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ESTIMATION AND TESTING PROCEDURES FOR THE RELIABILITY FUNCTIONS OF EXPONENTIATED DISTRIBUTIONS UNDER CENSORINGS

Ajit Chaturvedi
Department of Statistics, University of Delhi, Delhi, INDIA
Shantanu Vyas[1]
Department of Statistics, University of Delhi, Delhi, INDIA

1. Introduction

The reliability function $R(t)$ is defined as the probability of failure-free operation until time t. Thus, if the random variable (rv) X denotes lifetime of an item or system, then $R(t) = P(X > t)$. Another measure of reliability under stress-strength set-up is the probability $P = P(X > Y)$, which represents the reliability of item or system of random strength X subject to random stress Y. A lot of work has been done in the literature for the point estimation of $R(t)$ and 'P' under censoring and complete sample cases for individual distributions. For a brief review, one may refer to Pugh (1963), Basu (1964), Bartholomew (1957, 1963), Tong (1974, 1975), Johnson (1975), Kelley *et al.* (1976), Sathe and Shah (1981), Chao (1982), Constantine *et al.* (1986), Awad and Gharraf (1986), Tyagi and Bhattacharya (1989a,b), Chaturvedi and Rani (1997, 1998), Chaturvedi and Surinder (1999), Chaturvedi and Tomer (2002); Chaturvedi *et al.* (2002); Chaturvedi and Tomer (2003),Chaturvedi and Singh (2006, 2008) and others.

Various authors have considered inferential problems related to different exponentiated distributions.To cite a few, we refer to Mudholkar and Srivastava (1993); Mudholkar *et al.* (1995),Mudholkar and Hutson (1996), Jiang and Murthy (1999), Nassar and Eissa (2003, 2004), Pal *et al.* (2006, 2007), Gupta *et al.* (1998),Gupta and Kundu (1999, 2001a,b, 2002, 2003a,b), Gupta *et al.* (2002), Kundu *et al.* (2005), Kundu and Gupta (2005), Raqab and Ahsanullah (2001), Raqab (2002), Abdel-Hamid and Al-Hussaini (2009), AL-Hussaini (2010), AL-Hussaini and Hussein (2011), Shawky and Abu-Zinadah (2009),Kundu and Raqab (2005), Abdul-Moniem and Abdel-Hameed (2012), Tadikamalla (1980), Lai *et al.* (2003), Xie *et al.* (2002),Ljubo (1965) and others.

If $F(x)$ is the cumulative distribution function(cdf) of a positive random variable(rv) X, then for a parameter $\alpha(> 0)$,

$$G(x) = [F(x)]^{\alpha} \tag{1}$$

[1] Corresponding Author. E-mail: shanstatsdu@gmail.com

is also a cdf. Such distributions are referred to as exponentiated distributions denoting by $f(x)$, the probability distributions function(pdf) corresponding to $F(x)$,the pdf for the model (1) is

$$g(x;\alpha) = \alpha[F(x)]^{\alpha-1} f(x) \qquad (2)$$

The purpose of this paper is many-fold. For the family of exponentiated distributions (2), we consider estimation and testing procedures for 'α', R(t) and 'P' under type I and type II censoring. As far as estimation procedures are concerned, UMVUES and MLES are derived.
In Section 2 and Section 3, respectively, we provide point estimators under type II and type I censoring. In Section 4, we propose testing procedures.Finally in Section 5, we present numerical findings and in section 6, we propose some remarks and conclusions.

2. Point estimators under type II censoring

Let n items are put an a life test and failure times of first r units are observed. Let $X_{(1)} \le X_{(2)} \le X_{(3)} \le ... \le X_{(r)}, (0 < r \le n)$ be the lifetimes of first r units. Obviously, $(n-r)$ items survived until $X_{(r)}$. The joint pdf of n order statistics $X_{(1)} \le X_{(2)} \le X_{(3)} \le ... \le X_{(n)}$ is

$$g^*(x_{(1)}, x_{(2)}, ..., x_{(n)}; \alpha) = n!\alpha^n \prod_{i=1}^{n} f(x_{(i)}) F(x_{(i)})^{\alpha-1}$$

or alternatively, we can write

$$g^*(x_{(1)}, x_{(2)}, ..., x_{(n)}; \alpha) = n!\alpha^n \prod_{i=1}^{n} exp(-\alpha \left\{-logF(x_{(i)})\right\}) \left\{\frac{f(x_{(i)})}{F(x_{(i)})}\right\}. \qquad (3)$$

Let us make the transformation $y_{(i)} = -logF(x_{(i)})$. The Jacobian J of transformation is

$$|J| = \frac{F(x_{(i)})}{f(x_{(i)})}$$

From(3), the joint pdf of $Y_{(1)} \le Y_{(2)} \le Y_{(3)} \le ... \le Y_{(n)}$ is

$$h^*(y_{(1)}, y_{(2)}, ..., y_{(n)}; \alpha) = n!\alpha^n exp(-\alpha \sum_{i=1}^{n} y_{(i)}). \qquad (4)$$

The joint pdf of $Y_{(1)} \le Y_{(2)} \le Y_{(3)} \le ... \le Y_{(r)}$ is obtained by integrating out $Y_{(r+1)} \le Y_{(r+2)} \le ... \le Y_{(n)}$, which leads us to

$$h^{**}(y_{(1)}, y_{(2)}, ..., y_{(r)}; \alpha) = n(n-1)....(n-r+1)\alpha^r exp\left\{-\alpha(\sum_{i=1}^{r} y_{(i)} + (n-r)y_{(r)})\right\}. \qquad (5)$$

Since $F(x_i)$ is uniform over $(0,1)$,$-logF(x_i)$ follows exponential distribution with mean life $(1/\alpha$.
Lest us consider the tranformation $Z_i = (n-i+1)\left\{Y_{(i)} - Y_{(i-1)}\right\}, i = 1, 2, ..., r$

obviously, $\sum_{i=1}^{r} Z_i = S_r$, say,
where $S_r = \sum_{i=1}^{r} y_{(i)} + (n-r)y_{(r)}$.
Since $Z_i's$ are exponential random variables with mean life unity, from (5), the pdf of S_r is

$$f_1(s_r;\alpha) = \frac{\alpha^r}{\Gamma(r)} s_r^{r-1} exp(-\alpha s_r). \tag{6}$$

The following theorem provides UMVUES of powers of α.

