STATISTICA, anno LXXVI, n. 1, 2016

SHRINKAGE ESTIMATORS OF THE RELIABILITY CHARACTERISTICS OF A FAMILY OF LIFETIME DISTRIBUTIONS.

Ajit Chaturvedi [1]
Department of Statistics, University of Delhi, Delhi, India.
Shruti Nandchahal
Department of Statistics, University of Delhi, Delhi, India.

1. Introduction

The reliability function R(t) is defined as the probability of failure-free operation until time t. Thus, if the random variable (rv) X denotes the lifetime of an item or system, then $R(t) = P(X > t)$. Another measure of reliability under stress-strength set-up is the probability $P = P(X > Y)$, which represents the reliability of an item or system of random strength X subject to random stress Y. A lot of work has been done in the literature for the estimation and testing of parameter, R(t) and 'P' under censorings and complete sample case for individual distributions. For a brief review, one may refer to Pugh (1963), Basu (1964), Bartholomew (1957, 1963), Tong (1974, 1975), Johnson (1975), Kelley *et al.* (1976), Sathe and Shah (1981), Chao (1982), Constantine *et al.* (1986), Awad and Gharraf (1986), Tyagi and Bhattacharya (1989), Chaturvedi and Rani (1997, 1998), Chaturvedi and Surinder (1999), and others.

Thompson (1968) introduced the concept of 'shrinkage estimators'. A lot of work has been done in the literature in the direction of shrinkage estimators. For some citations, one may refer to George (1986), Ebrahimi and Hosmane (1987), Ghosh *et al.* (1987), Blattberg and George (1991), Clyde *et al.* (1998), Kubokawa (1998),Kolaczyk (1999), Longford (1999), Ahmed (2001), Royle and Link (2002), Şendur and Selesnick (2002), Fourdrinier *et al.* (2003), Pope and Szapudi (2008), Prakash and Singh (2008), Chen *et al.* (2009, 2010), Ledoit and Wolf (2012), Carreras and Brannath (2013), Liao (2013). Pandey (1983) proposed various shrinkage estimators for the mean of exponential distribution. Siu-Keung and Geoffrey (1996), Baklizi (2003) and Baklizi and Abu Dayyeh (2003) proposed shrinkage estimators of R(t) and 'P' for one-parameter exponential distribution. For estimating R(t), type I and type II censorings were considered. In order to estimate 'P', complete sample case was considered.

[1] Corresponding Author e-mail: ajitc2001@yahoo.co.in

Let the rv X follow the distribution having the probability density function (pdf)

$$f(x;a,\lambda,\underline{\theta}) = \lambda G'(x;a,\underline{\theta}) \exp\{-\lambda G(x;a,\underline{\theta})\} \; ; x > a \geq 0, \; \lambda > 0. \qquad (1)$$

Here, $G(x;a,\underline{\theta})$ is a function of x and may also depend on the (known) parameters 'a' and $\underline{\theta}$ - may be vector valued. Moreover, $G(x;a,\underline{\theta})$ is monotonically increasing in x with $G(a;a,\underline{\theta}) = 0, \; G(\infty;a,\underline{\theta}) = \infty$ and $G'(x;a,\underline{\theta})$ denotes the derivative of $G(x;a,\underline{\theta})$ with respect to x. We note that (1) represents a family of distributions and it covers the following lifetime distributions as specific cases:

1. For $G(x;a,\underline{\theta}) = x$ and $a = 0$, we get the one-parameter exponential distribution [Johnson *et al.* (1994), p.494].

2. For $G(x;a,\underline{\theta}) = x^p(p > 0)$ and $a = 0$, it gives Weibull distribution [Johnson *et al.* (1994), p.630].

3. For $G(x;a,\underline{\theta}) = x^2$ and $a = 0$, it leads to Rayleigh distribution [Sinha (1986), p.200].

4. For $G(x;a,\underline{\theta}) = \log(1 + x^b)(b > 0)$ and $a = 0$, it turns out to be Burr distribution [Burr (1942) and Burr and Cislak (1968)].

5. For $G(x;a,\underline{\theta}) = \log(\frac{x}{a})$, it is known as Pareto distribution [Johnson *et al.* (1994), p.233].

6. For $G(x;a,\underline{\theta}) = \log(1 + \frac{x}{v}), v > 0$ and $a = 0$ we get Lomax (1954) distribution.

7. For $G(x;a,\underline{\theta}) = \log(1 + \frac{x^b}{v}), b > 0$ and $a = 0$ it gives Burr distribution with scale parameter $v(> 0)$ [see Tadikamalla (1980)].

8. For $G(x;a,\underline{\theta}) = x^\gamma \exp(\nu x), \gamma > 0, \; \nu > 0$ and $a = 0$ it leads us to the modified Weibull distribution of Lai *et al.* (2003).

9. For $G(x;a,\underline{\theta}) = (x - a) + \frac{v}{\lambda} \log(\frac{x+v}{a+\lambda}), v > 0, \lambda > 0$, we get the generalized Pareto distribution of Ljubo (1965).

10. For $G(x;a,\underline{\theta}) = ax + \frac{\theta}{2}x^2, \theta > 0, a > 0, x > 0$ and $\lambda = 1$ we get the linear exponential distribution [see Mahmoud and Al-Nagar (2009)].

11. For $G(x;a,\underline{\theta}) = (1 + x^a)^\theta - 1, a > 0, \theta > 0, x \geq 0$ and $\lambda = 1$, we get the generalized power Weibull distribution [see Nikulin and Haghighi (2006)].

12. For $G(x;a,\underline{\theta}) = \frac{\beta}{a}(\exp(ax) - 1), a > 0, \beta > 0, x \geq 0$, we get the Gompertz distribution [see Khan and Zia (2009)].

13. For $G(x;a,\underline{\theta}) = \lambda(\exp(x^a) - 1), \lambda > 0, a > 0, x \geq 0$, we get the Chen distribution [see Chen (2000)].

14. For $G(x;a,\underline{\theta}) = x - a, x > a$, we get the two parameter exponential distribution [see Ahsanullah (1980)].

We note that

$$\begin{aligned} R(t) &= P(X > t) \\ &= P[G(X; a, \underline{\theta}) > G(t; a, \underline{\theta})] \\ &= \exp(-\lambda G(t; a, \underline{\theta})) \end{aligned} \tag{2}$$

and the hazard rate is

$$h(t) = \lambda G'(t; a, \underline{\theta}). \tag{3}$$

Let the rv X follow $f(x; a_1, \lambda_1, \theta_1)$ distribution and Y follow $f(y; a_2, \lambda_2, \theta_2)$ distribution. Then, we have

$$P = \frac{\lambda_1}{\lambda_1 + \lambda_2} \tag{4}$$

Now, we summarize the results of Chaturvedi *et al.* (2009). Suppose, n items are put on a test and the test is terminated after the first r ordered observations are recorded. Let $a \leq X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(r)},\ 0 < r < n$, be the lifetimes of first r ordered observations. Obviously, (n-r) items survived until $X_{(r)}$. Let $S_r = \sum_{i=1}^{r} G(X_{(i)}; a, \underline{\theta}) + (n-r)G(X_{(r)}; a, \underline{\theta})$. The likelihood function is

$$L(\lambda | x_{(1)}, x_{(2)}, \ldots, x_{(r)}, a, \underline{\theta}) = n(n-1)\ldots(n-r+1)\lambda^r \prod_{i=1}^{r} G'(X_{(i)}; a, \underline{\theta}) exp(-\lambda s_r) \tag{5}$$

S_r is complete and sufficient for the family of distributions given at (1) and the pdf of S_r is

$$g(s_r; a, \lambda, \underline{\theta}) = \frac{\lambda^r s_r^{r-1} \exp(-\lambda s_r)}{\Gamma(r)}; r > 0, \lambda > 0, 0 < s_r < \infty. \tag{6}$$

The maximum likelihood estimators (MLES) of $\lambda, R(t)$ and 'P' are, respectively,

$$\hat{\lambda}_{II} = \frac{r}{S_r}, \tag{7}$$

$$\hat{R}_{II}(t) = \exp\left\{-\frac{r}{S_r} G(t; a, \underline{\theta})\right\} \tag{8}$$

and

$$\hat{P}_{II} = \frac{\hat{\lambda}_{1II}}{\hat{\lambda}_{1II} + \hat{\lambda}_{2II}}, \tag{9}$$

where,

$$S_{r_1} = \sum_{i=1}^{r_1} G(X_{(i)}; a_1, \underline{\theta_1}) + (n - r_1) G(X_{(r_1)}; a_1, \underline{\theta_1})$$

and

$$T_{r_2} = \sum_{j=1}^{r_2} G(Y_{(j)}; a_2, \underline{\theta_2}) + (m - r_2) G(Y_{(r_2)}; a_2, \underline{\theta_2})$$

$\hat{\lambda}_{1II} = \frac{r_1}{S_{r_1}}$ and $\hat{\lambda}_{2II} = \frac{r_2}{T_{r_2}}$. For $q \in (-\infty, \infty)$, $q \neq 0$ the uniformly minimum variance unbiased estimator (UMVUE) of λ^q is

$$\tilde{\lambda}^q_{II} = \begin{cases} \frac{\Gamma(r)}{\Gamma(r-q)} S_r^{-q} & (q < r) \\ 0, & otherwise. \end{cases} \tag{10}$$

