1. Introduction

Successive sampling often applied to gather information in applied sciences and socio-economic researches where characters are liable to change over the time. Government agencies such as National Bureau of Statistics and other research based institutions collect information on regular basis to estimate important population parameters and to find the patterns of variations in these parameters over the period of time.

The theory of successive sampling appears to have started with the work of Jessen (1942). He utilized entire information collected on the previous occasion in order to produce the reliable estimates on current occasion. This theory was further extended by Patterson (1950), Rao and Graham (1964), Gupta (1979), Das (1982) and Chaturvedi and Tripathi (1983), among others. Sen (1971) developed estimators of the population mean on the current occasion using information on two auxiliary variables which were readily available on previous occasion. Sen (1972, 1973) extended his work for several auxiliary variables. Singh et al. (1991), and Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two occasion successive sampling. Singh (2001) extended his work for h-occasion successive sampling.

In many practical situations, information on an auxiliary variable may be readily available on the first as well as on the second occasion, for example, tonnage (or seat capacity) of each vehicle or ship may be known in transportation survey, many other examples may be cited where the information on auxiliary variables are readily available on both the occasion in two-occasion successive sampling. Utilizing the auxiliary information on both the occasions, Feng and Zou (1997), Biradar and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007), Singh and Priyanka (2008, 2010), Singh and Karna (2009), Singh and Vishwakarma

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(2009), Singh and Priyanka (2010), Singh et al. (2011), Singh and Prasad (2013),
Singh and Homa (2013) have proposed varieties of estimators of population mean
on current (second) occasions in two occasion successive sampling.

Singh and Shukla (1987) proposed an one parameter family of factor type ratio
estimators of population mean in single occasion sample survey and presented its
nice behaviours. The main attraction of the proposed class of estimators was, it
produces the precise estimate and at the same time controls the bias as well. This
Type of the behaviour was not found in any other class of estimators. Motivated
with these points, we wish to extend the possible aspects of one parameter family
of factor type ratio estimators in two occasion successive sampling. Hence, in
this paper we propose two classes of estimators of current population mean in
two-occasion successive sampling under the assumption that the information on a
stable auxiliary variable is readily available on both occasions. Properties of the
proposed classes of estimators have been studied and suitable recommendations
are made.

2. FORMULATION OF THE CLASSES OF ESTIMATORS

Let \( U = (U_1, U_2, ..., U_N) \) be the finite population of \( N \) units, which has been sam-
pled over two occasions. The character under study be denoted by \( x(y) \) on the
first (second) occasion respectively. It is assumed that the information on an aux-
iliary variable \( z \) (stable over occasion) whose population mean known and closely
related to \( x \) and \( y \) are available on first (second) occasion respectively. Let a sim-
ple random sample (without replacement) of size \( n \) is drawn on the first occasion.
A random sub sample \( m \) of \( m = n\lambda \) units is retained (matched) for its use on
the second occasion. While a fresh simple random sample (without replacement)
of size \( u \) of \( u = (n - m) = n\mu \) is drawn on the second occasion from the entire
population so that the sample size on the current (second) occasion is also \( n \). Here
\( \lambda \) and \( \mu \) (\( \lambda + \mu = 1 \)) are the fractions of matched and fresh samples respectively
at the current (second) occasion. The values of \( \lambda \) or \( \mu \) would be choosen optimally.
The following notations have been considered for the further use:

- \( \bar{X}, \bar{Y} \): The population means of the study variables \( x(y) \) on the first (second)
occaision respectively.
- \( \bar{Z} \): The population mean of the auxiliary variable \( z \).
- \( \bar{x}_n, \bar{x}_m, \bar{y}_n, \bar{y}_m, \bar{z}_n, \bar{z}_m \): The sample means of the respective variables based
on the sample sizes shown in suffices.
- \( \rho_{yx}, \rho_{yz}, \rho_{xz} \): The population correlation coefficients between the variables
shown in suffices.
- \( S_x^2 = (N - 1)^{-1} \sum_{i=1}^{N}(x_i - \bar{X})^2 \): The Population variance of the variable \( x \).
- \( S_y^2, S_z^2 \): The population variances of the variables \( y \) and \( z \) respectively.
- \( \beta_{yx} \): The population regression coefficient of \( y \) on \( x \).
- \( C_x, C_y, C_z \): The coefficient of variations of the variables shown in suffices.