THEOREM 1. *For $p \in (-\infty,\infty)(p \neq 0)$, the UMVUE of α^p is given by*

$$\widetilde{\alpha}_{II}^{p} = \begin{cases} \left\{\frac{\Gamma(r)}{\Gamma(r-p)}\right\} S_r^{-p} & (p < r) \\ \\ 0, & otherwise. \end{cases}$$

PROOF. From (6) and Fisher-Neymann factorization theorem[(see Rohtagi and Saleh, 2012,p.347)] S_r is sufficient for α. Moreover, since the distribution of S_r belongs to exponential family, it is also complete [see Rohtagi (1976,p.347)]. The theorem now follows from the fact that

$$E(S_r^{-p}) = \left\{\frac{\Gamma(r-p)}{\Gamma(p)}\right\}\alpha^p$$

In the following theorem we derive the UMVUE of R(t).

THEOREM 2. *The UMVUE of $R(t)$ is given by*

$$\widetilde{R}_{II}(t) = \begin{cases} 1 - \left[1 + \frac{logF(t)}{S_r}\right]^{r-1} & , -logF(t) < S_r \\ 0 & otherwise. \end{cases}$$

PROOF. We can write

$$\begin{aligned} R(t) &= 1 - [F(t)]^{\alpha} \\ &= 1 - exp[\alpha logF(t)] \\ &= 1 - \sum_{i=0}^{\infty} \frac{\{logF(t)\}^i}{i!}\alpha^i. \end{aligned}$$

Thus,

$$\widetilde{R}_{II}(t) = 1 - \sum_{i=0}^{\infty} \frac{\{logF(t)\}^i}{i!}\widetilde{\alpha}_{II}^{i}$$

Applying Theorem 1,

$$\begin{aligned} \widetilde{R}_{II}(t) &= 1 - \sum_{i=0}^{r-1} \frac{\{logF(t)\}^i}{i!}\widetilde{\alpha}_{II}^{i} \\ &= 1 - \sum_{i=0}^{r-1} \frac{\{logF(t)\}^i}{i!}\left\{\frac{\Gamma(r)}{\Gamma(r-i)}\right\} S_r^{-i} \\ &= 1 - \sum_{i=0}^{r-1} \binom{r-1}{i}\left\{\frac{logF(t)}{S_r}\right\}^i \end{aligned}$$

and the theorem follows.

The UMVUE of the pdf $g(x;\alpha)$ given at (2) is provided in the following corollary.

COROLLARY 3. *The UMVUE of $g(x;\alpha)$ at a specified point 'x' is*

$$\widetilde{g}_{II}(x;\alpha) = \begin{cases} (r-1)\left[1+\frac{logF(x)}{S_r}\right]^{r-2}\left\{\frac{f(x)}{S_rF(x)}\right\} & ,-logF(x) < S_r \\ 0, & otherwise. \end{cases}$$

PROOF. The expectation of $\int_t^\infty \widetilde{g}_{II}(x;\alpha)dx$ with respect to S_r is R(t). Thus,

$$\widetilde{R}_{II}(t) = \int_t^\infty \widetilde{g}_{II}(x;\alpha)dx$$

or,

$$\widetilde{g}_{II}(t;\alpha) = -\frac{d\widetilde{R}_{II}(t)}{dt} \tag{7}$$

The result now follows from Theorem 2 and (7).
Let X and Y be two independent random variables following the classes of distributions $g_1(x;\alpha_1)$ and $g_2(y;\alpha_2)$,respectively, where

$$g_1(x;\alpha_1) = \alpha_1[F_1(x)]^{\alpha_1-1}f_1(x)$$
$$g_2(y;\alpha_2) = \alpha_2[F_2(y)]^{\alpha_2-1}f_2(y)$$

Let n items on X and m items on Y are put on life tests and the truncation numbers for X and Y are r_1 and r_2, respectively. Denoting by

$$Y_1(i) = -logF_1(x_{(i)}), i = 1,2,...,r_1,$$

$$Y_2(j) = -logF_2(y_{(j)}), j = 1,2,...,r_2$$

$$S_{r_1} = \sum_{i=1}^{r_1} Y_{1(i)} + (n-r_1)Y_{1(r_1)}$$

and

$$T_{r_2} = \sum_{j=1}^{r_2} Y_{2(j)} + (m-r_2)Y_{2(r_2)}.$$

The UMVUE's of $g_1(x;\alpha_1)$ and $g_2(y;\alpha_2)$ are given respectively, by

$$\widetilde{g}_{1II}(x;\alpha_1) = \begin{cases} (r_1-1)\left[1+\frac{logF_1(x)}{S_{r_1}}\right]^{r_1-2}\left\{\frac{f_1(x)}{S_{r_1}F_1(x)}\right\}, & -logF_1(x) < S_{r_1} \\ 0, & otherwise \end{cases}$$

$$\widetilde{g}_{2II}(y;\alpha_2) = \begin{cases} (r_2-1)\left[1+\frac{logF_2(y)}{T_{r_2}}\right]^{r_2-2}\left\{\frac{f_2(y)}{T_{r_2}F_2(y)}\right\}, & -logF_2(y) < T_{r_2} \\ 0, & otherwise \end{cases}$$

In what follows we derive the UMVUE of 'P'. We denote by $F^{-1}(.)$, the inverse function of F(.).

