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TESTING THE VALIDITY OF THE EXPONENTIAL MODEL BASED ON TYPE II CENSORED DATA USING TRANSFORMED SAMPLE DATA

Hadi Alizadeh Noughabi [1]
Department of Statistics, University of Birjand, Birjand, Iran

1. Introduction

Alizadeh Noughabi and Arghami (2011a) were the first to propose a test for the exponential distribution using transformed data based on the complete samples. They transformed the original sample into a uniform sample over [0,1] and then used testing uniformity for this transformed sample. Next, Dhumal and Shirke (2012) suggested another test for uniformity of transformed observations and improved the exponentiality test using transformed data. Moreover, Alizadeh Noughabi and Arghami (2011b) proposed other transformations on the original sample and then proposed other tests for the exponential distribution. It is important to know that they showed these tests dont depend on the scale parameter of the exponential distribution and hence all these tests are exact. Also, to computing the test statistics based on transformed data, we dont need to estimate the scale parameter unlike the other tests such as KolmogorovSmirnov, Cramervon Mises, and Anderson-Darling.

Recently, again Alizadeh Noughabi and Arghami (2013) introduced a test for the normal distribution using transformed data based on the complete samples. Then, they compared this test with competing tests and showed that their test has a good performance.

In this article, we are going to construct a test for the exponential distribution using transformed data with Type-II censored data.

Goodness of fit tests has been widely used in practice and many researchers have proposed different tests. For example, see DAgostino and Stephens (1986), Huber-Carol et al. (2002), Thode (2002), Park and Park (2003), and Alizadeh Noughabi and Arghami (2012) for the complete samples and Balakrishnan et al. (2007), Lim and Park (2007), Lin et al. (2008), Habibi Rad et al. (2011) and Pakyari and Balakrishnan (2013) for the censored samples. It is noticeable that there are lots of tests for the complete samples but there are few tests for censored samples.

In the next section, we apply the popular classic goodness of fit tests. Therefore a summary of these tests are given as follows.
Let $X_1, \cdots, X_n$ be a random sample from a distribution F with density function

[1] Corresponding Author e-mail: alizadehhadi@birjand.ac.ir

f. Also, let $g(x;\theta)$ be a parametric family of distributions. We wish to test the hypothesis

$$H_0 : f(x) = g(x;\theta),$$

against the general alternative

$$H_1 : f(x) \neq g(x;\theta).$$

Cramer von Mises (1931) proposed a test statistic for the above hypotheses based on the empirical distribution function given by

$$CH = \frac{1}{12n} + \sum_{i=1}^{n} \left(\frac{2i-1}{2n} - p_{(i)} \right)^2,$$

where $p_{(i)} = G(x_{(i)};\hat{\theta})$ and $\hat{\theta}$ is an estimator of θ.
Kolmogorov-Smirnov (1933) based on maximum distance between the theoretical distribution and the empirical distribution suggested the following test statistic.

$$KS = \max\left\{D^+,\, D^-\right\},$$

where

$$D^+ = \max\left\{\frac{i}{n} - p_{(i)}\right\} \quad ; \quad D^- = \max\left\{p_{(i)} - \frac{i-1}{n}\right\}, \quad i = 1, ..., n.$$

Anderson-Darling (1954) used the weight $\psi(p) = [p(1-p)]^{-1}$ in the following integral

$$n \int_{-\infty}^{\infty} (F_n(x) - G(x))^2 \psi(G(x)) dG(x)$$

and then obtained a test statistic as

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left\{\log(p_{(i)}) + \log(1 - p_{(n-i+1)})\right\}.$$

In Section 2, some test statistics using transformed data for testing exponentiality based on Type-II censored data are introduced. In Section 3, by Monte Carlo simulations, the critical values of the proposed test are obtained and the power values of the proposed test are evaluated and compared with competing tests. We finally conclude that the tests using transformed data have a good performance in detecting non-exponentiality. In Section 4, the tests are employed to analyze real data sets in the literature. A brief conclusion in presented in Section 5.