The UMVUE of $R(t)$ is

$$\tilde{R}_{II}(t) = \begin{cases} [1 - \frac{G(t;a;\underline{\theta})}{S_r}]^{r-1}; & G(t;a;\underline{\theta}) < S_r \\ 0, & otherwise. \end{cases} \tag{11}$$

The UMVUE of $'P'$ is given by

$$\tilde{P}_{II} = \begin{cases} (r_2-1)\sum_{i=0}^{r_2-2}(-1)^i \binom{r_2-2}{i} (\frac{S_{r_1}}{T_{r_2}})^{i+1} B(i+1, r_1), & S_{r_1} < T_{r_2} \\ (r_2-1)\sum_{i=0}^{r_1-1}(-1)^i \binom{r_1-1}{i} (\frac{T_{r_2}}{S_{r_1}})^{i} B(i+1, r_2-1), & T_{r_2} < S_{r_1}. \end{cases} \tag{12}$$

Now, we consider the case of type I censoring. Let $a \leq X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the failure times of n items under test from (1). The test begins at time $X_{(0)} = a$ and the system operates till $X_{(1)} = x_{(1)}$ when the first failure occurs. The failed item is replaced by a new one and the system operates till the second failure occurs at time $X_{(2)} = x_{(2)}$, and so on. The experiment is terminated at time t_0. If $N(t_0)$ be the number of failures during the interval $[0, t_0]$, then

$$P[N(t_0) = r|t_0] = \frac{\{n\lambda G(t_0; a, , \underline{\theta})\}^r}{r!} \exp\{-n\lambda G(t_0; a, , \underline{\theta})\}. \tag{13}$$

The MLES of $\lambda, R(t)$ and 'P' are, respectively,

$$\hat{\lambda}_I = \frac{r}{nG(t_0; a, \underline{\theta})}, \tag{14}$$

$$\hat{R}_I(t) = \exp\left\{-\frac{rG(t; a, \underline{\theta})}{nG(t_0; a, \underline{\theta})}\right\} \tag{15}$$

and

$$\hat{P}_I = \frac{\hat{\lambda}_{1I}}{\hat{\lambda}_{1I} + \hat{\lambda}_{2I}}, \tag{16}$$

where $\hat{\lambda}_{1I} = \frac{r_1}{nG(t_0; a_1, \underline{\theta}_1)}$ and $\hat{\lambda}_{2I} = \frac{r_2}{mG(t_0; a_2, \underline{\theta}_2)}$. The UMVUES of λ^q, $R(t)$ and 'P' are, respectively,

$$\tilde{\lambda}^q_I = \begin{cases} \frac{r!}{(r-q)!}\{nG(t_0; a, \underline{\theta})\}^{-q} & (q \leq r) \\ 0, & otherwise \end{cases} \tag{17}$$

$$\tilde{R}_I(t) = \begin{cases} [1 - \frac{G(t;a;\underline{\theta})}{nG(t_0;a;\underline{\theta})}]^r; & G(t;a;\underline{\theta}) < nG(t_0;a;\underline{\theta}) \\ 0, & otherwise. \end{cases} \tag{18}$$

and

$$\tilde{P}_I = \begin{cases} r_2 \sum_{i=0}^{r_1} (-1)^i \binom{r_1}{i} (\frac{m}{n})^i B(i+1, r_2), & m < n \\ r_2 \sum_{i=0}^{r_2-1} (-1)^i \binom{r_2-1}{i} (\frac{n}{m})^{i+1} B(i+1, r_1+1), & n < m. \end{cases} \tag{19}$$

In Section 2, we propose shrinkage estimators for the powers of λ. We consider estimation of powers of λ because they come in expressions for the moments of different distributions and hazard-rate. In Sections 3 and 4, respectively, we develop shrinkage estimators of $R(t)$ and 'P'. Finally, in Section 5, numerical findings are presented.

2. Shrinkage Estimators of powers of λ

We first consider the shrinkage estimator of λ^q based on its MLE and type II censored data. Let λ_0 be the guess value of λ . We consider,

$$\hat{\hat{\lambda}}^q_{II} = \alpha_1 \hat{\lambda}^q_{II} + (1-\alpha_1)\lambda_0^q, \; 0 \leq \alpha_1 \leq 1. \tag{20}$$

The value of α_1 that minimizes the mean sum of squares due to error (MSE) of $\hat{\hat{\lambda}}^q_{II}$ is

$$\alpha_1 = \frac{(\lambda^q - \lambda_0^q)[E(\hat{\lambda}^q_{II}) - \lambda_0^q]}{E(\hat{\lambda}^{2q}_{II}) + \lambda_0^{2q} - 2\lambda_0^q E(\hat{\lambda}^q_{II})}. \tag{21}$$

Using (6) and (7) ,

$$E(\hat{\lambda}^q_{II}) = \frac{r^q \Gamma(r-q)}{\Gamma(r)} \lambda^q \qquad (r > q)$$

and

$$E(\hat{\lambda}^{2q}_{II}) = \frac{r^{2q} \Gamma(r-2q)}{\Gamma(r)} \lambda^{2q} \qquad (r > 2q).$$

Since, λ is unknown, we estimate it by $\hat{\lambda}_{II}$. Now, we propose shrinkage estimator using the p-value of the likelihood ratio test. Consider $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$. From (5), H_0 is rejected when $s_r < \frac{\chi^2_{(2r)}(1-\frac{\alpha}{2})}{2\lambda_0}$ or $s_r > \frac{\chi^2_{(2r)}(\frac{\alpha}{2})}{2\lambda_0}$. Let τ_1 be the observed value of $2\lambda_0 S_r$. Then, the p-value for this test is $z_1 = 2\min\{1 - F(\tau_1), F(\tau_1)\}$, where $F(\tau_1)$ is the cumulative distribution function of $2\lambda_0 S_r$. Since a large value of z_1 indicates that λ is close to its guess value λ_0 [see Siu-Keung and Geoffrey (1996)], we can use z_1 to form the shrinkage estimator

$$\hat{\hat{\lambda}}^q_{II(z_1)} = (1-z_1)\hat{\lambda}^q_{II} + z_1 \lambda_0^q. \tag{22}$$

Now, we consider shrinkage estimator of λ^q based on its UMVUE and type II censored data. We propose

$$\tilde{\tilde{\lambda}}_{II}^q = \alpha_2 \tilde{\lambda}_{II}^q + (1-\alpha_2)\lambda_0^q,\ 0 \leq \alpha_2 \leq 1. \tag{23}$$

The value of α_2 which minimizes variance of $\tilde{\tilde{\lambda}}_{II}^q$ is

$$\begin{aligned}\alpha_2 &= \frac{(\lambda^q - \lambda_0^q)[E(\tilde{\lambda}_{II}^q) - \lambda_0^q]}{E(\tilde{\lambda}_{II}^q)^2 + \lambda_0^{2q} - 2\lambda_0^q E(\tilde{\lambda}_{II}^q)} \\ &= \frac{(\lambda^q - \lambda_0^q)^2}{\left\{\frac{\Gamma(r)\Gamma(r-2q)}{\Gamma^2(r-q)}\right\}\lambda^{2q} - 2\lambda_0^q\lambda^q + \lambda_0^{2q}}.\end{aligned} \tag{24}$$

Since powers of λ are unknown, we replace them by their UMVUES given at (10).

Based on p-value z_1 already defined,

$$\tilde{\tilde{\lambda}}_{II(z_1)}^q = (1-z_1)\tilde{\lambda}_{II}^q + z_1\lambda_0^q. \tag{25}$$

Let us define the shrinkage estimator of λ^q based on its MLE and type 1 censored data to be

$$\hat{\hat{\lambda}}_I^q = \alpha_3\hat{\lambda}_I^q + (1-\alpha_3)\lambda_0^q,\ 0 \leq \alpha_3 \leq 1. \tag{26}$$

The value of α_3 which minimizes the MSE of $\hat{\hat{\lambda}}_I^q$ is given by

$$\alpha_3 = \frac{(\lambda^q - \lambda_0^q)[E(\hat{\lambda}_I^q) - \lambda_0^q]}{E(\hat{\lambda}_I^{2q}) + \lambda_0^{2q} - 2\lambda_0^q E(\hat{\lambda}_I^q)}. \tag{27}$$

Since, λ is unknown, we use its MLE. It is to be noted here that since this estimate involves Poisson rv, q must be a positive integer. Moreover, from (Philipson, 1963,pp. 243) for a positive integer p,

$$E(\hat{\lambda}_I^p) = \frac{1}{\{nG(t_0; a, \underline{\theta})\}^p}\sum_{j=1}^{p}\{nG(t_0; a, \underline{\theta})\}^j \sum_{i=1}^{j}\frac{(-1)^{j-1}i^r}{(j-i)!i!}.$$