THEOREM 4. *The UMVUE of 'P' is given by*

$$\tilde{P}_{II} = \begin{cases} (r_2-1)\int_o^{-T_{r_2}^{-1}logF_2(F_1^{-1}(e^{-S_{r_1}}))}(1-z)^{r_2-2} \\ \qquad \left[1-\left\{1+\frac{logF_1(F_2^{-1}(exp(-zT_{r_2})))}{S_{r_1}}\right\}^{r_1-1}\right]dz, \\ \qquad\qquad F_1^{-1}(e^{-S_{r_1}}) > F_2^{-1}(e^{-T_{r_2}}) \\ (r_2-1)\int_o^1(1-z)^{r_2-2}\left[1-\left\{1+\frac{logF_1(F_2^{-1}(e^{-zT_{r_2}}))}{S_{r_1}}\right\}^{r_1-1}\right]dz, \\ \qquad\qquad F_2^{-1}(e^{-T_{r_2}}) > F_1^{-1}(e^{-S_{r_1}}). \end{cases}$$

PROOF. From the arguments similar to those adopted in corollary 3,

$$\begin{aligned} \widetilde{P}_{II} &= \int_{y=0}^{\infty}\int_{x=y}^{\infty}\widetilde{g}_{1II}(x;\alpha_1)\widetilde{g}_{2II}(y;\alpha_2)dxdy \\ &= \int_{y=0}^{\infty}\widetilde{R}_{1II}(y)\left\{\frac{-d}{dy}\widetilde{R}_{2II}(y)\right\}dy \\ &= (r_2-1)\int_{y=M}^{\infty}\left[1-\left\{1+\frac{logF_1(x)}{S_{r_1}}\right\}^{r_1-1}\right] \\ &\qquad \left[\left\{1+\frac{logF_2(y)}{T_{r_2}}\right\}^{r_2-2}\left\{\frac{f_2(y)}{T_{r_2}F_2(y)}\right\}\right]dy \end{aligned} \tag{8}$$

where $M = max\left\{F_1^{-1}(exp(-S_{r_1})), F_2^{-1}(exp(-T_{r_2}))\right\}$
when $M = F_1^{-1}(exp(-S_{r_1}))$ putting $z = \frac{-logF_2(y)}{T_{r_2}}$ in (8),

$$\begin{aligned} \widetilde{P}_{II} =& (r_2-1)\int_0^{-T_{r_2}^{-1}logF_2(F_1^{-1}(exp(-S_{r_1})))}(1-z)^{r_2-2} \\ &\left[1-\left\{1+\frac{logF_1(F_2^{-1}(exp(-zT_{r_2})))}{S_{r_1}}\right\}^{r_1-1}\right]dz \end{aligned} \tag{9}$$

when $max\left\{F_1^{-1}(exp(-S_{r_1})), F_2^{-1}(exp(-T_{r_2}))\right\} = F_2^{-1}(exp(-T_{r_2})$, for the same transformation of variables,

$$\begin{aligned} \widetilde{P}_{II} = (r_2-1)\int_0^1(1-z)^{r_2-2} \\ \left[1-\left\{1+\frac{logF_1(F_2^{-1}(exp(-zT_{r_2})))}{S_{r_1}}\right\}^{r_1-1}\right]dz \end{aligned} \tag{10}$$

The theorem now follows on combining (9) and (10).

COROLLARY 5. *When X and Y belong to the same families of distributions*

(X and Y are independent random variables),

$$\widetilde{P}_{II} = \begin{cases} (r_2-1)\int_o^{\frac{S_{r_1}}{T_{r_2}}}(1-z)^{r_2-2}\left[1-\left\{1-\frac{T_{r_2}}{S_{r_1}}z\right\}^{r_1-1}\right]dz, \\ \qquad if \quad S_{r_1} < T_{r_2} \\ (r_2-1)\int_o^{1}(1-z)^{r_2-2}\left[1-\left\{1-\frac{T_{r_2}}{S_{r_1}}z\right\}^{r_1-1}\right]dz, \\ \qquad if \quad S_{r_1} > T_{r_2}. \end{cases}$$

In the following theorem, we derive the MLES of powers of α.

THEOREM 6. *For $p \in (-\infty, \infty)(p \neq 0)$,the MLE of α^p is given by*

$$\hat{\alpha}_{II}^p = \left(\frac{r}{S_r}\right)^p.$$

PROOF. Taking natural logarithm of the both sides of (5), differentiating it with respect to α, equating the differential coefficient equal to zero and solving for α, we get

$$\hat{\alpha}_{II} = \frac{r}{S_r}$$

The result now follows from the invariance property of MLES.

THEOREM 7. *The MLE of the Reliability function is*

$$\hat{R}_{II}(t) = 1 - [F(t)]^{\frac{r}{S_r}}.$$

PROOF. The result follows from the expression of R(t), Theorem 6 and the invariance property of the MLE.

COROLLARY 8. *The MLE of the pdf $g(x;\alpha)$ at a specified point 'x' is*

$$\hat{g}_{II}(x;\alpha) = \hat{\alpha}_{II}\,[F(x)]^{\hat{\alpha}_{II}-1} f(x).$$

PROOF. The proof is similar to that of Corollary 3.

THEOREM 9. *The MLE of 'P' is given by*

$$\hat{P}_{II} = \frac{r_2}{T_{r_2}}\int_1^0 \left\{1 - exp(logF_1(F_2^{-1}(z)))^{\frac{r_1}{S_{r_1}}}\right\} z^{\frac{r_2}{T_{r_2}}-1}dz.$$

PROOF.

$$\begin{aligned} \hat{P}_{II} &= \int_{y=0}^{\infty}\int_{x=y}^{\infty} \hat{g}_{1II}(x;\alpha_1)\hat{g}_{2II}(y;\alpha_2)dxdy \\ &= \int_{y=0}^{\infty} \hat{R}_{1II}(y)\left\{\frac{-d\hat{R}_{2II}(y)}{dy}\right\}dy \\ &= \int_{y=0}^{\infty}\left\{1 - exp\left[\frac{-r_1}{S_{r_1}}logF_1(y)\right]\right\}\left\{\frac{r_2}{T_{r_2}}[F_2(y)]^{\frac{r_2}{T_{r_2}}-1} f_2(y)\right\}dy \end{aligned}$$

The result now follows on substituting $F_2(y) = z$

COROLLARY 10. *When X and Y belong to same families of distributions(X and Y are independent random variables),*

$$\hat{P}_{II} = \frac{(r_1/S_{r_1})}{(r_1/S_{r_1} + r_2/T_{r_2})}.$$

3. POINT ESTIMATORS UNDER TYPE I CENSORING

Let $0 \le X_{(1)} \le X_{(2)} \le .... \le X_{(n)}$ be the failure times of n items under test from (2).Test begins at time 0 and operates till $X_{(1)} = x_{(1)}$, the first failure time. Failed item is replaced by a new one and system operates further till $X_{(2)} = x_{(2)}$, and so on.The experiment is terminated at time t_o.

