_i) - L(p_{i-1}))}{(p_i - p_{i-1})} = \dfrac{s_i}{f_i}$, $i = 1, 2, ..., k+1$; $\beta_i^* = \dfrac{(L(p_i) - y_i)}{(p_i - x_i)}$, $i = 1, 2, ..., k+1$; and $\beta_{k+1}^* = \infty$.

Using the above notation, the following modified coordinates can be derived:

$$x_0 = y_0 = y_1 = 0,$$

$$x_1 = f_1 - \frac{s_1}{\beta_1^*}, \quad (3')$$

$$x_i = \frac{(\beta_i^* p_i - \beta_{i-1}^* p_{i-1}) - \beta_i (p_i - p_{i-1})}{(\beta_i^* - \beta_{i-1}^*)} = \frac{(\beta_i^* p_i - \beta_{i-1}^* p_{i-1}) - s_i}{(\beta_i^* - \beta_{i-1}^*)}, \; i = 2,...,k \quad (4')$$

$$y_i = \frac{\beta_{i-1}^* \beta_i^* (p_i - p_{i-1}) - \beta_{i-1}^* L(p_i) + \beta_i^* L(p_{i-1})}{(\beta_i^* - \beta_{i-1}^*)}$$

$$= \frac{\beta_{i-1}^* \beta_i^* f_i - \beta_{i-1}^* (s_1 + ... + s_i) + \beta_i^* (s_1 + ... + s_{i-1})}{(\beta_i^* - \beta_{i-1}^*)}, \ i = 2,...,k \tag{5'}$$

$$y_{k+1} = \beta_k^* - \beta_k^* p_k + L(p_k) = \beta_k^* - \beta_k^* (f_1 + ... + f_k) + (s_1 + ... + s_k), \tag{6'}$$

$$x_{k+1} = x_{k+2} = y_{k+2} = 1.$$

Following Ogwang (2003, p. 419), the coordinates of the points of intersection of the tangents to the LC and, hence, the Gini index, are obtained by setting $\beta_i^* = a_i / \mu$, $i = 1, 2, ..., k+1$, where μ is the overall mean income. If the overall mean income is not known, it could be recovered from one of the group mean incomes using the formula $\mu_i = \beta_i \mu$, as pointed out by Ogwang (2003, p. 420). Thus, the β_i^*'s can be estimated using full or sparse information on mean incomes.

A careful inspection of the expressions for Silber's coordinates and the corresponding expressions for the modified coordinates (i.e. equations (3) and (3'), (4) and (4'), (5) and (5'), and (6) and (6')) reveals that the two sets of coordinates differ in the way the β_i^*'s are constructed. In Silber's algorithm, $\beta_i^* = \left(\frac{f_i + f_{i+1}}{s_1 + s_{i+1}} \right)$, which does not require mean income information. In the modified algorithm, $\beta_i^* = a_i / \mu$, which requires information on the upper limit of income bracket (a_{i-1}, a_i) as well as the overall mean income, μ. If μ is not known, it can be recovered from one of the known group mean incomes as described above. Thus, if the limits of the income brackets are provided and the overall mean income can be obtained either directly or indirectly as described above, the modified coordinate system could be substituted into Silber's algorithm to provide an estimate of the upper bound of the Gini index, which turns out to be identical to estimates of Gastwirth's, Fuller's, and Ogwang's upper bounds.

To see how the modified coordinate system yields an estimate of the upper bound which is identical to Gastwirth's, Fuller's, and Ogwang's upper bounds, we note that $G_U = d'Gw$ in equation (2) is, in fact, $1 - \sum_{i=1}^{k+1} \beta_{i-1}^{*\,-1} (y_i^2 - y_{i-1}^2)$, where $\sum_{i=1}^{k+1} \beta_{i-1}^{*\,-1} (y_i^2 - y_{i-1}^2)$ is twice the area below the tangents to the LC at the observed points (see Figure 1). Likewise, $G_L = e'Gs$ in equation (1) is, in fact,