The critical region of the likelihood ratio test for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$ under type I censoring is given by $\{r \leq a \text{ or } r \geq b\}$, where 'a' and 'b' are chosen such that

$$P_{H_0}(r \leq a) + P_{H_0}(r \geq b) = \alpha.$$

If z_2 be the p-value of the test, then the shrinkage estimator of λ^q is

$$\hat{\hat{\lambda}}_{I(z_2)}^q = (1-z_2)\hat{\lambda}_I^q + z_2\lambda_0^q. \tag{28}$$

The shrinkage estimator of λ^q based on its UMVUE and type I censored data is

$$\tilde{\tilde{\lambda}}_I^q = \alpha_4 \tilde{\lambda}_I^q + (1-\alpha_4)\lambda_0^q,\ 0 \le \alpha_4 \le 1, \tag{29}$$

where

$$\alpha_4 = \frac{(\lambda^q - \lambda_0^q)[E(\tilde{\lambda}_I^q) - \lambda_0^q]}{E(\tilde{\lambda}_I^q)^2 + \lambda_0^{2q} - 2\lambda_0^q E(\tilde{\lambda}_I^q)}. \tag{30}$$

Here,

$$E(\tilde{\lambda_I^q})^2 = \frac{(q!)^2}{\{nG(t_0; a, \underline{\theta})\}^{2q}} \exp\{-n\lambda G(t_0; a, \underline{\theta})\} \sum_{r=q}^{\infty} \binom{r}{q}^2 \frac{\{n\lambda G(t_0; a, \underline{\theta})\}^r}{r!}.$$

Based on p-value z_2 already defined,

$$\tilde{\tilde{\lambda}}_{I(z_2)}^q = (1-z_2)\tilde{\lambda}_I^q + z_2\lambda_0^q. \tag{31}$$

3. Shrinkage Estimators of $R(t)$

For $\hat{R}_{II}(t)$ defined at (18), we consider the shrinkage estimator of $R(t)$ based on its MLE and type II censoring to be

$$\hat{\hat{R}}_{II}(t) = \alpha_5 \hat{R}_{II}(t) + (1-\alpha_5)R_0,\ 0 \le \alpha_5 \le 1. \tag{32}$$

The value of α_5 which minimizes the MSE of $\hat{\hat{R}}_{II}(t)$ is given by

$$\alpha_5 = \frac{[R(t) - R_0][E\{\hat{R}_{II}(t)\} - R_0]}{[E\{\hat{R}_{II}(t)\}^2 - 2R_0 E\{\hat{R}_{II}(t)\} + R_0^2]}. \tag{33}$$

Using (6) and (8),

$$\begin{aligned} E\{\hat{R}_{II}(t)\} &= \frac{\lambda^r}{\Gamma(r)} \int_0^\infty s_r^{r-1} \exp\left\{\frac{-rG(t; a, \underline{\theta})}{s_r} + \lambda s_r\right\} ds_r \\ &= \frac{1}{\Gamma(r)} \int_0^\infty y^{r-1} \exp\left\{\frac{-r\lambda G(t; a, \underline{\theta})}{y} + y\right\} dy \end{aligned} \tag{34}$$

Using a result of Watson (1952), it follows from (34) that

$$E\{\hat{R}_{II}(t)\} = \frac{2\{r\lambda G(t; a, \underline{\theta})\}^{\frac{r}{2}}}{\Gamma(r)} K_r(2\sqrt{r\lambda G(t; a, \underline{\theta})})$$

where $K_r(.)$ is the modified Bessel function of second kind of order r. Similarly,

$$E\{\hat{R}_{II}(t)\}^2 = \frac{2\{2r\lambda G(t; a, \underline{\theta})\}^{\frac{r}{2}}}{\Gamma(r)} K_r(2\sqrt{2r\lambda G(t; a, \underline{\theta})}).$$

Since, λ is unknown, we estimate it by $\hat{\lambda}_{II}$. It is worth mentioning here that Baklizi (2003) obtained approximate expressions for $E\{\hat{R}_{II}(t)\}$ and $E\{\hat{R}_{II}(t)\}^2$, whereas, we have derived their exact expressions.

From (2), $R(t) = R_0$ is equivalent to $\lambda = \frac{log(1/R_0)}{G(t;a,\underline{\theta})}$. Thus, testing $H_0 : R(t) = R_0$ against $H_1 : R(t) \neq R_0$ is equivalent to $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$, where $\lambda_0 = \frac{log(1/R_0)}{G(t;a,\underline{\theta})}$. The shrinkage estimator of $R(t)$ is

$$\hat{\hat{R}}_{II(z_1)}(t) = (1 - z_1)\hat{R}_{II}(t) + z_1 R_0. \tag{35}$$

Here,

$$z_1 = 2\min\{1 - F(\tau_2), F(\tau_2)\} \tag{36}$$

and τ_2 is the observed value of $\frac{2\log(1/R_0)}{G(t;a;\underline{\theta})}S_r$.

For $\tilde{R}_{II}(t)$ defined at (11), we propose the shrinkage estimator of $R(t)$, based on its UMVUE, to be

$$\tilde{\tilde{R}}_{II}(t) = \alpha_6 \tilde{R}_{II}(t) + (1 - \alpha_6)R_0, 0 \leq \alpha_6 \leq 1. \tag{37}$$

The value of α_6 which minimizes the MSE of $\tilde{\tilde{R}}_{II}(t)$ is given by

$$\alpha_6 = \frac{[R(t) - R_0][E\{\tilde{R}_{II}(t)\} - R_0]}{[E\{\tilde{R}_{II}(t)\}^2 - 2R_0 E\{\tilde{R}_{II}(t)\} + R_0^2]}. \tag{38}$$

Using (6) and (11) ,

$$E\{\tilde{R}_{II}(t)\}^2 = \frac{\lambda^r}{\Gamma(r)} \int_{G(t;a,\underline{\theta})}^{\infty} \left\{1 - \frac{G(t;a,\underline{\theta})}{s_r}\right\}^{2r-2} s_r^{r-1} \exp\{-\lambda s_r\} ds_r \tag{39}$$

Putting $s_r = G(t;a,\underline{\theta})(u+1)$

$$E\{\tilde{R}_{II}(t)\}^2 = \frac{\{\lambda G(t;a,\underline{\theta})\}^r}{\Gamma(r)} \exp\{-\lambda G(t;a,\underline{\theta})\} I,$$

where for $a_i = (-1)^i \binom{2r-2}{i}$,

$$\begin{aligned} I &= \int_0^{\infty} \frac{u^{2r-2}}{(1+u)^{r-1}} \exp\{-\lambda u G(t;a,\underline{\theta})\} du \\ &= \sum_{i=0}^{2r-2} a_i \int_0^{\infty} \frac{(u+1)^i}{(u+1)^{r-1}} \exp\{-\lambda u G(t;a,\underline{\theta})\} du \\ &= \sum_{i=0}^{r-1} a_i \int_0^{\infty} \frac{\exp\{-\lambda u G(t;a,\underline{\theta})\}}{(u+1)^{r-1-i}} du + \sum_{i=r}^{2r-2} a_i \int_0^{\infty} (u+1)^{i-r+1} \exp\{-\lambda u G(t;a,\underline{\theta})\} du. \end{aligned}$$

Using the following results of Erdelyi (1954),

$$-E_i(-x) = \int_x^{\infty} \frac{\exp(-u)}{u} du$$

and

$$\int_0^\infty \frac{\exp(-up)}{(u+a)^n}du = \sum_{m=1}^{n-1} \frac{(m-1)!(-p)^{n-m-1}}{(n-1)!a^m} - \frac{(-p)^{n-1}}{(n-1)!}\exp(ap)E_i(-ap),$$

$$E\{\tilde{R}_{II}(t)\}^2 = \frac{\{\lambda G(t;a,\underline{\theta})\}^r}{\Gamma(r)}\exp\{-\lambda G(t;a,\underline{\theta})\}\left[\sum_{i=0}^{r-3} a_i \left\{\sum_{m=1}^{r-i-2} \frac{(m-1)!(-\lambda G(t;a,\underline{\theta}))^{r-m-i-2}}{(r-i-2)!}\right.\right.$$
$$\left. - \frac{(-\lambda G(t;a,\underline{\theta}))^{r-i-2}}{(r-i-2)!}\exp\{\lambda G(t;a,\underline{\theta})\}E_i(-\lambda G(t;a,\underline{\theta}))\right\} - a_{r-2}\exp\{\lambda G(t;a,\underline{\theta})\}$$
$$\left. E_i(-\lambda G(t;a,\underline{\theta})) + \frac{a_{r-1}}{\lambda G(t;a,\underline{\theta})} + \sum_{i=r}^{2r-2} \frac{a_i(i-r+1)!}{\{\lambda G(t;a\underline{\theta})\}^{i-r+2}} \sum_{k=0}^{i-r+1} \frac{\{\lambda G(t;a,\theta)\}^k}{k!}\right] \quad (40)$$

Since, the powers of λ are unknown, we replace them by their UMVUES.