THEOREM 11. *Let $N(t_o)$ be the number of items that failed before time t_o. Then, $N(t_o)$ follows the Poisson distribution.*

PROOF. For $Y'_{(i)}s, i = 1, 2, ..., n$ defined in section 2, lest us make the transformation $W_1 = Y_{(1)}, W_2 = Y_{(2)} - Y_{(1)}, ..., W_n = Y_{(n)} - Y_{(n-1)}$.
We have shown that $W'_i s$ are i.i.d. rv's having exponential distribution with mean α. By the definition of $N(t_o)$,

$$\begin{aligned}
&P[N(t_o) = r] \\
&\quad = P[X_{(r)} \le t_o] - P[X_{(r+1)} \le t_o] \\
&\quad = P[Y_{(r)} \le -logF(t_o)] - P[Y_{(r+1)} \le -logF(t_o)] \\
&\quad = P[n\alpha \sum_{i=1}^{r+1} W_i \ge -n\alpha logF(t_o)] - P[n\alpha \sum_{i=1}^{r} W_i \ge -n\alpha logF(t_o)] \\
&\quad = \frac{1}{\Gamma(r+1)} \int_{-n\alpha logF(t_o)}^{\infty} u^r e^{-u} du - \frac{1}{\Gamma(r)} \int_{-n\alpha logF(t_o)}^{\infty} u^{r-1} e^{-u} du
\end{aligned}$$

Hence,

$$P[N(t_o) = r] = \frac{exp\left\{-n\alpha logF(t_o)\right\}\left\{-n\alpha logF(t_o)\right\}^r}{r!} \tag{11}$$

and the theorem follows.

THEOREM 12. *For $p \in (0, \infty)$, the UMVUE of α^p is given by*

$$\widetilde{\alpha}_I^p = \begin{cases} \frac{r!}{(r-p)!}\left\{-nlogF(t_o)\right\}^{-p}, & if \quad p \le r \\ 0, & otherwise. \end{cases}$$

PROOF. It follows from (11) and factorization theorem that r is complete and sufficient for α. The theorem now follows from the result that

$$E\left[r(r-1)...(r-p+1)\right] = \left\{-n\alpha logF(t_o)\right\}^p$$

THEOREM 13. *The UMVUE of R(t) is given by*

$$\widetilde{R}_I(t) = \begin{cases} 1 - \left[1 - \frac{logF(t)}{nlogF(t_o)}\right]^r, & if \quad F(t) > F(t_o)^n \\ 0, & otherwise. \end{cases}$$

PROOF. Using theorem 8,

$$\widetilde{R}_I(t) = 1 - \sum_{i=0}^{r} \frac{\widetilde{\alpha}_I^i}{i!} \{logF(t)\}^i$$

$$= 1 - \sum_{i=0}^{r} (-1)^i \binom{r}{i} \left[\frac{logF(t)}{nlogF(t_o)}\right]^i$$

and the theorem follows.

COROLLARY 14. *The UMVUE of the sampled pdf at a specified point 'x' is*

$$\widetilde{g}_I(x;\alpha) = \begin{cases} \left[\frac{-rf(x)}{nlogF(t_o)F(x)}\right] \left\{1 - \frac{logF(x)}{nlogF(t_o)}\right\}^{r-1}, & if \quad F(x) > F(t_o)^n \\ 0, & otherwise. \end{cases}$$

PROOF. The result follows from Theorem 13 on using the techniques adopted in the proof of corollary 3.

As in section 2, let X and Y be two independent rv's following the classes of distributions $g_1(x;\alpha_1)$ and $g_2(y;\alpha_2)$, respectively. Let n items on X and m items on Y are put on life tests and r_1 and r_2 be the number of failures before time t_o and t_{oo}, respectively.It follows from corollary 14 that the UMVUE's of $g_1(x;\alpha_1)$ and $g_2(y;\alpha_2)$ at a specified point 'x' and 'y' respectively, are given by

$$\widetilde{g}_{1I}(x;\alpha_1) = \begin{cases} \left[\frac{-r_1 f_1(x)}{nlogF_1(t_o)F_1(x)}\right] \left\{1 - \frac{logF_1(x)}{nlogF_1(t_o)}\right\}^{r_1-1}, & if \quad F_1(x) > F_1(t_o)^n \\ 0, & otherwise. \end{cases}$$

and

$$\widetilde{g}_{2I}(y;\alpha_2) = \begin{cases} \left[\frac{-r_2 f_2(y)}{mlogF_2(t_{oo})F_2(y)}\right] \left\{1 - \frac{logF_2(y)}{mlogF_2(t_{oo})}\right\}^{r_2-1}, & if \quad F_2(y) > F_2(t_{oo})^m \\ 0, & otherwise. \end{cases}$$

THEOREM 15. *The UMVUE of 'P' is given by*

$$\widetilde{P}_I = \begin{cases} r_2 \int_o^{\frac{logF_2\left(F_1^{-1}(F_1(t_o))^n\right)}{mlogF_2(t_{oo})}} (1-z)^{r_2-1} \left[1 - \left\{1 - \frac{logF_1(F_2^{-1}(exp(mlogF_2(t_{oo})z)))}{nlogF_1(t_o)}\right\}^{r_1}\right] dz, \\ \qquad if \quad F_1^{-1}(F_1(t_o))^n > F_2^{-1}(F_2(t_{oo}))^m \\ r_2 \int_o^1 (1-z)^{r_2-1} \left[1 - \left\{1 - \frac{logF_1(F_2^{-1}(exp(mlogF_2(t_{oo})z)))}{nlogF_1(t_o)}\right\}^{r_1}\right] dz, \\ \qquad if \quad F_1^{-1}(F_1(t_o))^n < F_2^{-1}(F_2(t_{oo}))^m. \end{cases}$$