To estimate the population mean \( \bar{Y} \) on the current (second) occasion, two non
overlapping samples of size \( u \) and \( m \) are available. Therefore, we suggest some
relevant estimators based on these two samples. Knowing the attractive behaviours
of one parameter family of factor type ratio estimators of Singh and Shukla (1987),
we considered this estimator for estimation of population mean $\bar{Y}$ on current occasion based on fresh sample of size $u$.

$$T_u(d) = \bar{y}_u \left[ \frac{(A + C)\bar{Z} + fB\bar{z}_u}{(A + fB)\bar{Z} + C\bar{z}_u} \right]$$  \hspace{1cm} (1)

where $A = (d - 1)(d - 2)$, $B = (d - 1)(d - 4)$, $C = (d - 2)(d - 3)(d - 4)$, $f = \frac{n}{N}$ and $d$ is non negative constant identified to minimize the mean square error of the class of estimator $T_u(d)$.

**Remark 1.** The classes of estimators $T_u(d)$ reduce to following form of estimators for different value of $d$.

<table>
<thead>
<tr>
<th>Value of $d$</th>
<th>Estimator</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 1$</td>
<td>$T_u(1) = \bar{y}_u \left( \frac{\bar{Z}}{\bar{z}_n} \right)$</td>
<td>Ratio Estimator</td>
</tr>
<tr>
<td>$d = 2$</td>
<td>$T_u(2) = \bar{y}_u \left( \frac{\bar{Z}}{\bar{Z}} \right)$</td>
<td>Product Estimator</td>
</tr>
<tr>
<td>$d = 3$</td>
<td>$T_u(3) = \bar{y}_u \left( \frac{\bar{z}_n}{\bar{Z}} \right)$</td>
<td>Dual to Ratio Estimator</td>
</tr>
<tr>
<td>$d = 4$</td>
<td>$T_u(4) = \bar{y}_u$</td>
<td>Sample Mean</td>
</tr>
<tr>
<td>$d \to \infty$</td>
<td>$T_u(d) = T_u(1)$</td>
<td>Ratio Estimator</td>
</tr>
</tbody>
</table>

where

$$\bar{z}_n = \bar{y}_u \left[ \frac{N\bar{Z} - n\bar{z}_u}{(N - n)\bar{Z}} \right]$$

Again we considered two more estimators $T_{jm}(j = 1, 2)$ proposed by Chand (1975) and Kiregyera (1980) for the estimation of current population mean $\bar{Y}$ based on the matched sample of size $m$ and presented as

$$T_{1m} = \frac{\bar{y}_m}{\bar{x}_m} \left( \frac{\bar{x}_n}{\bar{z}_n} \bar{Z} \right)$$  \hspace{1cm} (2)

and

$$T_{2m} = \frac{\bar{y}_m}{\bar{x}_m} \left[ \bar{x}_n + b_{xz}^{(n)}(\bar{Z} - \bar{z}_n) \right]$$  \hspace{1cm} (3)

where $b_{xz}^{(n)}$ is the sample regression coefficient between the variable shown in suffix. As per the philosophy of successive sampling, the class of estimators $T_u$ is suitable to estimate the population mean on each occasion, while the estimators $T_{jm}(j = 1, 2)$ are more appropriate for estimating the change over two occasions. To address both the problems simultaneously, a suitable combination of $T_u$ and $T_{jm}$ are required.

Motivated with the above arguments the final estimators of current population mean $\bar{Y}$ is consider convex linear combinations of $T_u$ and $T_{jm}$ and presented as

$$T_j(d, \varphi_j) = \varphi_j T_u(d) + (1 - \varphi_j)T_{jm} ; \hspace{0.5cm} (j = 1, 2)$$  \hspace{1cm} (4)

where $\varphi_j (0 \leq \varphi_j \leq 1)$ is an unknown constant (scalar) to be determined under certain criterion.
3. Properties of the Proposed Classes of Estimators