The shrinkage estimator of $R(t)$ based on the p-value is

$$\tilde{\tilde{R}}_{II(z_1)}(t) = (1-z_1)\tilde{R}_{II}(t) + z_1 R_0. \quad (41)$$

For $\hat{R}_I(t)$ given at (15), the shrinkage estimator of $R(t)$ based on type I censoring is

$$\hat{\hat{R}}_I(t) = \alpha_7 \hat{R}_I(t) + (1-\alpha_7)R_0, 0 \le \alpha_7 \le 1. \quad (42)$$

where

$$\alpha_7 = \frac{[R(t)-R_0][E\{\hat{R}(t)\} - R_0]}{[E\{\hat{R}_I(t)\}^2 - 2R_0 E\{\hat{R}_I(t)\} + R_0^2]}. \quad (43)$$

Using (13) ,

$$E\{\hat{R}_I(t)\} = \exp\{-n\lambda G(t_0;a,\underline{\theta})\} \sum_{r=0}^{\infty} \frac{\left[n\lambda G(t_0;a,\underline{\theta})\exp\left\{-\frac{G(t;a,\underline{\theta})}{nG(t_0;a,\underline{\theta})}\right\}\right]^r}{r!}$$
$$= \exp\{-n\lambda G(t_0;a,\underline{\theta})\}\exp\left[n\lambda G(t_0;a,\underline{\theta})\exp\left\{-\frac{G(t;a,\underline{\theta})}{nG(t_0;a,\underline{\theta})}\right\}\right]$$

Similarly,

$$E\{\hat{R}_I(t)\}^2 = \exp\{-n\lambda G(t_0;a,\underline{\theta})\}\exp\left[n\lambda G(t_0;a,\underline{\theta})\exp\left\{-\frac{2G(t;a,\underline{\theta})}{nG(t_0;a,\underline{\theta})}\right\}\right]$$

Since, λ is unknown, it is to be estimated by its MLE.

The shrinkage estimator of $R(t)$ based on p-value is

$$\hat{\hat{R}}_{I(z_2)}(t) = (1-z_2)\hat{R}_I(t) + z_2 R_0. \quad (44)$$

The shrinkage estimator of $R(t)$ based on its UMVUE and type I censoring is

$$\tilde{\tilde{R}}_I(t) = \alpha_8 \tilde{R}_I(t) + (1-\alpha_8)R_0, 0 \le \alpha_8 \le 1, \quad (45)$$

where

$$\alpha_8 = \frac{[R(t) - R_0][E\{\tilde{R}(t)\} - R_0]}{[E\{\tilde{R}_I(t)\}^2 - 2R_0 E\{\tilde{R}_I(t)\} + R_0^2]}. \tag{46}$$

Using (18) ,

$$E\{\tilde{R}_I(t)\}^2 = \exp\{-n\lambda G(t_0; a, \underline{\theta})\} \sum_{r=0}^{\infty} \frac{1}{r!} \left[\left\{ 1 - \frac{G(t; a, \underline{\theta})}{n\lambda G(t_0; a, \underline{\theta})} \right\}^2 \{n\lambda G(t_0; a, \underline{\theta})\} \right]^r$$
$$= \exp\{-n\lambda G(t_0; a, \underline{\theta})\} \exp \left[\left\{ 1 - \frac{G(t; a, \underline{\theta})}{n\lambda G(t_0; a, \underline{\theta})} \right\}^2 \{n\lambda G(t_0; a, \underline{\theta})\} \right]$$

We propose the shrinkage estimator of $R(t)$ based on p-value to be

$$\tilde{\tilde{R}}_{I(z_2)}(t) = (1 - z_2)\tilde{R}_I(t) + z_2 R_0. \tag{47}$$

4. Shrinkage estimators of 'P'

For $\hat{P}_{II}$ defined in (9), we propose the shrinkage estimator of 'P' to be

$$\hat{\hat{P}}_{II} = \alpha_9 \hat{P}_{II} + (1 - \alpha_9)P_0, 0 \le \alpha_9 \le 1. \tag{48}$$

The value of α_9 , which minimizes the MSE of $\hat{\hat{P}}_{II}$ is given by

$$\alpha_9 = \frac{[P - P_0][E(\hat{P}_{II}) - P_0]}{[E(\hat{P}_{II})^2 - 2P_0 E(\hat{P}_{II}) + P_0^2]}. \tag{49}$$

We can write (9) as

$$\hat{P}_{II} = \left(1 + \frac{\hat{\lambda}_{2II}}{\hat{\lambda}_{1II}} \right)^{-1}$$
$$= \left[1 + \frac{\lambda_2 r_1}{\lambda_1 r_2} F(2r_1, 2r_2) \right]^{-1},$$

where the *rv* $F(2r_1, 2r_2)$ follows F-distribution with $(2r_1, 2r_2)$ degrees of freedom and having the pdf

$$f(F) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1}}{B(r_1, r_2)} . \frac{F^{r_1 - 1}}{\left[1 + \frac{r_1}{r_2} F\right]^{r_1 + r_2}};\ 0 < F < \infty.$$

Making the transformation

$$\left(1 + \frac{\lambda_2}{\lambda_1} F \right)^{-1} = Q$$

the pdf of Q comes out to be

$$f(q) = \frac{\left(\frac{r_2\lambda_2}{r_1\lambda_1}\right)^{r_2}}{B(r_1, r_2)} . \frac{q^{r_2-1}(1-q)^{r_1-1}}{\left[1+\left(\frac{r_2\lambda_2}{r_1\lambda_1}-1\right)q\right]^{r_1+r_2}};\ 0 < q < 1. \tag{50}$$

The distributions for which $r_1\lambda_1 = r_2\lambda_2$,

$$f(q) = \frac{1}{B(r_1, r_2)} .q^{r_2-1}(1-q)^{r_1-1};\ 0 < q < 1.$$

and

$$E(Q^l) = \frac{B(r_1, r_2 + l)}{B(r_1, r_2)}.$$

If $r_1\lambda_1 \neq r_2\lambda_2$, then from (50),

$$E(Q^l) = \frac{\left(\frac{r_2\lambda_2}{r_1\lambda_1}\right)^{r_2}}{B(r_1, r_2)} \int_0^1 \frac{q^{l+r_2-1}(1-q)^{r_1-1}}{\left[1+\left(\frac{r_2\lambda_2}{r_1\lambda_1}-1\right)q\right]^{r_1+r_2}} dq.$$

Putting

$$1+\left(\frac{r_2\lambda_2}{r_1\lambda_1}-1\right)q = u,$$

$$\begin{aligned} E(Q^l) &= \frac{\left(\frac{r_2\lambda_2}{r_1\lambda_1}\right)^{r_2}}{B(r_1,r_2)}(-1)^{r_1-1}\left(\frac{r_2\lambda_2}{r_1\lambda_1}-1\right)^{-l-r_1-r_2+1}\int_1^{\frac{r_2\lambda_2}{r_1\lambda_1}} u^{-r_1-r_2}(u-1)^{l+r_2-1}\left(u-\frac{r_2\lambda_2}{r_1\lambda_1}\right)^{r_1-1} du. \\ &= \frac{1}{B(r_1,r_2)}(-1)^{l+r_2-1}\left(\frac{r_1\lambda_1}{r_2\lambda_2}\right)^{l}\left(1-\frac{r_1\lambda_1}{r_2\lambda_2}\right)^{-r_1-r_2-l+1}\sum_{j=0}^{r_1-1}(-1)^j\binom{r_1-1}{j}\left(\frac{r_1\lambda_1}{r_2\lambda_2}\right)^j \\ &\cdot \sum_{k=0}^{l+r_2-1}(-1)^k\binom{l+r_2-1}{k} I\left(\frac{r_2\lambda_2}{r_1\lambda_1}; j+k-r_1-r_2\right)(c \neq 1) \end{aligned}$$

Here,

$$\begin{aligned} I(c, p) &= \int_1^c t^p dt \\ &= \begin{cases} (c^{p+1}-1)/(p+1); & p \neq 1 \\ logc; & p = -1 \end{cases} \end{aligned}$$

Since, λ_1 and λ_2 are unknown, they are estimated by their MLES.