PROOF. We have

$$\widetilde{P}_I = \int_{y=0}^{\infty}\int_{x=y}^{\infty} \widetilde{g}_{1I}(x;\alpha_1)\widetilde{g}_{2I}(y;\alpha_2)dxdy$$
$$= \int_{y=0}^{\infty} \widetilde{R}_{1I}(y)\left\{\frac{-d}{dy}\widetilde{R}_{2I}(y)\right\}dy$$
$$= \int_{y=max\left\{F_1^{-1}(F_1(t_o))^n, F_2^{-1}(F_2(t_{oo}))^m\right\}}^{\infty} \left\{1-\left[1-\frac{logF_1(y)}{nlogF_1(t_o)}\right]^{r_1}\right\}$$
$$.\left\{\frac{f_2(y)}{-logF_2(t_{oo})F_2(y)}\left[1-\frac{logF_2(y)}{mlogF_2(t_{oo})}\right]^{r_2-1}\right\}dy$$

The theorem now follows on putting $z = \frac{logF_2(y)}{mlogF_2(t_{oo})}$

COROLLARY 16. *When X and Y belong to the same families of distributions(X and Y are independent random variables),*

$$\widetilde{P}_I = \begin{cases} r_2\int_o^{\frac{n}{m}}(1-z)^{r_2-1}\left[1-\left\{1-\frac{m}{n}z\right\}^{r_1}\right]dz, & if \quad m<n \\ r_2\int_o^{1}(1-z)^{r_2-1}\left[1-\left\{1-\frac{m}{n}z\right\}^{r_1}\right]dz, & if \quad m>n. \end{cases}$$

THEOREM 17. *For $p \in (-\infty,\infty)$ $(p \neq 0)$, the MLE of α^p is given by*

$$\hat{\alpha}_I^p = \left\{\frac{r}{-nlogF(t_o)}\right\}^p.$$

PROOF. Taking logritham of (11), differentiating with respect to α, equating the differential coefficient to zero and solving for α, we get

$$\hat{\alpha}_I = \frac{r}{-nlogF(t_o)}$$

The theorem now follows from the invariance property of the MLE.

THEOREM 18. *The MLE of the reliability function is given by*

$$\hat{R}_I(t) = 1-[F(t)]^{\frac{-r}{nlogF(t_o)}}.$$

COROLLARY 19. *The MLE's of the sampled pdf's $g_1(x;\alpha_1)$ and $g_2(y;\alpha_2)$, at specified points 'x' and 'y', respectively, are given by*

$$\hat{g}_{1I}(x;\alpha_1) = \left\{\frac{r_1}{-nlogF_1(t_o)}\right\}[F_1(x)]^{\left\{\frac{r_1}{-nlogF_1(t_o)}-1\right\}}f_1(x)$$

and

$$\hat{g}_{2I}(y;\alpha_2) = \left\{\frac{r_2}{-mlogF_2(t_{oo})}\right\}[F_2(y)]^{\left\{\frac{r_2}{-mlogF_2(t_{oo})}-1\right\}}f_2(y).$$

THEOREM 20. *The MLE of 'P' is given by*

$$\hat{P}_I = \left\{\frac{-r_2}{mlogF(t_{oo})}\right\}\int_0^1\left\{1-\left(F_1\left(F_2^{-1}(z)\right)\right)^{\frac{-r_1}{nlogF_1(t_o)}}\right\}z^{\frac{-r_2}{mlogF(t_{oo})}-1}dz.$$

Corollary 21. *When X and Y belong to same families of distributions(X and Y are independent random variables),*

$$\hat{P}_I = \frac{r_1/nlogF(t_o)}{r_1/nlogF(t_o) + r_2/mlogF(t_{oo})}.$$

4. Testing procedures for different statistical hypotheses

In this section, we develop the test procedure for testing statistical hypotheses for the parameter α and 'P'.
Suppose we want to test the hypothesis $H_o : \alpha = \alpha_o$ against the alternative $H_1 : \alpha \neq \alpha_o$ under type II censoring. From (5)

$$Sup_{\alpha_o} L(\alpha\,|\underline{x}) = n(n-1)....(n-r+1)\alpha_o^r exp(-\alpha_o S_r)$$

and

$$Sup_{\alpha} L(\alpha\,|\underline{x}) = n(n-1)....(n-r+1)\left(\frac{r}{S_r}\right)^r exp(-r)$$

The likelihood ratio is given by

$$\lambda(\underline{x}) = \left(\frac{S_r \alpha_o}{r}\right)^r exp(-\alpha_o S_r + r) \tag{12}$$

The first term on the right side of (12) is monotonically increasing in S_r, whereas, the second one is monotonically decreasing. Using the fact that $2\alpha_o S_r$ follows χ^2_{2r} and denoting by β- the probability of Type I error, the critical region is given by,

$$\{0 < S_r < k_o\} \bigcup \left\{\acute{k_o} < S_r < \infty\right\}$$

where,

$$k_o = \frac{1}{2\alpha_o}\chi^2_{2r}\left(1 - \frac{\beta}{2}\right)$$

and

$$\acute{k_o} = \frac{1}{2\alpha_o}\chi^2_{2r}\left(\frac{\beta}{2}\right)$$

For type I censoring, a similar procedure can be used to find the critical region. Denoting by r, a poisson rv with parameter $n\alpha logF(t_o)$.
The critical region is given by $\left\{r < k_1 \; or \; r > \acute{k_1}\right\}$, r follows $Poisson(n\alpha logF(t_o))$
Now suppose we want to test the null hypothesis $H_o : \alpha \leq \alpha_o$ against $H_1 : \alpha > \alpha_o$ under type II censoring. It is easy to see that the family of sampled pdf has monotonic likelihood in S_r. Thus, the uniformly most powerful critical region is given by

$$S_r \leq \frac{1}{2\alpha_o}\chi^2_{2r}(1 - \beta)$$

Under type I censoring, the critical region is $r \geq \acute{\acute{k_1}}$, where

$$P(r \geq \acute{\acute{k_1}}) = \beta$$

Now suppose we want to test the null hypothesis $H_o : P = P_o$ against $H_1 : P \neq P_o$ under type II censoring. It is easy to see that

$$P = \frac{\alpha_1}{\alpha_1 + \alpha_2}$$

For $k = \frac{P_o}{1-P_o}$, H_o is equivalent to $H_o : \alpha_1 = k\alpha_2$, so that $H_1 : \alpha_1 \neq k\alpha_2$. For a generic[2] constant K,

$$L(\alpha_1, \alpha_2 \left| \underline{x}, \underline{y} \right.) = K\alpha_1^{r_1}\alpha_2^{r_2} exp\left\{-(\alpha_1 S_{r_1} + \alpha_2 T_{r_2})\right\}$$