3.1. Bias and Mean Square Error

Since the estimator $T_u(d)$ and $T_{jm}$ $(j = 1, 2; d > 0)$ are one-parameter family of factor-type ratio estimators and chain-type ratio, and chain-type ratio to regression estimators respectively and they are biased estimators of the population mean $\bar{Y}$. Therefore, the resulting classes of estimators are also biased estimators of $Y$, Singh and Shukla (1987) and Cochran (1977). The bias $B(.)$ and mean square errors $M(.)$ of the estimators $T_j(d, \varphi_j)$ are derived under large sample assumption and up to the first order of approximations using the following transformations:

$$y_u = (1 + e_0)\bar{Y}, \quad y_m = (1 + e_1)\bar{Y}, \quad \bar{x}_n = (1 + e_2)\bar{X}, \quad \bar{x}_m = (1 + e_3)\bar{X},$$
$$z_u = (1 + e_4)\bar{Z}, \quad \bar{z}_n = (1 + e_5)\bar{Z}, \quad s_{xz}(u) = (1 + e_6)S_{xz}, \quad s^2_Z(n) = (1 + e_7)S^2_Z$$

such that $E(e_k) = 0$ and $|e_k| \leq 1 \; \forall k = 1, 2, \ldots, 5$.

Under the above transformations, the estimators $T_u(d)$ and $T_{jm}$ $(j = 1, 2; d > 0)$ take the following forms:

$$T_u(d) = \bar{Y}(1 + e_0) \left[ \frac{(A + C)\bar{Z} + fB\bar{Z}(1 + e_4)}{(A + fB)\bar{Z} + C\bar{Z}(1 + e_4)} \right]$$

where $\theta_1(d) = \frac{fB}{A + fB + C}$ and $\theta_2(d) = \frac{C}{A + fB + C}$

$$T_{1m} = \bar{Y}(1 + e_1) \left[ \frac{\bar{X}(1 + e_2)}{\bar{Z}(1 + e_3)} \right]$$

$$T_{2m} = \bar{Y}(1 + e_1)(1 + e_3)^{-1}(1 + e_2)(1 + e_5)^{-1}$$

Thus we have the following theorems:

**Theorem 2.** Bias of the estimators $T_j(d)$ $(j = 1, 2; d > 0)$ to the first order of approximations is obtained as

$$B(T_j(d)) = \varphi_j B(T_u(d)) + (1 - \varphi_j)B(T_{jm}) ; \quad (j = 1, 2)$$

where

$$B(T_u(d)) = \bar{Y} \left( 1 - \frac{1}{N} \right) (\theta_2 - \theta_1)(\theta_2C^2_Z - \rho_{yz}C_yC_z)$$

$$B(T_{1m}) = \bar{Y} \left[ \frac{1}{N} \left( \frac{1}{m} - \frac{1}{n} \right) (C^2_x - \rho_{yz}C_yC_z) \right]$$

$$B(T_{2m}) = \bar{Y} \left[ \frac{1}{N} \left( \frac{1}{m} - \frac{1}{n} \right) (C^2_x - \rho_{yz}C_yC_z) \right]$$
\[ B(T_{2m}) = \bar{Y} \left( \frac{1}{m} - \frac{1}{n} \right) (C_x^2 - \rho_{yz} C_y C_z) + \left( \frac{1}{n} - \frac{1}{N} \right) (\rho_{xz} C_z^2 - \rho_{yz} \rho_{xz} C_y C_z) \]  

(11)

**Proof.** The bias of the estimators \( T_j(d) \) are given by

\[
B(T_j(d)) = E[T_j(d) - \bar{Y}] = \varphi_j E(T_u(d) - \bar{Y}) + (1 - \varphi_j) E(T_{jm} - \bar{y})
\]

(12)

where \( B(T_u(d)) = E[T_u(d) - \bar{Y}] \) and \( B(T_{jm}) = E[T_{jm} - \bar{Y}] \)

To derive the \( B(T_u(d)) \), we proceed as follows:

\[
E[T_u(d) - \bar{Y}] = E[\bar{Y}(1 + e_0) (1 + \theta_1 e_4)(1 + \theta_2 e_4)^{-1}]
\]

(13)

Now we expand the right hand side of equation (13) binomially, taking expectations and retaining the terms up-to the first order of approximations, we have the expression of the bias of the estimator \( T_u(d) \) as given in equation (9).