Now, we propose shrinkage estimator of 'P' based on the p-value related to the likelihood ratio test of the hypothesis $H_0 : P = P_0$ against the alternative $H_1 : P \neq P_0$. For $k = P_0/(1-P_0)$, these hypotheses are equivalent to $H_0 : \lambda_1 = k\lambda_2$ against the alternative $H_1 : \lambda_1 \neq k\lambda_2$. Denoting by, Θ_0 and Θ, respectively, the parametric space restricted by the null hypothesis and the entire parametric space, it can be seen that the likelihood ratio criterion is

$$\begin{aligned}\Phi =& \frac{\sup\limits_{\Theta_0} L(\lambda_1, \lambda_2 | \underline{x}, \underline{y})}{\sup\limits_{\Theta} L(\lambda_1, \lambda_2 | \underline{x}, \underline{y})} \\ =& \frac{\left(\frac{S_{r_1}}{T_{r_2}}\right)^{r_1}}{\left(1 + k\frac{S_{r_1}}{T_{r_2}}\right)^{r_1+r_2}}.\end{aligned}$$

Thus, the critical region is

$$w = \left\{ k\frac{S_{r_1}}{T_{r_2}} < k_1 \ or \ k\frac{S_{r_1}}{T_{r_2}} > k_2 \right\}$$

where k_1 and k_2 are determined so that

$$w = \left[(\underline{x}, \underline{y}) : \left\{ 0 < \frac{S_{r_1}}{T_{r_2}} < \frac{kr_1}{r_2} F_{1-\frac{\alpha}{2}}(2r_1, 2r_2) \right\} \cup \left\{ \frac{kr_1}{r_2} F_{\frac{\alpha}{2}}(2r_1, 2r_2) < \frac{S_{r_1}}{T_{r_2}} < \infty \right\} \right].$$

If z_3 be the p-value of the test, then the shrinkage estimator of 'P' is given by

$$\hat{\hat{P}}_{II(z_3)} = (1 - z_3)\hat{P}_{II} + z_3 P_0. \tag{51}$$

For $\tilde{P}_{II}$ given in (12), we define the shrinkage estimator of 'P' to be

$$\tilde{\tilde{P}}_{II} = \alpha_{10}\tilde{P}_{II} + (1 - \alpha_{10})P_0, 0 \leq \alpha_{10} \leq 1. \tag{52}$$

The value of α_{10}, which minimizes the MSE of $\tilde{\tilde{P}}_{II}$ is given by

$$\alpha_{10} = \frac{[P - P_0][E(\tilde{P}_{II}) - P_0]}{[E(\tilde{P}_{II})^2 - 2P_0E(\tilde{P}_{II}) + P_0^2]}. \tag{53}$$

Denoting by

$$a_i = (-1)^i (r_2 - 1) \binom{r_2 - 2}{i} B(i + 1, r_1)$$

and

$$b_i = (-1)^i (r_2 - 1) \binom{r_1 - 1}{i} B(i + 1, r_2 - 1),$$

and from (12),

$$\begin{aligned}E(\tilde{P}_{II}^2) =& \sum_{i=0}^{r_2-2} \sum_{j=0}^{r_2-2} a_i a_j E \left\{ \left(\frac{S_{r_1}}{T_{r_2}} \right)^{i+j+2} I(S_{r_1} < T_{r_2}) \right\} \\ &+ \sum_{i=0}^{r_1-1} \sum_{j=0}^{r_1-1} b_i b_j E \left\{ \left(\frac{T_{r_2}}{S_{r_1}} \right)^{i+j} I(T_{r_2} < S_{r_1}) \right\}\end{aligned} \tag{54}$$

We have,

$$E\left\{\left(\frac{S_{r_1}}{T_{r_2}}\right)^{i+j+2} I(S_{r_1} < T_{r_2})\right\} = \left(\frac{\lambda_2}{\lambda_1}\right)^{i+j+2} \left(\frac{r_1}{r_2}\right)^{r_1+i+j+2} \int_0^1 \frac{F^{r_1+i+j+1}}{B(r_1, r_2)\left(1+\frac{r_1}{r_2}F\right)^{r_1+r_2}} dF. \quad (55)$$

Similarly,

$$E\left\{\left(\frac{T_{r_2}}{S_{r_1}}\right)^{i+j} I(T_{r_2} < S_{r_1})\right\} = \left(\frac{\lambda_1}{\lambda_2}\right)^{i+j} \left(\frac{r_2}{r_1}\right)^{r_2+i+j} \int_0^1 \frac{F^{r_2+i+j-1}}{B(r_2, r_1)\left(1+\frac{r_2}{r_1}F\right)^{r_1+r_2}} dF. \quad (56)$$

Since λ_1 and λ_2 are unknown, they are estimated by their UMVUES.

Based on the p-value z_3 already defined, the shrinkage estimator of 'P' is given by

$$\tilde{\tilde{P}}_{II(z_3)} = (1 - z_3)\tilde{P}_{II} + z_3 P_0. \quad (57)$$

Under Type I censoring, the MLE of 'P' is given by

$$\hat{P}_I = \frac{\hat{\lambda}_{1I}}{\hat{\lambda}_{1I} + \hat{\lambda}_{2I}},$$

and the UMVUE of 'P' is

$$\tilde{P}_I = \begin{cases} r_2 \sum_{i=0}^{r_1} (-1)^i \binom{r_1}{i} \left(\frac{m}{n}\right)^i B(i+1, r_2), & m < n \\ r_2 \sum_{i=0}^{r_2-1} (-1)^i \binom{r_2-1}{i} \left(\frac{n}{m}\right)^{i+1} B(i+1, r_1+1), & n < m. \end{cases}$$

We observe that the estimators of 'P' can be presented as the ratio of two Poisson $rv's$. Therefore, the distributions of the estimators cannot be obtained. We, therefore, conduct simulation study for the shrinkage estimators of 'P' under type I censoring. The results are presented in Table 10.

5. Numerical Findings

A simulation study is conducted to investigate the performance of the above estimators.The indices of our simulations for Section 2 are:

λ: the true value of the parameter and is taken to be 1, 1.5 and 0.8
λ_0: prior guess value of λ and is taken to be 0.50, 0.80, 1.00, 1.20, 1.50 and 2.00
q: power of λ and is taken to be 1, 2 and 3
CP: the censoring proportion and is taken to be 0.25, 0.50 and 0.75
t_0: truncation time point and taken as 0.40, 0.80 and 1.00

For each combination of λ and λ_0, 1000 samples of size 40 were generated from the distribution given in (1), taking $G(x; a, \underline{\theta}) = x$. The shrinkage estimators for λ^q are calculated under both type II and type I censorings (considering the above values of CP and t_0 respectively) and the relative efficiencies of these estimators to the corresponding maximum likelihood estimators are calculated as the ratio of the mean squared error of the MLE to the mean squared error of the shrinkage estimator. Similarly, the relative efficiencies of these estimators to the UMVUES are computed. Table 1, 2 and 3 presents the relative efficiencies of the shrinkage estimators of λ^q for λ =1, 1.5 and 0.8 respectively, under type II censoring. Similar results for type I censoring are presented in table 4.

Under type II censoring, we can observe that $\hat{\hat{\lambda}}^q_{II}$ performs the best, followed by $\hat{\hat{\lambda}}^q_{II(z_1)}$ and $\tilde{\tilde{\lambda}}^q_{II(z_1)}$ is the worst estimator. One point to be noted is that $\tilde{\tilde{\lambda}}^q_{II}$ is equally efficient as $\tilde{\lambda}^q_{II}$ (It can be seen from the formula also). Also, as q increases, the relative efficiencies of the estimators $\hat{\hat{\lambda}}^q_{II}$ and $\hat{\hat{\lambda}}^q_{II(z_1)}$ increases when $\lambda = \lambda_0$. Same result is shown by $\tilde{\tilde{\lambda}}^q_{II(z_1)}$ estimator for higher censoring proportions only.
Under type I censoring, for $q = 1$, $\hat{\hat{\lambda}}^q_I$ are $\tilde{\tilde{\lambda}}^q_I$ equally efficient as can be seen from the formula (as $\hat{\lambda}^q_I = \tilde{\lambda}^q_I$ for q=1). However, for $q = 2$ and $q = 3$, the estimator $\tilde{\tilde{\lambda}}^q_I$ performs better than $\hat{\hat{\lambda}}^q_I$ except when $\lambda_0 = 0.5$.
Also, we can observe that the shrinkage estimators under both type I and type II censorings seem to perform better for small sample sizes than for large sample sizes when $\lambda = \lambda_0$.

The indices of our simulations for Section 3 are:
R: The true value of reliability and is taken to be 0.50,0.75,0.90 and 0.95
λ_0: obtained from Ro (the prior guess of R) and is taken to be 0.50, 0.80,1.00, 1.20, 1.50 and 2.00
CP: The censoring proportion and is taken to be 0.25, 0.50 and 0.75
t_0: truncation time point and taken as 0.40, 0.80 and 1.00
For each combination of R and λ_0, 1000 samples of size 40 were generated from the distribution given in (1), taking $\lambda = 1$ and $G(x; a, \underline{\theta}) = x$. The shrinkage estimators for $R(t)$ are calculated under both type II and type I censorings and their relative efficiencies are computed. Table 5 and 6 presents the relative efficiencies of the shrinkage estimators for $R(t)$ under type II censoring. Similar results for type I censoring are presented in table 7.

Under type II censoring, we can observe that $\hat{\hat{R}}_{II}(t)$ has the highest relative efficiency which is followed by $\tilde{\tilde{R}}_{II(z_3)}(t)$ and $\hat{\hat{R}}_{II(z_3)}(t)$ whereas $\tilde{\tilde{R}}_{II}(t)$ has the least relative efficiency except for the cases when λ is far away from λ_0. Also, all the shrinkage estimators perform better for small sample sizes as compared to large sample sizes when $\lambda = \lambda_0$. Under type I censoring, $\tilde{\tilde{R}}_I(t)$ performs better than $\hat{\hat{R}}_I(t)$ when $\lambda = \lambda_0$. However, for other values of λ, there is no clear trend for which estimator dominates. Here also, the relative efficiencies of the estimators decreases as the sample size increases when $\lambda = \lambda_0$.