$1-\sum_{i=1}^{k+1}\beta_i^2(L(p_i)^2-L(p_{i-1})^2)$, where $\sum_{i=1}^{k+1}\beta_i^2(L(p_i)^2-L(p_{i-1})^2)$ is twice the area below the line segments joining the observed points on the LC. Ogwang (2003) showed that twice the area of the triangle with vertices at $(p_{i-1},L(p_{i-1}))$, $(p_i,L(p_i))$, and (x_i,y_i) in Figure 1, which represents the grouping correction for income bracket (a_{i-1},a_i), $i=1,2,...,k+1$, is equal to $(p_i-p_{i-1})^2(\beta_i^*-\beta_i)(\beta_i-\beta_{i-1}^*)(\beta_i^*-\beta_{i-1}^*)^{-1}$. Thus,

$$\sum_{i=1}^{k+1}{\beta^*}_{i-1}^{-1}({y_i}^2-{y_{i-1}}^2)=\sum_{i=1}^{k+1}\beta_i^{-1}(L(p_i)^2-L(p_{i-1})^2)$$
$$-\sum_{i=1}^{k+1}(p_i-p_{i-1})^2(\beta_i^*-\beta_i)(\beta_i-\beta_{i-1}^*)(\beta_i^*-\beta_{i-1}^*)^{-1} \quad (7)$$

Taking one minus the expression on either side of equation (7) yields

$$1-\sum_{i=1}^{k+1}{\beta_{i-1}^*}^{-1}(y_i^2-y_{i-1}^2)=1-\sum_{i=1}^{k+1}\beta_i^{-1}(L(p_i)^2-L(p_{i-1})^2)$$
$$+\sum_{i=1}^{k+1}(p_i-p_{i-1})^2(\beta_i^*-\beta_i)(\beta_i-\beta_{i-1}^*)(\beta_i^*-\beta_{i-1}^*)^{-1} \quad (8)$$

From equation (8), it can be deduced that

$$G_U=G_L+\sum_{i=1}^{k+1}(p_i-p_{i-1})^2(\beta_i^*-\beta)(\beta_i-\beta_{i-1}^*)(\beta_i^*-\beta_{i-1}^*)^{-1} \quad (9)$$

If we substitute $\beta_i^*=a_i/\mu$ in equation (9), it is easy to verify that the resulting grouping correction is identical to that derived by Gastwirth, Fuller, and Ogwang, which establishes the equivalence. Detailed derivations of these results are available from the authors on request.

3. AN ILLUSTRATIVE EXAMPLE

In order to compare Silber's upper bound with the modified upper bound, we used United States data originally collected by the United States Bureau of the Census, and were previously used by Gastwirth (1972), Mehran (1975), and Ogwang (2003) to demonstrate the computations of their respective bounds. For details pertaining to these data that are divided into 10 income brackets, see Gastwirth (1972, Table 2) and Mehran (1975, p. 66). For this data set, the overall mean income, the group mean incomes, and the limits of the income brackets are known. However, for purposes of computing Silber's bounds, this information is ignored.

Table 1 presents Silber's coordinates of the points of intersection of the tangents to the LC at the observed points as well as the modified coordinates. Table 2 reports the actual numerical estimates of the bounds when each of the two coordinate systems is incorporated into Silber's algorithm. Differences between the two sets of coordinates, and, hence, differences in the resulting upper bound estimates are apparent from the two tables. For purposes of comparisons, the estimates of Gastwirth's, Fuller's, and Ogwang's bounds are also reported in Table 2. As expected, all the methods yield the same value of the lower bound, 0.3883. Table 2 shows that if the modified coordinates are incorporated into Silber's algorithm, the resulting estimate of the upper bound is 0.4083, which is identical to the corresponding estimates based on Gastwirth's, Fuller's, and Ogwang's formulas. In contrast, if Silber's coordinates are incorporated into his algorithm, the resulting estimate of the upper bound is 0.4061, which is slightly less than 0.4083. Since Silber's coordinates require less information about the income brackets, one would expect the resulting bounds to be wider. The fact that Silber's bounds are narrower for this data set is, therefore, contrary to expectations. Our experience with several data sets indicates that there are cases where Silber's coordinates do indeed yield wider bounds as expected. It can also be seen from Table 2 that Mehran's estimate of the upper bound, which is based on the same information requirements as Silber's, is 0.4087 (see also Mehran, 1975, p. 66). Clearly, Mehran's bounds are wider than other bounds that have more stringent information requirements as expected.