Similarly, the bias of the estimators \( T_{jm} \); \( (j=1, 2) \) are written as

\[
E[T_{1m} - \bar{Y}] = E[\bar{Y}(1 + e_1)(1 + e_3)^{-1}(1 + e_3)^{-1} - \bar{Y}]
\]

(14)

\[
E[T_{2m} - \bar{Y}] = E[\bar{Y}(1 + e_1)(1 + e_3)^{-1}(1 + e_2) - e_5 (1 + e_0)(1 + e_7)^{-1}(1 + e_1)(1 + e_3)^{-1} \frac{\bar{Y} \beta_{xz}}{X} - \bar{Y}]
\]

(15)

Expanding the right hand side of equation (14) and (15) binomially, taking expectations retaining the terms up-to the first order of approximations, we have the expressions of the bias of the estimators \( T_{1m} \) and \( T_{2m} \) as shown in equations (10) and (11) respectively.

**Theorem 3. mean square error of the estimators** \( T_j(d) \) \( (j=1, 2) \) **to the first degree of approximation are obtained as**

\[
M(T_j(d)) = \varphi_j^2 M(T_u(d))_{min} + (1 - \varphi_j)^2 M(T_{jm}); \quad (j = 1, 2)
\]

(16)

where

\[
M(T_u(d))_{min} = \left( \frac{1}{u} - \frac{1}{N} \right) (1 - \rho_{yz}^2)
\]

(17)

\[
M(T_{1m}) = S_y^2 \left( \frac{1}{m} (2 - 2 \rho_{yz}) + \frac{1}{n} (2 \rho_{yz} - 2 \rho_{yz}) - \frac{1}{N} (2 - 2 \rho_{yz}) \right)
\]

(18)

\[
M(T_{2m}) = S_y^2 \left( \frac{1}{m} (2 - 2 \rho_{yz}) + \frac{1}{n} (2 \rho_{yz} - 1 - \rho_{yz}^2) - \frac{1}{N} (1 - \rho_{yz}^2) \right)
\]

(19)
Proof. Since the estimators $T_u(d)$ and $T_{jm}$ $(j=1, 2)$ are based on two different non overlapping samples, the covariance terms are of order $O(N^{-1})$, therefore, they are ignored for large population size. It is obvious that mean square errors of the estimators $T_j(d)$ $(j=1, 2)$ are given by

$$M(T_j(d)) = E[T_j(d) - \bar{Y}]^2 = E[\varphi_j(T_u(d) - \bar{Y}) + (1 - \varphi_j)(T_{jm} - \bar{Y})]^2$$

where $M(T_u(d)) = E(T_u(d) - \bar{Y})^2$ and $M(T_{jm}) = E(T_{jm} - \bar{Y})^2$. To derive the $M(T_u(d))$, we proceed as follows:

$$E(T_u(d) - \bar{Y})^2 = E(\bar{Y}(e_0 + \theta_1e_4 - \theta_2e_4) - \bar{Y})^2$$

Now we expand the right hand side of equation (21) binomially, taking expectations and retaining the terms up-to the first order of approximations, we have the expression of the mean square error of the estimator $T_u(d)$ given as

$$M(T_u(d)) = \bar{Y}^2 \left(\frac{1}{u} - \frac{1}{N}\right) (C_y^2 + \theta^2C_z^2 + 2\theta\rho_{yz}C_yC_z)$$

where $\theta(d) = \theta_1(d) - \theta_2(d)$.

The expression for mean square error of estimator $T_u(d)$ derive in equation (22) is a function of unknown constant $d$. To derive the optimum value of $d$, differentiating $M(T_u(d))$ with respect to $d$ and writing $\theta'$ for the differential coefficient of $\theta$ and equate it to zero, we have

$$\frac{\partial M(T_u(d))}{\partial d} = 2 \left(\frac{1}{u} - \frac{1}{N}\right) \bar{Y}^2 \theta' \left[C_y^2 + \rho_{yz}C_yC_z\right] = 0$$

where $\theta' = \frac{\partial \theta}{\partial d}$.

Further from equation (24), we get

$$\frac{fB - C}{A + fB + C} = -V$$

where $V = \rho_{yz}^2$.