2. The proposed tests

Given the first r order statistics of a random sample of size n, $X_{(1:n)}, ..., X_{(r:n)}$, from a population with unknown distribution F and probability function f over a non-negative support with mean θ. We are interested in a goodness of fit test for

$$H_0 : \; f(x) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right),$$

versus

$$H_1: \ f(x) \neq \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right),$$

where θ is unknown.
In order to obtain a test statistic, the following theorem is applied.

Theorem 1. *Let X_1 and X_2 be two observations under Type-II censored sample from a continuous distribution F. Then $U = \frac{X_1}{X_1+X_2}$ is distributed as $g(u)$ if and only if F is exponential, where*

$$g(u) = \begin{cases} 4\left(1-\frac{r}{n}\right)u + \frac{r}{n} & 0 \le u \le 0.5, \\ 4\left(\frac{r}{n}-1\right)(u-1) + \frac{r}{n} & 0.5 < u \le 1, \end{cases}$$

and r is the sample size of Type-II censored sample.

Remark 2. *When $r = n$, the above theorem reduce to the theorem presented by Alizadeh Noughabi and Arghami (2011a) based on the complete samples.*

Let $X_{(1:n)}, ..., X_{(r:n)}$ be a Type-II censored sample. First, we transform the sample data to

$$U_{ij} = \frac{X_{(i:n)}}{X_{(i:n)} + X_{(j:n)}} \ , \ i \neq j \ , \ i,j = 1, 2, ..., r.$$

By the above theorem, under the null hypothesis, each U_i has a distribution with density $g(u)$, and it seems to be appropriate to use the classic tests to test that the distribution of $U_i^{,}$s are $g(u)$ and thus the exponentiality of the distribution of $X_i^{,}$s. Therefore, summary of the proposed tests can be presented as

$$X_{(1:n)}, \ldots, X_{(r:n)} \to U_{ij} = \frac{X_{(i:n)}}{X_{(i:n)} + X_{(j:n)}} \ , \ i \neq j \ \Rightarrow \begin{cases} CH \\ KS \\ A^2 \end{cases}$$

where

$$CH = \frac{1}{12r'} + \sum_{i=1}^{r'} \left(\frac{2i-1}{2r'} - p_{(i:n')}\right)^2,$$

$$KS = \max\left\{D^+, D^-\right\},$$

$$D^+ = \max\left\{\frac{i}{r'} - p_{(i:n')}\right\} \quad ; \quad D^- = \max\left\{p_{(i:n')} - \frac{i-1}{r'}\right\}, \qquad i = 1, ..., r',$$

$$A^2 = -r' - \frac{1}{r'} \sum_{i=1}^{r'} (2i-1)\left\{\log(p_{(i:n')}) + \log(1 - p_{(r-i+1:n')})\right\},$$

and $n' = n(n-1)$, $r' = r(r-1)$, $p_{(i:n')} = G(u_{(i:n')})$, $i = 1, ..., r'$. Also, $G(u)$ is the distribution function corresponding to the density function g above, given by

$$G(u) = \begin{cases} 2\left(1-\frac{r}{n}\right)u^2 + \frac{r}{n}u & 0 \le u \le 0.5, \\ 1 + 2\left(\frac{r}{n}-1\right)(u-1)^2 + \frac{r}{n}(u-1) & 0.5 < u \le 1. \end{cases}$$

Large values of the test statistics indicate that our censored sample follows a non-exponential distribution.

Remark 3. *When $r = n$, our procedure reduce to the method introduced by Alizadeh Noughabi and Arghami (2011a) for testing exponentiality based on the complete samples.*

3. Simulation study

For different sample sizes and different values of r, Monte Carlo simulations with 30,000 replicates used to obtain the critical values of our procedure. Since the test statistics dont depend on the scale parameter, we obtain the critical values from Type-II censored samples of the exponential distribution with mean 1. These values are presented in Table 1.

We chose the competitor tests the same tests discussed in Park (2005). The test statistics of these tests are as follows.