TABLE 1

*Computations pertaining to the proposed bounds**

		Observed points				Silber		Modified Silber	
i	$a_i(\$x10^3)$	p_i	$L(p_i)$	β_i	$\beta_i^* = a_i/\mu$	x_i	y_i	x_i	y_i
0	0	0.00000	0.00000	-	0.00000	0.0000	0.0000	0.0000	0.0000
1	1	0.04824	0.00323	0.06700	0.12351	0.0250	0.0000	0.0221	0.0000
2	2	0.13077	0.01815	0.18078	0.24702	0.0956	0.0098	0.0925	0.0087
3	3	0.20292	0.03994	0.30201	0.37054	0.1655	0.0264	0.1708	0.0280
4	4	0.27194	0.06925	0.42466	0.49405	0.2368	0.0522	0.2417	0.0543
5	5	0.33809	0.10550	0.54800	0.61756	0.3052	0.0854	0.3092	0.0877
6	6	0.41407	0.15618	0.66702	0.74107	0.3783	0.1301	0.3836	0.1336
7	7	0.49254	0.21813	0.78947	0.86458	0.4723	0.1986	0.4618	0.1915
8	10	0.70658	0.43763	1.02551	1.23512	0.6569	0.3763	0.6136	0.3228
9	15	0.89769	0.71857	1.47004	1.85268	0.8318	0.5923	0.8250	0.5839
10	∞	1.00000	1.00000	2.75076	∞	1.0000	0.9147	1.0000	0.9081
11	-	-	-	-	-	1.0000	1.0000	1.0000	1.0000

* Note that $\mu = 8096.4$

TABLE 2

Estimates of the bounds of the Gini index

Proposal	Lower bound	Upper bound
Gastwirth (1972)	0.3883	0.4083
Mehran (1975)	0.3883	0.4087
Fuller (1979)	0.3883	0.4083
Silber(1990)	0.3883	0.4061
Ogwang (2003)	0.3883	0.4083
Modified Silber	0.3883	0.4083

As suggested by a referee, knowledge of individual data should allow a better understanding of the behaviour of the different bounds of the Gini index. This issue can be addressed by examining whether the empirical estimate of the Gini index based on individual data lies within the bounds or by conducting a Monte Carlo study where individual data are simulated. We note that the actual value of the Gini index computed from the full sample of approximately 60,000 observations, from which the grouped data were constructed, reported by Gastwirth (1972, p. 310), is 0.4014 which lies within the estimated bounds in all cases. As expected, the empirical estimate of the Gini index based on individual data will always lie within the bounds. McDonald and Ransom (1981) conducted a Monte Carlo study to assess the reliability of Gastwirth's bounds and found that there is no guarantee that the estimated bounds will contain the population Gini index. Furthermore, increasing the number of income brackets reduces the bounds as well as the probability with which the estimated bounds include the population Gini index. Since Gastwirth's bounds are identical to those proposed by other researchers, including the present proposal, sampling variability issues raised by McDonald and Ransom are also relevant in these cases.

4. CONCLUDING REMARKS

In this note, we derived modified coordinates of the points of intersection of the tangents to the LC at the observed points, assuming that there is information on the limits of the income brackets and full or sparse information on mean incomes. We also showed that if the modified coordinates are incorporated into Silber's algorithm, the resulting estimate of the upper bound is identical to estimates of the upper bounds proposed by Gastwirth, Fuller, and Ogwang.

Department of Economics TOMSON OGWANG
Brock University
St. Catharines, Ontario, Canada

Economics Program BAOTAI WANG
University of Northern British Columbia
Prince George, BC, Canada

REFERENCES

F. COWELL, (1991), *Grouping bounds for inequality measures under alternative informational assumptions*, "Journal of Econometrics", 48, pp. 1-14.