On simplifying the above equation, we have the cubic equation in the form of $d$. This equation can be solved for $d$ in order to get optimum value say $d = d_0$. Since there will be three possible values of $d$. Although MSE will be minimum for all these three values of $d$ a criterion for selecting suitable $d$ can be set as follows: Out of all the possible values of optimum $d$, select the $d = d_0$ as the most suitable choice, which makes $|B(T_u(d))|$ smallest. Using equation (24) the minimum mean square error of the estimator $M(T_u(d))_{\text{min}}$ is given in (17).

Similarly the mean square errors of the estimators $T_{jm}$: $(j=1, 2)$ is written as

$$E(T_{jm} - \bar{Y})^2 = E[\bar{Y}(e_1 + e_2 - e_3 - e_5) - \bar{Y}]^2$$
\begin{align*}
E(T_{2m} - \bar{Y})^2 &= E \left[ \bar{Y} \left( e_1 + e_2 - e_3 \right) - \frac{\bar{Y}}{X} \beta_{xz} e_5 \right]^2 
\end{align*}

Expanding the right hand side of equations (25) and (26) binomially, taking expectations both sides, retaining the terms up-to the first order of approximations, we have the expression of the mean square error of the estimator \( T_{1m} \) and \( T_{2m} \) as shown in equation (18) and (19).

**Remark 4.** The above results are derived under the assumptions that the coefficients of variation of variables \( x, y \) and \( z \) are approximately equal. Reddy (1978) as described that the coefficients of variation is a stable quantity over the period of time. Since \( x \) and \( y \) are same study variable over two occasions and \( z \) is an auxiliary variables correlated to \( x \) and \( y \), therefore their coefficient of variations are assumed equal and the results are derived under these assumption.

### 3.2. Minimum mean square errors of the classes of estimator

Since the mean square errors of the estimator \( T_j(d); \ (j = 1, 2) \) in equation (16) are the functions of the unknown constants (scalar) \( \varphi_j \), therefore, we have minimized them with respect to \( \varphi_j \) and subsequently the optimum value of \( \varphi_j \) is obtained as

\begin{align*}
\varphi_{j_{opt}} = \frac{M(T_{jm})}{M(T_u(d)) + M(T_{jm})}; \ (j = 1, 2; \ d > 0) 
\end{align*}

From equation (27), substituting the value of \( \varphi_{j_{opt}} \) in equation (16) we get the optimum mean square errors of the estimator \( T_j \) as

\begin{align*}
M(T_j(d))_{opt} = \frac{M(T_u(d))_{min} M(T_{jm})}{M(T_u(d))_{min} + M(T_{jm})}; \ (j = 1, 2; \ d > 0) 
\end{align*}

Further substituting the values from equations (17) - (19) in equations (27) and (28), the simplified values of \( \varphi_{j_{opt}} \) and \( M(T_j(d))_{opt} (j = 1, 2) \) are obtained as

\begin{align*}
\varphi_{1_{opt}} &= \frac{\mu_1(A_0 - \mu_1 A_b)}{A_1 - \mu_1 A_9 - \mu_1^2 A_8} 
\end{align*}

\begin{align*}
M(T_1)_{opt} &= \frac{A_{16} + \mu_1^2 A_{13} - \mu_1 A_{15}}{A_1 - \mu_1^2 A_8 - \mu_1 A_9} \frac{S_y^2}{n} 
\end{align*}

\begin{align*}
\varphi_{2_{opt}} &= \frac{\mu_2(A_{18} + \mu_2 A_{17})}{A_1 - \mu_2 A_{20} + \mu_2^2 A_{20}} 
\end{align*}

\begin{align*}
M(T_2)_{opt} &= \frac{A_{23} - \mu_1^2 A_{22} + \mu_2^2 A_{24}}{A_1 + \mu_2^2 A_{19} - \mu_2 A_{20}} \frac{S_y^2}{n} 
\end{align*}

where \( \mu_j = \frac{u_j}{n} (j = 1, 2) \) are the fractions of fresh sample.