The indices of our simulations for Section 4 are:
P: the true value of $P = P(X > Y)$ and is taken to be 0.65, 0.70 and 0.80
P_0: the initial estimate of 'P' and is taken to be
0.55, 0.60, 0.65, 0.70, 0.75 and 0.80 when $P = 0.65$
0.60, 0.65, 0.70, 0.75, 0.80 and 0.85 when $P = 0.70$
0.70, 0.75, 0.80, 0.85, 0.90 and 0.95 when $P = 0.80$
r_1:number of X observations and is taken to be 20 and 30
r_2:number of Y observations and is taken to be 20 and 30

For each combination of P and P_0 , 1000 samples of size 40 were generated for X from the distribution given in (1), taking $\lambda_1 = 1$ and $G(x; a_1, \underline{\theta_1}) = x$ and 1000 samples of size 40 were generated for Y from the same distribution with $\lambda_2 = \frac{1}{P} - 1$ and $G(y; a_2, \theta_2) = y$. The shrinkage estimators for 'P' are calculated under type II censoring and their relative efficiencies are computed. Table 8 and 9 presents the relative efficiencies of the shrinkage estimators for 'P' under type II censoring.

From the tables, we can observe that $\tilde{\tilde{P}}_{II}$ has the highest relative efficiency. The shrinkage estimators can be arranged in terms of overall performance as follows (from best to worst); $\tilde{\tilde{P}}_{II} - \tilde{\tilde{P}}_{II(z_3)} - \hat{\hat{P}}_{II} - \hat{\hat{P}}_{II(z_3)}$.
Under Type I censoring, the shrinkage estimators for 'P' are directly calculated using simulation. Table 10 gives the respective relative efficiencies.

From the tables, we observe that $\tilde{\tilde{P}}_I$ performs better than $\hat{\hat{P}}_I$ when $P = P_0$ or when P is close to P_0.

TABLE 1

Relative efficiencies of the estimators of λ^q (when $\lambda = 1$) under Type II censoring

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		$q = 1$				$q = 2$				$q = 3$			
0.25	0.5	0.4312	0.7026	1	0.2264	0.3823	0.7703	1	0.1476	0.3824	0.8331	1	0.0809
0.25	0.8	1.1697	1.0526	1	0.3812	1.1199	1.0390	1	0.2444	1.1031	1.0468	1	0.1318
0.25	1.0	2.5666	1.5818	1	0.4834	3.6165	1.7348	1	0.3324	7.6240	1.9067	1	0.1980
0.25	1.2	1.4088	1.2181	1	0.3696	1.1673	1.2504	1	0.2443	0.7427	1.2653	1	0.1421
0.25	1.5	0.8987	0.9646	1	0.2915	0.5392	0.8888	1	0.1796	0.1920	0.7630	1	0.0967
0.25	2.0	0.8830	0.9659	1	0.2746	0.6362	0.9118	1	0.1632	0.1766	0.7972	1	0.0849
0.50	0.5	0.6903	0.9179	1	0.6816	0.8130	0.9556	1	0.6830	1.1992	0.9805	1	0.6969
0.50	0.8	1.0897	0.9371	1	0.7409	1.3882	0.9579	1	0.7470	2.2290	0.9849	1	0.7647
0.50	1.0	1.9747	1.3521	1	1.0339	2.8206	1.3552	1	1.0675	5.4858	1.2965	1	1.0918
0.50	1.2	1.2460	1.1374	1	0.8313	1.3398	1.1956	1	0.8429	1.6193	1.2528	1	0.8653
0.50	1.5	0.8911	0.9522	1	0.6808	0.7976	0.9502	1	0.6279	0.7386	0.9844	1	0.5725
0.50	2.0	0.8119	0.9166	1	0.6634	0.6744	0.8266	1	0.5694	0.4778	0.7142	1	0.4429
0.75	0.5	0.6141	0.9204	1	0.3886	0.6375	0.9510	1	0.3973	0.7394	0.9724	1	0.4204
0.75	0.8	1.0864	0.9992	1	0.4643	1.1426	0.9843	1	0.4678	1.3386	0.9793	1	0.4885
0.75	1.0	1.7689	1.2762	1	0.5449	1.8899	1.2898	1	0.5590	2.3647	1.2805	1	0.5952
0.75	1.2	1.2696	1.0725	1	0.4450	1.2132	1.1048	1	0.4508	1.2921	1.1524	1	0.4783
0.75	1.5	0.9894	0.9741	1	0.4025	0.9021	0.9549	1	0.3859	0.8373	0.9326	1	0.3748
0.75	2.0	0.9438	0.9816	1	0.4086	0.8971	0.9519	1	0.3953	0.7841	0.8880	1	0.3856

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II}$ with respect to $\hat{\lambda}^q_{II}$.

EF2 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II(z_1)}$ with respect to $\hat{\lambda}^q_{II}$.

EF3 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II}$ with respect to $\tilde{\lambda}^q_{II}$.

EF4 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II(z_1)}$ with respect to $\tilde{\lambda}^q_{II}$.

TABLE 2

Relative efficiencies of the estimators of λ^q (when $\lambda = 1.5$) under Type II censoring

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		$q = 1$				$q = 2$				$q = 3$			
0.25	0.50	0.3900	0.8867	1	0.2564	0.3223	0.9359	1	0.1589	0.3255	0.9641	1	0.0839
0.25	0.80	0.4524	0.6869	1	0.2262	0.4048	0.7475	1	0.1474	0.4035	0.8093	1	0.0811
0.25	1.00	0.6291	0.7194	1	0.2559	0.5775	0.7436	1	0.1650	0.5649	0.7859	1	0.0902
0.25	1.20	1.1697	1.0526	1	0.3812	1.1199	1.0390	1	0.2444	1.1031	1.0468	1	0.1318
0.25	1.50	2.5666	1.5818	1	0.4834	3.6165	1.7348	1	0.3324	7.6240	1.9067	1	0.1980
0.25	2.00	1.0528	1.0505	1	0.3214	0.7121	1.0091	1	0.2049	0.3236	0.9285	1	0.1145
0.50	0.50	0.6873	0.9806	1	0.7125	0.7780	0.9937	1	0.7025	1.1155	0.9983	1	0.7082
0.50	0.80	0.7034	0.9069	1	0.6741	0.8314	0.9441	1	0.6753	1.2330	0.9733	1	0.6913
0.50	1.00	0.7793	0.8236	1	0.6219	0.9617	0.8768	1	0.6365	1.4814	0.9332	1	0.6664
0.50	1.20	1.0897	0.9371	1	0.7409	1.3882	0.9579	1	0.7470	2.2290	0.9849	1	0.7647
0.50	1.50	1.9747	1.3521	1	1.0339	2.8206	1.3552	1	1.0675	5.4858	1.2965	1	1.0918
0.50	2.00	1.0265	1.0183	1	0.7325	0.9972	1.0522	1	0.7072	1.0360	1.1199	1	0.6878
0.75	0.50	0.6532	0.9988	1	0.4149	0.6421	0.9995	1	0.4123	0.7197	0.9998	1	0.4291
0.75	0.80	0.6105	0.8816	1	0.3758	0.6413	0.9225	1	0.3888	0.7495	0.9538	1	0.4148
0.75	1.00	0.6717	0.7807	1	0.3518	0.7241	0.8246	1	0.3703	0.8620	0.8731	1	0.4018
0.75	1.20	1.0864	0.9992	1	0.4643	1.1426	0.9843	1	0.4678	1.3386	0.9793	1	0.4885
0.75	1.50	1.7689	1.2762	1	0.5449	1.8899	1.2898	1	0.5590	2.3647	1.2805	1	0.5952
0.75	2.00	1.1100	1.0133	1	0.4177	1.0315	1.0277	1	0.4125	1.0409	1.0593	1	0.4225

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II}$ with respect to $\hat{\lambda}^q_{II}$.

EF2 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II(z_1)}$ with respect to $\hat{\lambda}^q_{II}$.

EF3 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II}$ with respect to $\tilde{\lambda}^q_{II}$.

EF4 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II(z_1)}$ with respect to $\tilde{\lambda}^q_{II}$.