M. FULLER, (1979), *The estimation of Gini coefficients from grouped data: upper and lower bounds*, "Economics Letters", 3, pp. 187-192.

J.L. GASTWIRTH, (1972), *The estimation of the Lorenz curve and Gini index*, "Review of Economics and Statistics", 54, pp. 306-316.

G.M. GIORGI, A. PALLINI, (1987), *About a general method for the lower and upper distribution-free bounds on Gini's concentration ratio from grouped data*, "Statistica", XLVII, pp. 171-184.

A.M. KRIEGER, (1983), *Bounding moments from grouped data and the importance of reporting group means*, "Sankhya Series B", 45, pp. 309-319.

J.B. MCDONALD, M.R. RANSOM, (1981), *An analysis of the bounds for the Gini coefficient*, "Journal of Econometrics", 17, pp. 177-188.

F. MEHRAN, (1975), *Bounds on the Gini index based on observed points of the Lorenz curve*, "Journal of the American Statistical Association", 70, pp. 64-66.

D. MURRAY, (1978), *Extreme values for Gini coefficients calculated from grouped data*, "Economics Letters", 1, pp. 389-393.

T. OGWANG, U.L.G. RAO, (1996), *A new functional form for approximating the Lorenz curve*, "Economics Letters", 52, pp. 21-29.

T. OGWANG, (2003), *Bounds of the Gini index using sparse information on mean incomes*, "Review of Income and Wealth", 49, pp. 415-423.

H.K. RYU, D.J. SLOTTJE, (1999), *Parametric approximations of the Lorenz curve*, in J. SILBER (Ed.), "Handbook on Income Inequality Measurement", Kluwer Academic Publishers, Boston, pp. 291-312.

J. SILBER, (1990), *On a new algorithm to derive bounds of Gini's concentration ratio from grouped observations*, "Statistica", L, pp. 215-220.

RIASSUNTO

Una modifica dell'algoritmo di Silber allo scopo di derivare confini sulla proporzione della concentrazione di Gini dalle osservazioni aggruppate

Fu Silber (1990) che inventò un algoritmo per derivare i confini del rapporto di concentrazione di Gini per dati raggruppati, il quale non prevede l'utilizzo di informazioni sui limiti degli intervalli di reddito, né dei redditi medi di gruppo, né dell'intera media del reddito. Nel caso del limite superiore, l'algoritmo di Silber prevede la determinazione delle coordinate dei punti d'intersezione delle tangenti alla curva di Lorenz (CL) ai punti osservati, i quali vengono poi usati coll'operatore **G**-matrice. In questa nota, sono derivate delle coordinate modificate dei punti d'intersezione delle tangenti alla CL ai punti osservati ipotizzando che ci saranno informazioni sui limiti degli intervalli di reddito e informazioni complete o parziali sulle medie dei redditi. Sarà peraltro dimostrato che se le coordinate modificate vengono incorporate nell'algoritmo di Silber, la stima risultante del limite superiore sarà uguale a stime di tale limite proposte da Gastwirth, Fuller e Ogwang.

SUMMARY

A modification of Silber's algorithm to derive bounds on Gini's concentration ratio from grouped observations

Silber (1990) devised an algorithm to derive the bounds of Gini's concentration ratio from grouped data, which does not require information on the limits of the income brackets, the group mean incomes, or the overall mean income. In the case of the upper bound, Silber's algorithm entails determining the coordinates of the points of intersection of the tangents to the Lorenz Curve (LC) at the observed points, which are then used in conjunction with the **G**-matrix operator. In this note we derive modified coordinates of the points of intersection of the tangents to the LC at the observed points assuming that there is information on the limits of the income brackets and full or sparse information on mean incomes. We also show that if the modified coordinates are incorporated into Silber's algorithm, the resulting estimate of the upper bound is identical to estimates of the upper bound proposed by Gastwirth, Fuller, and Ogwang.