\[ A_1 = 1 - \rho_{yz}^2, \ A_2 = (2 - 2 \rho_{yx}), \ A_3 = (2 \rho_{yx} - 2 \rho_{yz}), \ A_4 = (2 - 2 \rho_{yx}), \ A_5 = A_3 - f A_4, \]
$A_{10} = A_{1}A_{2},$  
$A_{11} = A_{1}A_{3},$  
$A_{12} = A_{1}A_{4},$  
$A_{13} = fA_{11} - f^{2}A_{12},$  
$A_{14} = A_{10} + A_{11} - fA_{12},$  
$A_{15} = A_{11} - fA_{12} + fA_{10} + A_{13},$  
$A_{16} = (1 + \rho^{2}_{yx} - 2\rho_{yx}),$  
$A_{17} = A_{16} + fA_{1},$  
$A_{18} = A_{2} - A_{17},$  
$A_{19} = A_{16} + 2fA_{1},$  
$A_{20} = A_{1} - A_{2} + A_{19},$  
$A_{21} = A_{1}A_{16},$  
$A_{22} = fA_{21} + f^{2}A_{21},$  
$A_{23} = A_{10} - A_{21} - fA_{2}^{2},$  
$A_{24} = A_{21} - fA_{10} + fA_{1}^{2} + A_{22},$  
$f = \frac{n}{N}$

4. Optimum replacement strategies of the classes of estimators $T_{j}(d)$

The optimum mean square errors $M(T_{j})_{opt}$ ($j=1, 2$) in equations (30) and (32) are functions of $\mu_{j}$ ($j=1, 2$) (fractions of sample to be drawn afresh at the second occasion) which play important role in reducing the cost of the survey, therefore, to determine the optimum values of $\mu_{j}$ so that $\bar{Y}$ may be estimated with maximum precision and minimum cost, we minimize $M(T_{1})_{opt}$ and $M(T_{2})_{opt}$ with respect to $\mu_{1}$ and $\mu_{2}$ respectively which results in a quadratic equations in $\mu_{1}$ and $\mu_{2}$, which are shown as

$$\mu_{1}^{2}D_{1} - 2\mu_{1}D_{2} - D_{3} = 0 \quad (33)$$

and

$$\mu_{2}^{2}D_{4} - 2\mu_{2}D_{5} + D_{6} = 0 \quad (34)$$

Solving the equation (33) and (34) for $\mu_{1}$ and $\mu_{2}$ respectively, the solutions of $\mu_{j}$ (say $\hat{\mu}_{j}$) ($j = 1, 2$) are given as

$$\hat{\mu}_{1} = \frac{D_{2} \pm \sqrt{D_{2}^{2} + D_{4}D_{6}}}{D_{4}} \quad (35)$$

and

$$\hat{\mu}_{2} = \frac{D_{5} \pm \sqrt{D_{5}^{2} + D_{4}D_{6}}}{D_{4}} \quad (36)$$

where, $D_{1} = (A_{9}A_{13} + A_{8}A_{15}),$  
$D_{2} = (A_{1}A_{13} + A_{8}A_{14}),$  
$D_{3} = (A_{9}A_{14} - A_{1}A_{15}),$  
$D_{4} = (A_{20}A_{22} - A_{19}A_{24}),$  
$D_{5} = (A_{1}A_{22} + A_{19}A_{23}),$  
$D_{6} = (A_{1}A_{24} + A_{20}A_{23}).$

From equations (35) and (36) it is clear that the real values of $\hat{\mu}_{j}$ ($j = 1, 2$) exist, if, the quantities under square roots are greater than or equal to zero. For any combinations of correlations, which satisfy the condition of real solutions, two real values of $\hat{\mu}_{j}$ ($j = 1, 2$) are possible. Hence, while choosing the values of $\hat{\mu}_{j}$, it should be remembered that $0 \leq \hat{\mu}_{j} \leq 1$, all other values of $\hat{\mu}_{j}$ are said to be inadmissible. If both the values of $\hat{\mu}_{j}$ are admissible, the lowest one is the best choice as it reduces the cost of the survey. From equation (35) and (36), substituting the admissible value of $\hat{\mu}_{j}$ ($j = 1, 2$) (say $\hat{\mu}_{1}^{j}$) in equation (30) and (32), we have the optimum values of mean square errors of the classes of estimator $T_{j}(d)$, which is shown below:
\[ M(T_1)_{\text{opt}}^* = \left[ \frac{A_{14} + \mu_1(0)^2 A_{13} - \mu_1^0 A_{15}}{A_1 - \mu_1(0)^2 A_8 - \mu_1^0 A_9} \right] \frac{S_y^2}{n} \] (37)

\[ M(T_2)_{\text{opt}} = \left[ \frac{A_{23} + \mu_2(0)^2 A_{22} + \mu_2^0 A_{24}}{A_1 + \mu_2(0)^2 A_{19} - \mu_2^0 A_{20}} \right] \frac{S_y^2}{n} \] (38)