Brain and Shapiro (1983) introduced two test statistics for testing exponentality based on Type-II censored data as

$$z = \left(\frac{12}{r-2}\right)^{\frac{1}{2}} \frac{\sum_{i=1}^{r-1}\left(i - \frac{r}{2}\right) Y_{i+1}}{\sum_{i=1}^{r-1} Y_{i+1}},$$

$$Z = z^2 + \left(\frac{5}{4(r+1)(r-2)(r-3)}\right)^{\frac{1}{2}} \times \frac{12\sum_{i=1}^{r-1}\left(i-\frac{r}{2}\right)^2 Y_{i+1} - r(r-2)\sum_{i=1}^{r-1} Y_{i+1}}{\sum_{i=1}^{r-1} Y_{i+1}},$$

where $Y_1 = nX_{(1:n)}$, and

$$Y_i = (n-i+1)\left(X_{(i:n)} - X_{(i-1:n)}\right), \ i = 2, ..., r.$$

They showed that their tests outperform other tests.
Samanta and Schwarz (1988) suggested the following test statistic.

$$W = \frac{\left(\sum_{i=1}^{r} Y_i\right)^2}{r\sum_{i=2}^{r+1}\sum_{j=2}^{r+1} \frac{\min(i,j)-1}{r-\min(i,j)+2} Y_{i-1}Y_{j-1}}.$$

Then they showed that this test has a good performance in compared to z and Z. Park (2005), based on Kullback-Leibler information, proposed the following test statistic for testing exponentiality with Type-II censored data.

$$T(n,m,r) = -H(n,m,r) + \frac{r}{n}\left(\log\left(\frac{1}{r}\left(\sum_{i=1}^{r} X_{(i:n)} + (n-r)X_{(r:n)}\right)\right) + 1\right),$$

where

$$H(n,m,r) = \frac{1}{n}\sum_{i=1}^{r} \log\left\{\frac{n}{2m}(X_{(i+m)} - X_{(i-m)})\right\} - \left(1 - \frac{r}{n}\right)\log\left(1 - \frac{r}{n}\right),$$

and the window size m is a positive integer $\leq r/2$, and $X_{(i)} = X_{(1)}$ if $i \leq 1$, $X_{(i)} = X_{(r)}$ if $i \geq 1$. He showed that his test in compared to the above tests is powerful against the alternatives with monotone increasing hazard functions.

TABLE 1
Critical points of the test statistics.

		CH		KS		A^2	
n	r	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$
10	5	0.3579	0.4690	0.2452	0.2732	3.1747	4.3112
	6	0.4138	0.5445	0.2146	0.2397	3.6317	4.8296
	7	0.4695	0.6390	0.1907	0.2163	4.0503	5.3521
	8	0.5313	0.7181	0.1724	0.1959	4.4779	6.0651
	9	0.5641	0.7742	0.1556	0.1771	4.8015	6.3920
20	10	0.6986	0.9246	0.1551	0.1732	6.3554	8.4829
	12	0.8176	1.0579	0.1365	0.1521	7.1971	9.2688
	14	0.9096	1.2750	0.1219	0.1397	7.9555	10.8311
	16	1.0316	1.4306	0.1110	0.1273	8.7424	11.8295
	18	1.1240	1.5221	0.1008	0.1153	9.6122	12.7565
30	15	1.0220	1.3917	0.1199	0.1370	9.3136	12.644
	18	1.1931	1.6310	0.1064	0.1211	10.6331	14.4550
	21	1.3625	1.8944	0.0956	0.1099	11.8650	15.8995
	24	1.5597	2.1718	0.0878	0.1014	13.3246	18.0105
	27	1.6544	2.3828	0.0793	0.0924	14.2343	19.6695
40	20	1.3579	1.8214	0.1014	0.1150	12.4552	16.7472
	24	1.5703	2.1616	0.0897	0.1023	13.9935	19.1044
	28	1.8197	2.5688	0.0814	0.0939	15.8597	21.4964
	32	2.0908	2.9822	0.0748	0.0870	17.6982	24.2804
	36	2.2637	3.1889	0.0682	0.0791	19.4433	26.1698
50	25	1.6814	2.3085	0.0888	0.1019	15.5352	21.1774
	30	1.9761	2.7462	0.0787	0.0911	17.5046	23.8932
	35	2.2931	3.1701	0.0717	0.08245	19.7519	26.1328
	40	2.6245	3.7617	0.0662	0.07752	22.3549	30.9224
	45	2.8820	4.0067	0.0607	0.07009	24.4596	32.9978

To facilitate comparisons of the power values of the present tests with the power values of the published tests, we chose the same alternatives presented in Park (2005). These alternatives according to the type of hazard function are as:

1. Monotone decreasing hazard: Gamma (shape parameter 0.5), Weibull (shape parameter 0.5, 0.8).

2. Monotone increasing hazard: Uniform, Gamma (shape parameter 1.5, 2), Weibull (shape parameter 2), Beta (shape parameters: 1 and 2, 2 and 1).