Under H_o,

$$\hat{\alpha}_{1II} = \frac{k(r_1 + r_2)}{kS_{r_1} + T_{r_2}}, \qquad \hat{\alpha}_{2II} = \frac{(r_1 + r_2)}{kS_{r_1} + T_{r_2}}$$

Thus,

$$Sup_{H_o} L(\alpha_1, \alpha_2 \left| \underline{x}, \underline{y} \right.) = \frac{K}{(S_{r_1} + T_{r_2}/k)^{r_1+r_2}} exp\left(-(r_1 + r_2)\right)$$

Over the entire parametric space

$$\Theta = \left\{(\alpha_1, \alpha_2) / \alpha_1, \alpha_2 > 0\right\},$$

$$Sup_{\Theta} L(\alpha_1, \alpha_2 \left| \underline{x}, \underline{y} \right.) = \frac{K}{S_{r_1}^{r_1} T_{r_2}^{r_2}} exp\left(-(r_1 + r_2)\right).$$

Denoting by $F_{2r_1,2r_2}(.)$, the F-statistic with $(2r_1, 2r_2)$ degrees of freedom, the critical region is given by,

$$\left\{\left(\frac{S_{r_1}}{T_{r_2}} < k_2\right) \cup \left(\frac{S_{r_1}}{T_{r_2}} > k_2'\right)\right\}$$

where,

$$k_2 = \frac{r_1}{kr_2} F_{2r_1,2r_2}\left(1 - \frac{\beta}{2}\right)$$

and

$$k_2' = \frac{r_1}{kr_2} F_{2r_1,2r_2}\left(\frac{\beta}{2}\right).$$

5. Numerical findings

5.1. Simulation Studies

5.1.1. Estimation of parameters and Reliability functions

In order to validate the theoretical findings devloped under Type-II censoring, we have generated (by using inverse cumulative density method) 100 observations $(Y_i's)$ using the transformation $y_i = -(1/\alpha)logG(x_i, \alpha)$ with $\alpha = 1.5$ and using

[2] By generic constant we represent here a group of normalizing constants which includes all constants arising at each step. This help us to get rid of writing different normalizing constants at each step.

the fact that distribution function $G(x_i, \alpha)$ follows $U(0,1)$. Here it is assumed that $X_i's$ represent the life-spans of 100 experimental units and distributed as Weibull distribution with shape parameter $\gamma = 1.25$ and scale parameter $\lambda = 10$. In particular, using the distribution function of Weibull distribution with $\gamma = 1.25$ $\lambda = 10$ we obtained the Reliability function **R (t) = 0.8599** for t =2.5 Hours.
In order to obtain MLE of parameter $Alpha(\alpha)$ i.e. $\hat{\alpha}$ and MLE and UMVUE of R(t) under Type II censoring, we Set r = 75 and obtained these estimates. Then we replicated this computation 1000 times and obtained the mean values of these estimates as follows: $\left[\hat{\alpha},\ \hat{R}(t),\ \tilde{R}(t)\right] = [1.5266,\ 0.8586,\ 0.8585]$
To estimate the measure of Stress-Strength Reliability function $P = P(X_1 > X_2)$, we generated two Sample of observations of sizes 110 and 120 ($Y_{ji}'s; j = 1,2, i = 110,120$) using the transformation $y_{ji} = -(1/\alpha_j) log G(x_j, \alpha_j); j = 1,2$ with $\alpha_1 = \alpha_2 = 1.5$ and using the fact that distribution function $G(x_j, \alpha_j)$ follows $U(0,1); j = 1,2$. Here it is assumed that X_j for $j = 1,2$ represent the life-spans of 110 and 120 experimental units on the life-test respectively and distributed as Weibull distribution with shape parameters $\gamma_1 = \gamma_2 = 1.25$ and scale parameters $\lambda_1 = \lambda_2 = 10$ respectively.
In order to obtain UMVUE of $P = P(X_1 > X_2)$ under Type II censoring, we set $r_1 = r_2 = 80$ and calculated $S_{r_1} = 54.3398$ and $T_{r_2} = 50.2478$. Using these values of S_{r_1} and T_{r_2} we obtained the UMVUE of Stress-Strength Reliability function. We repeated this process 1000 times and obtained the mean value of the UMVUE as $\tilde{P} = 0.4991$.
While obtaining MLE of $P = P(X_1 > X_2)$ under Type II censoring, we set $r_1 = r_2 = 80$ and calculated $S_{r_1} = 53.1585$ and $T_{r_2} = 52.4389$. Using these values of S_{r_1} and T_{r_2} we obtained the MLE of Stress-Strength Reliability function. We replicated this process 1000 times and obtained the mean value of the MLE as $\hat{P} = 0.4997$.
Assuming that both X_1 and X_2 belongs to the Weibull distribution with shape parameters $\gamma_1 = \gamma_2 = 1.25$ and scale parameters $\lambda_1 = \lambda_2 = 10$ respectively and taking $\alpha_1 = \alpha_2 = 1.5$, we calculated P = 0.5.
The average values of these estimates in vector form are given as follows:
$\left[\hat{P}, \tilde{P}\right]$=[MLE, UMVUE]=[0.4997, 0.4991]
In order to validate the theoretical findings devloped under Type-I censoring, we have generated 100 observations ($Y_i's$) using the transformation $y_i = -(1/\alpha) log G(x_i, \alpha)$ with $\alpha = 1.5$ and using the fact that distribution function $G(x_i, \alpha)$ follows $U(0,1)$. Here it is assumed that $X_i's$ represent the life-spans of 100 experimental units and distributed as Weibull distribution with shape parameter $\gamma = 1.25$ and scale parameter $\lambda = 10$.
In particular for $t = 2.5$ hours we obtained **R(t)=0.8599**. We set up the termination time $t_o = 1.5$ hours and obtained $r = 89$. Using this value of r we obtained MLE of α and MLE and UMVUE of $R(t)$. We replicated this process 1000 times and obtained the average values in vector form as follows:
$\left[\hat{\alpha},\ \hat{R}(t),\ \tilde{R}(t)\right] = [0.4769,\ 0.8643,\ 0.8920]$
To estimate the measure of Stress-Strength Reliability function $P = P(X_1 > X_2)$ we generated two Sample of observations of sizes 110 and 120 ($Y_{ji}'s; j = 1,2, i = 110,120$) using the transformation $y_{ji} = -(1/\alpha_j) log G(x_j, \alpha_j); j = 1,2$ with $\alpha_1 = \alpha_2 = 1.5$ and using the fact that distribution function $G(x_j, \alpha_j)$ follows