5. Efficiency Comparison

The percent relative efficiencies of the classes of estimators \( T_j(d, \varphi_j)(j = 1, 2) \) with respect to (i) sample mean estimator \( \bar{y}_n \) when there is no matching and (ii) natural successive sampling estimator \( \hat{\bar{y}} = \varphi \bar{y}_n + (1 - \varphi^*)\bar{y}_m \) when no auxiliary information is used at any occasion where \( \bar{y}_m = \bar{y}_n + \beta_{yx}(\bar{x}_n - \bar{x}_m) \) have been computed for different choices of correlations and presented in Tables 1-2. Since \( \bar{y}_n \) and \( \hat{\bar{y}} \) are unbiased estimators of \( \bar{Y} \), therefore, following Sukhatme et al. (1984), the variance of \( \bar{y}_n \) and optimum variance of \( \hat{\bar{y}} \) are given by

\[ V(\bar{y}_n) = \left( \frac{1}{n} - \frac{1}{N} \right) S_y^2 \] (39)

\[ V(\hat{\bar{y}}) = \left[ 1 + \sqrt{1 - \rho_{yz}^2} \right] \frac{S_y^2}{2n} \frac{S_y^2}{N} \] (40)

For different choices of correlations \( \rho_{yz}, \rho_{yx} \) and \( f \) Tables 1-2 present the optimum values of \( \mu_j \) \( (j = 1, 2) \) and the percent relative efficiencies \( E_1 \) and \( E_2 \) of \( T \) with respect to \( \bar{y}_n \) and \( \hat{\bar{y}} \) respectively, where \( E_1 = \frac{V(\bar{y}_n)}{M(T_1)_{\text{opt}}^*} \times 100 \) and \( E_2 = \frac{V(\hat{\bar{y}})}{M(T_2)_{\text{opt}}^*} \times 100 \).

6. Interpretation of results

From Table 1 it is clear that

(a) For fixed value of \( \rho_{yz} \) and different values of \( f \), the values of \( \mu_1, E_1 \) and \( E_2 \) are increasing with the increasing values of \( \rho_{yx} \). This behaviour is in agreement with Sukhatme et al. (1984), results which explain that more the value of \( \rho_{yx} \), more the fraction of fresh sample required at the current occasion.

(b) For fixed value of \( \rho_{yx} \), the values of \( E_1 \) and \( E_2 \) are increasing while the values of \( \mu_1 \) do not follow any definite pattern with the increasing value of \( \rho_{yz} \).

(c) Minimum value of \( E_1 \) is observed as 0.2222, which indicates that the fraction to be replaced at the current occasion is as low as about 22 percent of the total sample size, which leads to appreciable amount of the survey cost.

From Table 2 it is observed that
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| 0.2 |
|-----|--------|--------|-----|-----|-----|-----|-----|-----|
| 0.6 | $\mu_1$ | 0.3215 | 0.3077 | 0.2492 | * | 0.9656 | 0.7660 |
| $E_1$ | 109.14 | 114.75 | 121.13 | ** | 156.32 | 174.86 |
| $E_2$ | 103.44 | 105.14 | 105.98 | ** | 117.24 | 113.21 |
| 0.7 | $\mu_1$ | 0.3439 | 0.3532 | 0.3545 | 0.3173 | * | 0.7432 |
| $E_1$ | 130.06 | 137.58 | 146.86 | 158.56 | ** | 212.49 |
| $E_2$ | 123.28 | 126.06 | 128.51 | 130.23 | ** | 137.57 |
| 0.8 | $\mu_1$ | 0.3334 | 0.3500 | 0.3689 | 0.3881 | 0.3815 | 0.7790 |
| $E_1$ | 166.32 | 176.82 | 190.07 | 207.69 | 232.94 | 283.10 |
| $E_2$ | 157.64 | 162.01 | 166.31 | 170.59 | 174.70 | 183.294 |
| 0.9 | $\mu_1$ | 0.2786 | 0.2963 | 0.3185 | 0.3474 | 0.3865 | 0.3782 |
| $E_1$ | 275.62 | 294.37 | 318.32 | 350.69 | 398.77 | 484.55 |
| $E_2$ | 262.84 | 272.46 | 282.95 | 295.00 | 310.15 | 332.69 |