3. Non-monotone hazard: Log-normal (shape parameter: 0.6, 1.0, 1.2), Beta (shape parameters: 0.5 and 1).

The power values of the present tests and also the other tests for different values of n and r at significance level $\alpha = 0.10$, were computed by Monte Carlo simulations. The power results of the tests are summarized in Tables 2, 3 and 4.

It is evident from tables that there is not a uniform most powerful test against all alternatives. Therefore, we discuss on the power values of the tests in term of type of alternative. A uniform superiority of the proposed tests to all competitors against the alternatives with monotone decreasing hazard functions is clearly evident. For this type of alternatives, the proposed test A^2 has the most power and the Park's test T has the smallest power. For these alternatives, all proposed tests (CH, KS, A^2) have a high power and the difference power between these tests with the competitors are substantial.
For alternatives with increasing hazard functions the Park's test T has the most power. However, the difference power between the proposed tests and T is not substantial and when r increases the difference is decreases. Also, the proposed tests have better powers than z and Z tests.
Finally, for the alternative with a non-monotone hazard function, no single test uniformly dominates other tests and it is seen that sometimes the proposed tests are powerful and sometimes the other tests. For example, the test T has higher power than others against log-normal(0,1) and the proposed tests are powerful than the competitors against Beta(0.5,1).

4. Illustrative examples

Two real examples are analyzed below to show the behavior of the proposed tests when only a Type-II censored sample is available.

Example 4. *We apply the data presented by Balakrishnan and Chen (1999) which are from a life expectancy experiment. They considered twenty three ball bearings in the experiment and recorded the data corresponding to the millions of revolutions before failure for each of the bearings. The experiment was terminated once the twentieth ball failed. The data are*
17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.48, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.65, 68.88, 84.12, 93.12, 96.64, 105.12, 105.84.
The proposed procedures are employed for investigating that this dataset follows an exponential distribution. In this example, $n = 23$, $r = 20$ *and the values of*

TABLE 2
Power comparison at 10% significance level: monotone decreasing hazard alternatives.

			Tests						
alternative	n	r	z	Z	W	T	CH	KS	A^2
Gamma(0.5)	10	5	0.182	0.172	0.246	0.035	0.332	0.306	0.447
		6	0.204	0.178	0.268	0.044	0.374	0.346	0.484
		7	0.233	0.199	0.280	0.061	0.397	0.370	0.499
		8	0.254	0.210	0.291	0.026	0.416	0.393	0.520
		9	0.288	0.241	0.320	0.033	0.451	0.437	0.553
Weibull(0.5)	10	5	0.234	0.206	0.320	0.036	0.413	0.378	0.524
		6	0.294	0.240	0.381	0.067	0.475	0.440	0.576
		7	0.361	0.280	0.429	0.111	0.539	0.495	0.621
		8	0.433	0.340	0.478	0.061	0.597	0.558	0.674
		9	0.527	0.420	0.560	0.108	0.668	0.641	0.732
Weibull(0.8)	10	5	0.113	0.117	0.126	0.055	0.141	0.134	0.188
		6	0.124	0.117	0.137	0.054	0.146	0.135	0.197
		7	0.135	0.120	0.148	0.057	0.154	0.143	0.199
		8	0.145	0.138	0.157	0.043	0.148	0.138	0.201
		9	0.162	0.147	0.177	0.038	0.164	0.156	0.212
Gamma(0.5)	20	10	0.333	0.262	0.352	0.087	0.552	0.524	0.635
		12	0.379	0.291	0.376	0.139	0.573	0.548	0.657
		14	0.428	0.325	0.397	0.190	0.597	0.572	0.682
		16	0.470	0.349	0.426	0.151	0.639	0.618	0.722
		18	0.509	0.384	0.447	0.181	0.670	0.646	0.744
Weibull(0.5)	20	10	0.476	0.368	0.489	0.157	0.675	0.641	0.733
		12	0.564	0.429	0.550	0.266	0.734	0.702	0.785
		14	0.673	0.523	0.625	0.388	0.788	0.759	0.832
		16	0.758	0.610	0.701	0.398	0.840	0.816	0.873
		18	0.838	0.711	0.779	0.522	0.899	0.881	0.920
Weibull(0.8)	20	10	0.154	0.142	0.161	0.051	0.184	0.173	0.227
		12	0.156	0.141	0.159	0.056	0.188	0.178	0.233
		14	0.188	0.158	0.176	0.067	0.191	0.179	0.241
		16	0.211	0.176	0.201	0.056	0.199	0.187	0.247
		18	0.240	0.196	0.222	0.060	0.219	0.207	0.268