TABLE 3

Relative efficiencies of the estimators of λ^q (when $\lambda = 0.8$) under Type II censoring

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		$q=1$				$q=2$				$q=3$			
0.25	0.50	0.5525	0.6915	1	0.2392	0.5035	0.7272	1	0.1548	0.4958	0.7771	1	0.0850
0.25	0.80	2.5666	1.5818	1	0.4834	3.6165	1.7348	1	0.3324	7.6240	1.9067	1	0.1980
0.25	1.00	1.2381	1.1405	1	0.3481	0.9314	1.1374	1	0.2270	0.5118	1.1053	1	0.1301
0.25	1.20	0.8987	0.9646	1	0.2915	0.5392	0.8888	1	0.1796	0.1920	0.7630	1	0.0967
0.25	1.50	0.8660	0.9559	1	0.2751	0.5762	0.8899	1	0.1641	0.1562	0.7611	1	0.0856
0.25	2.00	0.9562	0.9923	1	0.2757	0.9007	0.9764	1	0.1633	0.3913	0.9315	1	0.0846
0.50	0.50	0.7480	0.8455	1	0.6315	0.9079	0.8950	1	0.6426	1.3782	0.9444	1	0.6695
0.50	0.80	1.9747	1.3521	1	1.0339	2.8206	1.3552	1	1.0675	5.4858	1.2965	1	1.0918
0.50	1.00	1.1451	1.0868	1	0.7887	1.1744	1.1393	1	0.7864	1.3138	1.2082	1	0.7940
0.50	1.20	0.8911	0.9522	1	0.6808	0.7976	0.9502	1	0.6279	0.7386	0.9844	1	0.5725
0.50	1.50	0.8070	0.9094	1	0.6553	0.6793	0.8272	1	0.5602	0.5160	0.7352	1	0.4388
0.50	2.00	0.9026	0.9721	1	0.7057	0.8272	0.9230	1	0.6542	0.6238	0.8307	1	0.5758
0.75	0.50	0.6321	0.7887	1	0.3488	0.6799	0.8397	1	0.3683	0.8078	0.8902	1	0.4001
0.75	0.80	1.7689	1.2762	1	0.5449	1.8899	1.2898	1	0.5590	2.3647	1.2805	1	0.5952
0.75	1.00	1.1997	1.0461	1	0.4325	1.1305	1.0738	1	0.4343	1.1764	1.1202	1	0.4558
0.75	1.20	0.9894	0.9741	1	0.4025	0.9021	0.9549	1	0.3859	0.8373	0.9326	1	0.3748
0.75	1.50	0.9332	0.9730	1	0.4052	0.8705	0.9347	1	0.3880	0.7399	0.8606	1	0.3712
0.75	2.00	0.9825	0.9983	1	0.4148	0.9854	0.9946	1	0.4107	0.9756	0.9839	1	0.4238

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II}$ with respect to $\hat{\lambda}^q_{II}$.

EF2 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II(z_1)}$ with respect to $\hat{\lambda}^q_{II}$.

EF3 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II}$ with respect to $\tilde{\lambda}^q_{II}$.

EF4 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II(z_1)}$ with respect to $\tilde{\lambda}^q_{II}$.

TABLE 4

Relative efficiencies of the estimators of λ^q (when $\lambda = 1$) under Type I censoring

t_0	λ_0	EF1	EF2	EF1	EF2	EF1	EF2
		$q=1$		$q=2$		$q=3$	
0.40	0.50	0.6461	0.6461	0.6346	0.6029	0.6397	0.6216
0.40	0.80	1.2914	1.2914	1.2013	1.2629	1.1936	1.2311
0.40	1.00	1.7668	1.7668	1.6159	1.8480	1.6944	1.8080
0.40	1.20	1.4073	1.4073	1.1390	1.3691	1.0067	1.2771
0.40	1.50	1.1874	1.1874	0.9278	1.0823	0.7983	0.9753
0.40	2.00	1.1360	1.1360	0.9602	1.0596	0.9685	0.9969
0.80	0.50	0.7341	0.7341	0.7227	0.7177	0.7185	0.7505
0.80	0.80	1.3621	1.3621	1.2415	1.3386	1.1937	1.2848
0.80	1.00	1.4568	1.4568	1.2901	1.4623	1.2468	1.3934
0.80	1.20	1.3152	1.3152	1.1206	1.2930	1.0488	1.2244
0.80	1.50	1.2051	1.2051	1.0248	1.1630	0.9891	1.1025
0.80	2.00	1.1289	1.1289	0.9989	1.0842	0.9993	1.0430
1.00	0.50	0.6913	0.6913	0.7002	0.6753	0.7101	0.7123
1.00	0.80	1.3767	1.3767	1.2750	1.3688	1.1996	1.3145
1.00	1.00	1.4551	1.4551	1.2830	1.4619	1.2200	1.3692
1.00	1.20	1.2947	1.2947	1.1164	1.2760	1.0357	1.1983
1.00	1.50	1.1830	1.1830	1.0250	1.1481	1.0002	1.0906
1.00	2.00	1.1101	1.1101	1.0001	1.0723	1.0000	1.0352

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_I$ with respect to $\hat{\lambda}^q_I$.

EF2 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_I$ with respect to $\tilde{\lambda}^q_I$.

TABLE 5

Relative efficiencies of the estimators for $R(t)$ under Type II censoring

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		$R=0.50$				$R=0.75$			
0.25	0.50	0.4826	0.6671	0.5184	0.7726	0.4543	0.6877	0.5730	0.8006
0.25	0.80	1.2134	1.0662	1.1221	1.3182	1.1874	1.0575	1.1782	1.3549
0.25	1.00	2.2886	1.5048	1.2024	1.6099	2.4157	1.5488	1.2441	1.6898
0.25	1.20	1.5347	1.1952	0.9509	1.2630	1.4593	1.2089	0.9444	1.3050
0.25	1.50	1.1307	0.9932	0.9715	1.0372	0.9988	0.9787	0.9906	1.0470
0.25	2.00	1.0397	0.9846	1.0000	0.9948	0.9431	0.9754	1.0000	0.9940
0.50	0.50	0.6648	0.8923	0.9962	0.9241	0.6743	0.9071	1.0000	0.9346
0.50	0.80	1.0052	0.9295	0.7865	1.0041	1.0404	0.9328	0.8052	1.0156
0.50	1.00	1.7236	1.3249	1.2220	1.3668	1.8366	1.3440	1.2452	1.4041
0.50	1.20	1.2417	1.1043	0.9311	1.1203	1.2344	1.1225	0.9518	1.1370
0.50	1.50	0.9763	0.9622	0.9487	0.9670	0.9269	0.9555	0.9315	0.9532
0.50	2.00	0.9055	0.9612	0.9746	0.9644	0.8515	0.9387	0.9769	0.9408
0.75	0.50	0.6165	0.8996	0.9563	0.9153	0.6134	0.9120	0.9654	0.9277
0.75	0.80	1.0814	1.0116	0.9927	1.1068	1.0801	1.0040	0.9986	1.1137
0.75	1.00	1.7607	1.2625	1.0880	1.2841	1.7554	1.2711	1.0897	1.3006
0.75	1.20	1.3471	1.0571	0.9917	1.0593	1.2963	1.0654	0.9957	1.0663
0.75	1.50	1.1032	0.9837	0.9793	0.9837	1.0325	0.9785	0.9712	0.9762
0.75	2.00	1.0129	0.9921	0.9965	0.9930	0.9585	0.9870	0.9933	0.9885

EF1 denotes the relative efficiency of $\hat{\hat{R}}_{II}$ with respect to $\hat{R}_{II}$.
EF2 denotes the relative efficiency of $\hat{\hat{R}}_{II(z_3)}$ with respect to $\hat{R}_{II}$.
EF3 denotes the relative efficiency of $\tilde{\tilde{R}}_{II}$ with respect to $\tilde{R}_{II}$.
EF4 denotes the relative efficiency of $\tilde{\tilde{R}}_{II(z_3)}$ with respect to $\tilde{R}_{II}$.

TABLE 6

Relative efficiencies of the estimators for $R(t)$ under Type II censoring(continued)

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		$R=0.90$				$R=0.95$			
0.25	0.50	0.4394	0.6971	0.5969	0.8125	0.4356	0.6999	0.6033	0.8159
0.25	0.80	1.1764	1.0543	1.2033	1.3704	1.1730	1.0534	1.2108	1.3748
0.25	1.00	2.5065	1.5695	1.2635	1.7270	2.5350	1.5758	1.2685	1.7381
0.25	1.20	1.4273	1.2148	0.9410	1.3251	1.4181	1.2165	0.9395	1.3311
0.25	1.50	0.9364	0.9701	0.9883	1.0518	0.9178	0.9673	0.9876	1.0533
0.25	2.00	0.9034	0.9698	1.0000	0.9937	0.8934	0.9678	1.0000	0.9936
0.50	0.50	0.6831	0.9140	1.0012	0.9390	0.6866	0.9160	1.0015	0.9403
0.50	0.80	1.0687	0.9353	0.8121	1.0206	1.0790	0.9362	0.8152	1.0221
0.50	1.00	1.9166	1.3498	1.2484	1.4196	1.9452	1.3511	1.2505	1.4240
0.50	1.20	1.2399	1.1318	0.9628	1.1442	1.2427	1.1347	0.9657	1.1463
0.50	1.50	0.9041	0.9532	0.9208	0.9454	0.8974	0.9527	0.9198	0.9429
0.50	2.00	0.8262	0.9253	0.9688	0.9262	0.8189	0.9209	0.9774	0.9213
0.75	0.50	0.6135	0.9173	0.9689	0.9327	0.6138	0.9189	0.9699	0.9341
0.75	0.80	1.0833	1.0009	1.0001	1.1163	1.0847	1.0000	1.0024	1.1171
0.75	1.00	1.7622	1.2744	1.0873	1.3079	1.7651	1.2753	1.0879	1.3100
0.75	1.20	1.2785	1.0697	0.9959	1.0695	1.2736	1.0711	0.9956	1.0705
0.75	1.50	1.0043	0.9757	0.9652	0.9719	0.9965	0.9749	0.9666	0.9705
0.75	2.00	0.9478	0.9838	0.9910	0.9857	0.9456	0.9827	0.9902	0.9848

EF1 denotes the relative efficiency of $\hat{\hat{R}}_{II}(t)$ with respect to $\hat{R}_{II}(t)$.
EF2 denotes the relative efficiency of $\hat{\hat{R}}_{II(z_3)}(t)$ with respect to $\hat{R}_{II}(t)$.
EF3 denotes the relative efficiency of $\tilde{\tilde{R}}_{II}(t)$ with respect to $\tilde{R}_{II}(t)$.
EF4 denotes the relative efficiency of $\tilde{\tilde{R}}_{II(z_3)(t)}$ with respect to $\tilde{R}_{II(t)}$.