| 0.3 |
|-----|--------|--------|-----|-----|-----|-----|-----|-----|
| 0.6 | $\mu_1$ | 0.3504 | 0.3529 | 0.3421 | 0.2642 | * | 0.8004 |
| $E_1$ | 101.50 | 106.49 | 113.41 | 121.65 | ** | 166.60 |
| $E_2$ | ** | ** | ** | ** | ** | 101.47 |
| 0.7 | $\mu_1$ | 0.3571 | 0.3703 | 0.3827 | 0.3849 | 0.2741 | 0.8103 |
| $E_1$ | 119.31 | 126.61 | 135.76 | 147.72 | 163.57 | 201.86 |
| $E_2$ | 112.20 | 114.50 | 116.36 | 117.56 | 116.79 | 120.63 |
| 0.8 | $\mu_1$ | 0.3377 | 0.3556 | 0.3769 | 0.4021 | 0.4254 | * |
| $E_1$ | 151.55 | 161.55 | 174.26 | 191.32 | 216.31 | ** |
| $E_2$ | 142.51 | 146.09 | 149.36 | 152.25 | 154.51 | ** |
| 0.9 | $\mu_1$ | 0.2805 | 0.2986 | 0.3216 | 0.3521 | 0.3959 | 0.4637 |
| $E_1$ | 228.00 | 244.57 | 265.94 | 295.23 | 339.58 | 422.13 |
| $E_2$ | 214.40 | 221.16 | 227.95 | 234.95 | 242.56 | 252.04 |

Note: * indicate $\mu_1$ do not exist and ** indicate no gain.
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(a) For fixed value of $\rho_{yz}$ and different values of $f$, the values of $\mu_2, E_1$ and $E_2$ are increasing with increasing value of $\sigma$. This behaviour is in agreement with Sukhatme et al. (1984), results which explain that more the value of $\rho_{yz}$, more the fraction of fresh sample required at the current occasion.

(b) For fixed value of $\rho_{yz}$, the values of $E_1$ and $E_2$ are increasing while the values of $\mu_2$ are decreasing with the increasing values of $\rho_{yz}$. This behaviour is highly desirable in terms of percent relative efficiencies as well as the cost of the survey. It concludes that if the information on highly correlated auxiliary variable is available, its use at estimation stage not only pay in terms of enhance precision of estimates but also reduces the cost of the survey as well.

(c) Minimum value of $\mu_2$ is observed as 0.2846, which indicates that the fraction to be replaced at the current occasion is as low as about 28 percent of the total sample size, which leads to reduction in cost to an appreciable amount.

(d) For fixed values of $\rho_{yx}$ and $\rho_{yz}$, the values of $\mu_2$ are equal for different values of $f$.

7. Conclusions

It is clear from the above interpretation the use of an auxiliary character is highly rewarding in terms of the proposed classes of estimators. It is also clear that if a highly correlated auxiliary variable is used, relatively, only a smaller fraction of the sample on the current (second) occasion is required to be replaced by a fresh sample, which is reducing the cost of the survey. If one has to make choice between $T_1(d)$ and $T_2(d)$ it is visible from empirical results that for same combination of correlation, the PRE of $T_2(d)$ is more than the PRE of $T_1(d)$, which produces more reliable estimates than $T_1(d)$, hence it may be preferred for practical application.

Acknowledgements

Authors are thankful to the Reviewer for his valuable suggestions. Authors are also thankful to the Indian School of Mines, Dhanbad for providing financial assistance and necessary infrastructure to carry out the present research work.

References


**Summary**

The goal of this paper is to consider the problem of estimation of the population mean on current (second) occasion in two-occasion successive sampling. Some classes of estimators have been proposed and their detailed behaviours are examined. The dominance of the suggested classes of estimators has been established over sample mean estimator when there is no matching from the previous occasion and the natural successive sampling estimator. Optimum replacement strategies have been discussed. Empirical studies are carried out to study the performances of the proposed classes of estimator and suitable recommendations have been made.

*Keywords:* Successive sampling; Auxiliary information; Bias; Mean square error, Optimum replacement policy.