TABLE 3
Power comparison at 10% significance level: monotone increasing hazard alternatives.

			Tests						
alternative	n	r	z	Z	W	T	CH	KS	A^2
Uniform	10	5	0.107	0.107	0.119	0.164	0.130	0.133	0.103
		6	0.126	0.117	0.142	0.194	0.153	0.154	0.121
		7	0.153	0.133	0.172	0.228	0.191	0.193	0.148
		8	0.206	0.165	0.236	0.318	0.239	0.243	0.188
		9	0.306	0.228	0.356	0.448	0.309	0.315	0.243
Weibull(2)	10	5	0.134	0.117	0.320	0.457	0.339	0.336	0.176
		6	0.166	0.126	0.409	0.536	0.431	0.415	0.271
		7	0.195	0.144	0.469	0.573	0.533	0.510	0.389
		8	0.230	0.168	0.549	0.705	0.620	0.597	0.504
		9	0.295	0.211	0.647	0.773	0.702	0.689	0.592
Gamma(1.5)	10	5	0.102	0.101	0.133	0.211	0.144	0.144	0.072
		6	0.108	0.099	0.143	0.219	0.162	0.157	0.091
		7	0.106	0.096	0.147	0.221	0.180	0.173	0.115
		8	0.111	0.098	0.155	0.259	0.200	0.200	0.143
		9	0.121	0.104	0.169	0.275	0.222	0.218	0.156
Gamma(2)	10	5	0.109	0.096	0.203	0.319	0.227	0.220	0.104
		6	0.121	0.098	0.229	0.347	0.262	0.249	0.143
		7	0.127	0.103	0.254	0.367	0.319	0.304	0.214
		8	0.139	0.111	0.274	0.437	0.366	0.358	0.267
		9	0.159	0.121	0.313	0.478	0.408	0.400	0.306
Beta(1,2)	10	5	0.102	0.105	0.098	0.128	0.109	0.114	0.100
		6	0.104	0.101	0.107	0.141	0.120	0.121	0.101
		7	0.114	0.107	0.116	0.156	0.131	0.131	0.115
		8	0.120	0.115	0.123	0.184	0.147	0.152	0.121
		9	0.150	0.138	0.165	0.235	0.172	0.177	0.138
Beta(2,1)	10	5	0.178	0.139	0.454	0.575	0.446	0.443	0.255
		6	0.252	0.185	0.596	0.683	0.577	0.561	0.402
		7	0.348	0.250	0.741	0.786	0.709	0.684	0.569
		8	0.472	0.338	0.858	0.908	0.811	0.802	0.715
		9	0.636	0.475	0.947	0.970	0.900	0.892	0.824
Uniform	20	10	0.140	0.129	0.152	0.185	0.143	0.144	0.115
		12	0.179	0.147	0.205	0.241	0.181	0.183	0.146
		14	0.255	0.196	0.291	0.307	0.251	0.249	0.198
		16	0.394	0.277	0.447	0.466	0.337	0.336	0.271
		18	0.578	0.400	0.653	0.666	0.455	0.448	0.371
Weibull(2)	20	10	0.284	0.209	0.597	0.710	0.601	0.578	0.451
		12	0.355	0.254	0.699	0.783	0.739	0.710	0.624
		14	0.443	0.329	0.784	0.836	0.831	0.805	0.757
		16	0.557	0.420	0.867	0.925	0.908	0.891	0.863
		18	0.644	0.512	0.925	0.952	0.951	0.939	0.921
Gamma(1.5)	20	10	0.126	0.105	0.190	0.285	0.210	0.203	0.130
		12	0.139	0.105	0.206	0.304	0.246	0.236	0.172
		14	0.151	0.115	0.221	0.319	0.290	0.280	0.228
		16	0.168	0.124	0.240	0.372	0.348	0.337	0.289
		18	0.179	0.133	0.257	0.373	0.363	0.349	0.307
Gamma(2)	20	10	0.172	0.132	0.352	0.500	0.392	0.375	0.266
		12	0.198	0.142	0.390	0.534	0.484	0.464	0.375
		14	0.224	0.163	0.432	0.569	0.568	0.544	0.483
		16	0.270	0.190	0.474	0.662	0.649	0.625	0.582
		18	0.302	0.220	0.514	0.679	0.696	0.675	0.637
Beta(1,2)	20	10	0.106	0.107	0.107	0.132	0.114	0.117	0.101
		12	0.115	0.106	0.120	0.153	0.128	0.131	0.110
		14	0.136	0.121	0.143	0.178	0.152	0.155	0.131
		16	0.178	0.149	0.192	0.234	0.187	0.188	0.160
		18	0.246	0.189	0.271	0.306	0.230	0.228	0.191
Beta(2,1)	20	10	0.421	0.313	0.788	0.838	0.733	0.707	0.591
		12	0.575	0.426	0.907	0.919	0.862	0.842	0.773
		14	0.730	0.586	0.973	0.968	0.945	0.931	0.896
		16	0.876	0.745	0.997	0.996	0.984	0.979	0.965
		18	0.954	0.871	1.000	1.000	0.997	0.996	0.991