$U(0,1); j = 1,2$ Here it is assumed that X_j for $j = 1,2$ represent the life-spans of 110 and 120 experimental units on the life-test respectively and distributed as Weibull distribution with scale parameters $\gamma_1 = \gamma_2 = 1.25$and shape parameters $\lambda_1 = \lambda_2 = 10$ respectively.
In order to obtain UMVUE of $P = P(X_1 > X_2)$ under Type I censoring, we set termination times $t_o = t_{oo} = 1.5$ and obtained $r_1 = 99$ and $r_2 = 107$. Using these values of r_1 and r_2 we obtained the MLE and UMVUE of Stress-Strength Reliability function. We replicated this process 1000 times and obtained the mean values of the MLE and UMVUE in vector form as follows:
$\left[\hat{P}, \tilde{P}\right]$=[MLE, UMVUE]=[0.4997, 0.4998]

5.1.2. Testing of hypothesis

In order to check the authenticity of theory developed under Type-II censoring, we generated a sample of 100 observations $(Y_i' s)$ using the transformation $Y_i = -(1/\alpha_o) log G(x_i, \alpha_o)$ with $\alpha_o = 1.5$, and using the fact that distribution function $G(x_i, \alpha_o)$ follows $U(0,1)$.
Suppose we want to test the hypothesis $H_o : \alpha = \alpha_o$ against $H_1 : \alpha \neq \alpha_o$ under type II censoring scheme. From the above generated sample we calculated $S_r = 51.4347$. Using the Chi-square table, we obtained $k_o = 39.3282$ and $k_o^{'} = 61.9335$ at $\beta = 5$ percent level of significance. Hence we may accept $H_o : \alpha = \alpha_o$ at 5 percent level of significance. Now for testing composite hypothesis $H_o : \alpha \leq \alpha_o$ against the composite alternative $H_1 : \alpha > \alpha_o$, using the above generated sample we obtained $k_o^{''} = 40.8973$. Hence we may accept $H_o : \alpha \leq \alpha_o$ at 5 percent level of significance as $S_r = 51.4347$. For testing the null hypothesis $H_o : P = P_o$ against $H_1 : P \neq P_o$ under type II sampling scheme, we generated two Sample of observations of sizes 110 and 120 $(Y_{ji}' s; j = 1,2, i = 110, 120)$ using the transformation $y_{ji} = -(1/\alpha_j) log G(x_j, \alpha_j); j = 1,2$ with $\alpha_1 = \alpha_2 = 1.5$ and using the fact that distribution function $G(x_j, \alpha_j)$ follows $U(0,1); j = 1,2$
Using the above generated data we calculated $S_{r_1}/T_{r_2} = 1.0965$. Using the F-table, we obtained $k_2 = 0.7327$ and $k_2^{'} = 1.3648$ at $\beta = 5$ percent level of significance. Hence we may accept $H_o : P = P_o$ at 5 percent level of significance.
In order to check the authenticity of theory developed under Type-I censoring, we generated a sample of 100 observations $(Y_i' s)$ using the transformation $y_i = -(1/\alpha_o) log G(x_i, \alpha_o)$ with $\alpha_o = 1.25$ and using the fact that distribution function $G(x_i, \alpha_o)$ follows $U(0,1)$. Here it is assumed that $X_i' s$ represent the life-spans of 100 experimental units and distributed as Weibull distribution with shape parameter $\gamma = 1.75$ and scale parameter $\lambda = 8$.
Suppose we want to test the null hypothesis $H_o : \alpha = \alpha_o$ against alternative $H_1 : \alpha \neq \alpha_o$ under type I censoring scheme.Using the above generated data we obtained r = 97 for $t_o = 2.5$.
Now using the fact that r follows $Poisson(-n\alpha log F(t_o; a, \theta))$, from Poisson table we obtained $k_1 = 78$ and $k_1^{'} = 116$ at $\beta = 5$ percent level of significance. Hence we may accept $H_o : \alpha = \alpha_o$ at 5 percent level of significance.
For testing composite hypothesis $H_o : \alpha \leq \alpha_o$ against the composite alternative $H_1 : \alpha > \alpha_o$, we used the above generated sample and proceeding as before obtained $k_1^{''} = 113$. Hence we may accept $H_o : \alpha \leq \alpha_o$ at 5 percent level of

significance.