TABLE 4
Power comparison at 10% significance level: non-monotone hazard alternatives.

alternative	n	r	z	Z	W	T	CH	KS	A^2
Log-normal(0.6)	10	5	0.109	0.096	0.471	0.659	0.551	0.535	0.292
		6	0.131	0.103	0.521	0.710	0.643	0.617	0.428
		7	0.139	0.106	0.558	0.730	0.728	0.700	0.568
		8	0.143	0.110	0.562	0.826	0.776	0.760	0.667
		9	0.160	0.125	0.576	0.833	0.805	0.794	0.710
Log-normal(1.0)	10	5	0.095	0.091	0.149	0.255	0.172	0.168	0.070
		6	0.095	0.087	0.147	0.250	0.185	0.178	0.093
		7	0.103	0.092	0.145	0.246	0.191	0.179	0.113
		8	0.113	0.100	0.140	0.241	0.185	0.180	0.124
		9	0.133	0.113	0.158	0.227	0.166	0.163	0.110
Log-normal(1.2)	10	5	0.098	0.095	0.106	0.165	0.105	0.104	0.042
		6	0.109	0.097	0.110	0.153	0.105	0.101	0.050
		7	0.116	0.104	0.120	0.141	0.097	0.093	0.056
		8	0.136	0.125	0.130	0.120	0.090	0.090	0.058
		9	0.193	0.163	0.185	0.108	0.073	0.072	0.047
Beta(0.5,1)	10	5	0.156	0.158	0.213	0.037	0.315	0.290	0.428
		6	0.167	0.160	0.217	0.047	0.328	0.312	0.442
		7	0.163	0.163	0.184	0.057	0.321	0.310	0.444
		8	0.146	0.174	0.149	0.032	0.315	0.322	0.446
		9	0.135	0.187	0.122	0.047	0.308	0.328	0.438
Log-normal(0.6)	20	10	0.220	0.165	0.791	0.955	0.915	0.896	0.824
		12	0.239	0.172	0.808	0.968	0.965	0.953	0.931
		14	0.273	0.205	0.816	0.971	0.983	0.977	0.972
		16	0.308	0.224	0.821	0.989	0.991	0.989	0.988
		18	0.303	0.233	0.801	0.987	0.995	0.994	0.993
Log-normal(1.0)	20	10	0.109	0.094	0.225	0.425	0.318	0.301	0.204
		12	0.107	0.087	0.206	0.415	0.344	0.327	0.251
		14	0.109	0.087	0.182	0.391	0.354	0.335	0.287
		16	0.127	0.100	0.176	0.404	0.353	0.342	0.308
		18	0.144	0.120	0.173	0.346	0.297	0.286	0.264
Log-normal(1.2)	20	10	0.102	0.091	0.130	0.238	0.151	0.143	0.087
		12	0.107	0.094	0.124	0.221	0.150	0.144	0.102
		14	0.130	0.106	0.127	0.200	0.134	0.128	0.101
		16	0.174	0.139	0.161	0.202	0.117	0.114	0.095
		18	0.267	0.221	0.244	0.187	0.084	0.080	0.068
Beta(0.5,1)	20	10	0.276	0.228	0.290	0.067	0.505	0.487	0.601
		12	0.268	0.230	0.261	0.098	0.506	0.492	0.610
		14	0.258	0.232	0.220	0.121	0.490	0.484	0.605
		16	0.212	0.246	0.162	0.073	0.471	0.482	0.604
		18	0.147	0.268	0.094	0.107	0.444	0.469	0.586

the test statistics are

$$CH = 6.743,\ KS = 0.2051,\ A^2 = 40.481\,.$$

Example 5. *The following data were originally presented by Xia et al.(2009) and then were analyzed by Saraoglu et al. (2012) under progressively Type-II censoring. These data are breaking strengths of jute fiber for different gauge lengths. Here, we assume that only the 24/30 smallest breaking strengths for 20 mm gauge length were observed.*
36.75, 45.58, 48.01, 71.46, 83.55, 99.72, 113.85, 116.99, 119.86, 145.96, 166.49, 187.13, 187.85, 200.16, 244.53, 284.64, 350.70, 375.81, 419.02, 456.60, 547.44, 578.62, 581.60, 585.57.
In this example, we want to know this dataset can be modeled by an exponential distribution. In this case, $n = 30$*,* $r = 24$ *and the values of the proposed test statistics are*

$$CH = 0.4683,\ KS = 0.0586,\ A^2 = 5.657.$$

At the 5% significance level, the critical values for CH*,* KS *and* A^2 *are 2.118, 0.1001 and 17.613, respectively. Since the values of the test statistics are smaller than the critical points, the exponential hypothesis is not rejected at the significance level of 0.05. So, there is evidence that shows that the data can be modeled by using an exponential distribution. This result agrees with the conclusion reached by other tests mentioned above.*

5. Concluding remarks

In this paper, we have developed a new procedure for testing exponentiality based on Type-II censored data using transformed data. Then, based on the famous classic tests, three tests have been proposed. We have shown by Monte Carlo simulation study that the proposed tests outperform all other considered tests for alternatives with decreasing monotone hazard functions, which are used in many real-life applications. For other alternatives also the proposed tests have a considerable power against some distributions. Moreover, the data examples showed that the proposed tests are reasonable when used in real observations.

Here, exponentiality tests have been constructed based on Type-II censored data. For complete samples these tests reduce to the procedure introduced by Alizadeh Noughabi and Arghami (2011a). As we know, in reliability and life-testing experiments there are different censoring schemes and hence testing exponentiality for observations of other censoring schemes like the progressively Type-II and hybrid censored data is an important issue. Some authors developed tests for the progressively Type-II censoring such as Balakrishnan et al. (2004), Balakrishnan et al. (2007), Habibi Rad et al. (2011), and Pakyari and Balakrishnan (2012, 2013). Moreover, we know that there is not any goodness of fit test under hybrid censored data. Therefore, in future work, it would be interesting to derive a goodness of fit test for the observations obtained by the hybrid censoring scheme. We are currently working on the proposed tests here to extend for observations obtained under progressively censoring or hybrid censoring and hope to report these finding in future papers.

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References

H. Alizadeh Noughabi N. R. Arghami (2011a). *Testing exponentiality using transformed data.* Journal of Statistical Computation and Simulation, 81, 511–516.

H. Alizadeh Noughabi N. R. Arghami (2011b). *Testing exponentiality based on characterizations of the exponential distribution.* Journal of Statistical Computation and Simulation, 81, 1641–1651.