5.2. Real data

We consider the following data consisting of 100 observations on breaking stress of carbon fibers (in Gba).This data set is given by Nicholas and Padgett (2006).
3.70 2.74 2.73 2.50 3.60 3.11 3.27 2.87 1.47 3.11 4.42 2.41 3.19 3.22 1.69
3.28 3.09 1.87 3.15 4.90 3.75 2.43 2.95 2.97 3.39 2.96 2.53 2.67 2.93 3.22
3.39 2.81 4.20 3.33 2.55 3.31 3.31 2.85 2.56 3.56 3.15 2.35 2.55 2.59 2.38
2.81 2.77 2.17 2.83 1.92 1.41 3.68 2.97 1.36 0.98 2.76 4.91 3.68 1.84 1.59
3.19 1.57 0.81 5.56 1.73 1.59 2.00 1.22 1.12 1.71 2.17 1.17 5.08 2.48 1.18
3.51 2.17 1.69 1.25 4.38 1.84 0.39 3.68 2.48 0.85 1.61 2.79 4.70 2.03 1.80
1.57 1.08 2.03 1.61 2.12 1.89 2.88 2.82 2.05 3.65
Using the method of Maximum likelihood estimation, we fitted Exponentiated Weibull distribution(with shape parameters α and θ and scale parameter λ) to the above data. A quasi-Newton algorithm was used to solve the likelihood equations which gave following estimates of parameters:

$$[\hat{\alpha}, \hat{\gamma}, \hat{\lambda}] = [1.17262, 2.57902, 14.188]$$

Using the estimated value of α, we generated $Y_i's$ using the transformation $y_i = -(1/\hat{\alpha})logG(x_i)$.
Type-II censoring: We set $r = 75$ here and obtained $S_r = 69.893$. For p=2, we have $\alpha^p = 1.375$, MLE of α^p is given by $\hat{\alpha}^p_{II} = 1.1515$ and UMVUE is given by $\tilde{\alpha}^p_{II} = 1.1058$.
For reliability estimation, we set t=1.5 and obtained R(t)=0.8646.
The MLE and UMVUE of R(t) is further obtained and given by $\hat{R}_{II}(t) = 0.8396$ and $\tilde{R}_{II}(t) = 0.8393$.
Type-I censoring: We set up termination time as $t_o = 2.25$ hours here and obtained $r = 91$. For p=2, we have $\alpha^p = 1.375$, MLE of α^p is given by $\hat{\alpha}^p_I = 1.19404$ and UMVUE is given by $\tilde{\alpha}^p_I = 1.15497$.
For reliability estimation, we set t=1.5 and obtained R(t)=0.8646.
The MLE and UMVUE of R(t) is further obtained and given by $\hat{R}_I(t) = 0.8448$ and $\tilde{R}_{II}(t) = 0.8478$.
Below we present estimators of different powers of α and Reliability function R(t) under the two censoring schemes considered by us in tabulated form:

TABLE 1
Estimates of powers of α under Type-II censoring

p ↓	**r** →	75	80	85	90
2	$\hat{\alpha}^p_{II}$	1.1515	1.4072	1.4572	1.4248
	$\tilde{\alpha}^p_{II}$	1.1058	1.3548	1.4062	1.3776
4	$\hat{\alpha}^p_{II}$	1.3259	1.9801	2.1234	2.0300
	$\tilde{\alpha}^p_{II}$	1.1572	1.7432	1.8837	1.8130

TABLE 2
MLE's of R(t) obtained under Type-II censoring

t ↓	**R(t)**	**r** →	75	80	85	90
1.5	0.8646		0.8396	0.8677	0.8724	0.8694
2	0.7142	$\hat{R}_{II}(t)$	0.6800	0.7166	0.7230	0.7191
2.5	0.5281		0.4940	0.5294	0.5358	0.5319
3	0.3437		0.3169	0.3441	0.3491	0.3461

TABLE 3
UMVUE's of R(t) obtained under Type-II censoring

t ↓	**R(t)**	**r** →	75	80	85	90
1.5	0.8646		0.8392	0.8678	0.8725	0.8694
2	0.7142	$\tilde{R}_{II}(t)$	0.6800	0.7166	0.7230	0.7191
2.5	0.5281		0.4940	0.5294	0.5358	0.5319
3	0.3437		0.3169	0.3441	0.3491	0.3461

TABLE 4
Estimates of powers of α obtained under Type I censoring (p=1,2)

t_o	2.25	2.5	2.75
r	91	93	94
$\hat{\alpha}_I$	1.0927	1.4522	1.9410
$\tilde{\alpha}_I$	1.0807	1.4366	1.9204
$\hat{\alpha}_I^2$	1.194	2.1088	3.7675
$\tilde{\alpha}_I^2$	1.155	2.0413	3.6481

TABLE 5
MLE's of R(t) obtained under Type-I censoring

		t_o	2.25	2.5	2.75
$t\downarrow$	$R(t)\downarrow$	r	91	93	94
1.5	0.8646		0.8449	0.9159	0.9634
2	0.7142	$\hat{R}_I(t)$	0.6887	0.7879	0.8742
2.5	0.5281		0.5033	0.6054	0.7115
3	0.3437		0.3245	0.4063	0.5019

TABLE 6
UMVUE'S of R(t) obtained under Type-I censoring

		t_o	2.25	2.5	2.75
$t\downarrow$	$R(t)\downarrow$	r	91	93	94
4.5	0.03864		0.02515	0.01982	0.015184
5	0.01335	$\tilde{R}_I(t)$	0.00865	0.00680	0.005205
5.5	0.00384		0.00248	0.00680	0.001492

6. Remarks and conclusion

In the Real data analysis part, we see that estimates of Reliability function R(t) obtained under Type-I censoring are closer to the true value as compared to Type-II censoring.Now if we look at Table 1, we find that for larger values of r, the estimators of powers of α are closer to the true value. Similarly if we look at Table 2 and Table 3, we find that for larger values of r, the estimators of Reliability function R(t) are closer to the true value. Further, if we look at Tables 4,5 and 6, we find that estimates of powers of α and Reliability function R(t) are closer to the true value for small values of t_o as comapared to the large values.

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Summary

Exponentiated distributions are considered. Two measures of reliability are considered, $R(t) = P(X > t)$ and $P = P(X > Y)$. Point estimation and testing procedures are developed for different parametric functions under Type II and Type I censoring. Uniformly minimum variance unbiased estimators (UMVUES) and maximum likelihood estimators (MLES) are derived. A new technique of obtaining these estimators is introduced.

Keywords: Exponentiated distributions; Point estimation; Testing procedures; Type I and Type II censoring.