H. Alizadeh Noughabi N. R. Arghami (2012). *General treatment of goodness-of-fit tests based on Kullback-Leibler information.* Journal of Statistical Computation and Simulation, 83, 1556–1569.

H. Alizadeh Noughabi N. R. Arghami (2013). *Testing normality using transformed data.* Communication in Statistics: Theory and Methods, 42, 3065–3075.

H. Alizadeh Noughabi *A goodness of fit test for Laplace distribution against symmetric alternatives, using transformed data.* (Submitted).

T. W. Anderson, D. A. Darling (1954). *A test of goodness of fit.* Journal of American Statistical Association, 49, 765–769.

N. Balakrishnan, W. Chen (1999). *Handbook of tables for order statistics from Lognormal distributions with applications.* Springer.

N. Balakrishnan, A. Habibi Rad, N. R. Arghami (2007). *Testing exponentiality based on KullbackLeibler information with progressively type-II censored data.* IEEE Transactions on Reliability, 56, 301–307.

N. Balakrishnan, H.K.T. Ng, N. Kannan (2004). *Goodness-of-fit tests based on spacings for progressively Type-II censored data from a general location-scale distribution.* IEEE Transactions on Reliability, 53, 349–356.

C. W. Brain, S. S. Shapiro (1983). *A regression test for exponentiality: censored, complete samples.* Technometrics, 25, 69–76.

R .B. D'Agostino, M. A. Stephens (1986). *Goodness-of-Fit Techniques.* New York, Marcel Dekker.

B. R. Dhumal, D. T. Shirke (2012). *A modified test for testing exponentiality using transformed data.* Journal of Statistical Computation and Simulation, 84, 397–403.

A. Habibi Rad, F. Yousefzadeh, N. Balakrishnan (2011). *Goodness-of-fit test based on KullbackLeibler information for progressively Type-II censored data.* IEEE Transactions on Reliability, 60, 570–579.

C. Huber-Carol, N. Balakrishnan, M. S. Nikulin, M. Mesbah (2002). *Goodness-of-fit tests and model validity*. Boston, Basel, Berlin: Birkhuser.

A. N. Kolmogorov (1933). *Sulla Determinazione Empirica di une legge di Distribuzione*. Giornale dellIntituto Italiano degli Attuari, 4, 83–91.

J. Lim, S. Park (2007). *Censored Kullback-Leibler information and goodness-of-fit test with type II censored data*. Journal of Applied Statistics, 34, 1051–1064.

C-T. Lin, Y-L. Huang, N. Balakrishnan (2008). *A new method for goodness-of-fit testing based on Type-II right censored samples*. IEEE Transactions on Reliability, 57, 633–642.

R. Pakyari, N. Balakrishnan (2012). *A general purpose approximate goodness-of-fit test for progressively Type-II censored data*. IEEE Transactions on Reliability, 61, 238–244.

R. Pakyari, N. Balakrishnan (2013). *Goodness-of-fit tests for progressively Type-II censored data from location-scale distributions*. Journal of Statistical Computation and Simulation, 83, 167–178.

S. Park (2005). *Testing exponentiality based on the Kullback-Leibler information*. IEEE Transactions on Reliability, 54, 22–26.

S. Park, D. Park (2003). *Correcting moments for goodness of fit tests based on two entropy estimates*. Journal of Statistical Computation and Simulation, 73, 685–694.

M. Samanta, C. J. Schwarz (1998). *The Shapiro-Wilk test for exponentiality based on censored data*. Journal of American Statistical Association, 83, 528–531.

C. von Mises (1931). *Wahrscheinlichkeitsrechnung und ihre Anwendung in der Statistik und theoretischen Physik*. Deuticke, Leipzig and Vienna.

Summary

In this article, we propose a new approach to testing exponentiality for Type-II censored samples. Our method converts the original observations into transformed data which follow from a special distribution. It is shown that the proposed tests do not depend on the scale parameter of the exponential distribution and hence the tests are exact. Power performance of the proposed tests is compared with the existing tests against various alternatives. The results are impressive and the proposed tests have higher power than the competing tests. Finally, two real data from reliability literature as illustrate examples are presented.

Keywords: Testing exponentiality, Type-II censoring, Power study, Monte Carlo simulation.