TABLE 7

Relative efficiencies of the estimators for $R(t)$ under Type I censoring

t_0	λ_0	EF1	EF2	EF1	EF2	EF1	EF2	EF1	EF2
		$R = 0.50$		$R = 0.75$		$R = 0.90$		$R = 0.95$	
0.40	0.50	0.6629	0.7064	0.7111	0.7347	0.7368	0.7465	0.7450	0.7499
0.40	0.80	1.2075	1.2394	1.2759	1.2991	1.3164	1.3268	1.3297	1.3351
0.40	1.00	2.9467	2.9666	2.9424	2.9719	2.9710	2.9858	2.9835	2.9913
0.40	1.20	1.4074	1.3897	1.3132	1.3131	1.2794	1.2804	1.2703	1.2710
0.40	1.50	0.7791	0.7190	0.7070	0.6860	0.6783	0.6711	0.6701	0.6667
0.40	2.00	0.7992	0.7282	0.7679	0.7425	0.7587	0.7501	0.7565	0.7524
0.80	0.50	0.5768	0.5701	0.5584	0.5550	0.5500	0.5487	0.5475	0.5468
0.80	0.80	0.7939	0.7572	0.7851	0.7702	0.7810	0.7756	0.7798	0.7772
0.80	1.00	2.6556	2.7642	2.5158	2.5577	2.4623	2.4773	2.4475	2.4547
0.80	1.20	1.9695	1.9481	1.7929	1.7861	1.7227	1.7205	1.7029	1.7018
0.80	1.50	1.6900	1.6835	1.5191	1.5187	1.4522	1.4522	1.4334	1.4334
0.80	2.00	1.5147	1.5208	1.3436	1.3463	1.2782	1.2792	1.2601	1.2605
1.00	0.50	0.6792	0.6749	0.6785	0.6764	0.6781	0.6773	0.6780	0.6776
1.00	0.80	1.3749	1.3470	1.3424	1.3319	1.3291	1.3254	1.3253	1.3235
1.00	1.00	1.6720	1.7077	1.5883	1.6018	1.5548	1.5596	1.5454	1.5477
1.00	1.20	1.4731	1.4976	1.3819	1.3904	1.3458	1.3486	1.3355	1.3369
1.00	1.50	1.3353	1.3511	1.2467	1.2516	1.2121	1.2137	1.2024	1.2032
1.00	2.00	1.2491	1.2599	1.1634	1.1664	1.1308	1.1316	1.1217	1.1221

EF1 denotes the relative efficiency of $\hat{\hat{R}}_I(t)$ with respect to $\hat{R}_I(t)$.
EF2 denotes the relative efficiency of $\tilde{\tilde{R}}_I(t)$ with respect to $\tilde{R}_I(t)$.

TABLE 8

Relative efficiencies of the estimators for 'P' under Type II censoring

r_1	r_2	P_0	EF1	EF2	EF3	EF4	P_0	EF1	EF2	EF3	EF4
			$P = 0.65$					$P = 0.70$			
20	20	0.55	1.0121	0.9420	5.9052	1.4354	0.60	0.9854	0.9588	13.3118	1.5228
20	20	0.60	1.2474	1.0786	23.0536	2.0709	0.65	1.3861	1.1716	49.0326	2.0088
20	20	0.65	1.5541	1.2660	14914.8	2.6950	0.70	1.8053	1.3173	4779.9	2.4522
20	20	0.70	1.4210	1.2218	26.1854	2.3816	0.75	1.5164	1.1551	72.9049	2.2493
20	20	0.75	1.0357	1.0150	6.4050	1.6080	0.80	1.0842	0.9574	16.6895	1.5540
20	20	0.80	0.7820	0.8798	2.8418	1.1690	0.85	0.8927	0.9085	7.2634	1.1297
20	30	0.55	0.9186	0.8786	3.9075	1.3965	0.60	0.9906	0.9475	1.9704	1.5185
20	30	0.60	1.2748	1.0860	4.0214	1.9248	0.65	1.3136	1.1214	1.7700	1.9720
20	30	0.65	1.7550	1.3397	2.8278	2.4656	0.70	1.6410	1.2526	1.5454	2.2898
20	30	0.70	1.5326	1.2520	1.9901	2.2001	0.75	1.4033	1.1353	1.3177	1.9483
20	30	0.75	1.0638	1.0055	1.4392	1.4927	0.80	1.0404	0.9756	1.1613	1.3569
20	30	0.80	0.7842	0.8579	1.1106	1.1080	0.85	0.8446	0.9102	1.0657	1.0615
30	20	0.55	1.0967	0.9667	5.8190	1.6049	0.60	0.9367	0.9329	13.8883	1.5188
30	20	0.60	1.3284	1.0911	23.2760	2.7757	0.65	1.3809	1.1776	55.5514	2.5931
30	20	0.65	1.7411	1.3348	1.3E+11	2.9952	0.70	2.0888	1.4334	3.20E+10	3.3305
30	20	0.70	1.3981	1.2209	23.2771	1.8293	0.75	1.3809	1.0919	55.5602	2.0360
30	20	0.75	0.8209	0.8784	5.8192	1.2318	0.80	0.8765	0.8568	13.8896	1.2705
30	20	0.80	0.6332	0.8052	2.5863	1.0391	0.85	0.8077	0.9197	6.1731	1.0390
30	30	0.55	0.9815	0.8979	5.5508	1.5451	0.60	0.9464	0.9089	12.3771	1.5973
30	30	0.60	1.3720	1.1209	21.9693	2.6064	0.65	1.3199	1.1099	47.7006	2.6192
30	30	0.65	2.2106	1.5546	86361.2	2.6214	0.70	1.8832	1.3638	23750.8	2.6315
30	30	0.70	1.2509	1.1213	23.2921	1.5920	0.75	1.3498	1.1188	57.0328	1.5783
30	30	0.75	0.6974	0.7898	5.7687	1.1357	0.80	0.8339	0.8624	13.7274	1.1259
30	30	0.80	0.6334	0.8558	2.5635	1.0162	0.85	0.7610	0.9228	6.0498	1.0105

EF1 denotes the relative efficiency of $\hat{\hat{P}}_{II}$ with respect to $\hat{P}_{II}$.
EF2 denotes the relative efficiency of $\hat{\hat{P}}_{II(z_3)}$ with respect to $\hat{P}_{II}$.
EF3 denotes the relative efficiency of $\tilde{\tilde{P}}_{II}$ with respect to $\tilde{P}_{II}$.
EF4 denotes the relative efficiency of $\tilde{\tilde{P}}_{II(z_3)}$ with respect to $\tilde{P}_{II}$.

TABLE 9
Relative efficiencies of the estimators for 'P' under Type II censoring(continued)

r_1	r_2	P_0	EF1	EF2	EF3	EF4
				$P = 0.80$		
20	20	0.70	0.8422	0.8751	29.2511	1.3479
20	20	0.75	1.3274	1.1651	87.3638	1.8756
20	20	0.80	1.9782	1.3239	1047.55	2.5060
20	20	0.85	1.5054	1.0631	451.873	2.0181
20	20	0.90	1.0573	0.9261	68.1037	1.2382
20	20	0.95	1.0426	0.9979	26.4597	1.0023
20	30	0.70	0.7933	0.8328	1.0016	1.3615
20	30	0.75	1.3144	1.1481	1.0000	1.9387
20	30	0.80	2.0246	1.3189	1.0000	2.3808
20	30	0.85	1.4749	1.0373	1.0000	1.7976
20	30	0.90	1.0824	0.9444	1.0000	1.1416
20	30	0.95	1.0877	0.9995	1.0000	1.0005
30	20	0.70	0.7612	0.8284	36.0374	1.2332
30	20	0.75	1.1931	1.0923	144.128	1.8218
30	20	0.80	2.2163	1.4332	5.80E+09	3.1601
30	20	0.85	1.2806	0.9766	144.219	2.0786
30	20	0.90	0.9700	0.9333	36.0494	1.1071
30	20	0.95	1.0057	0.9995	16.0213	1.0003
30	30	0.70	0.7026	0.7821	31.7038	1.2210
30	30	0.75	1.1393	1.0721	107.588	1.8667
30	30	0.80	2.2581	1.4084	3879.76	3.2945
30	30	0.85	1.2477	0.9521	241.598	1.6907
30	30	0.90	1.0492	0.9657	48.5012	1.0407
30	30	0.95	1.0893	1.0000	20.2647	1.0000

EF1 denotes the relative efficiency of $\hat{\hat{P}}_{II}$ with respect to $\hat{P}_{II}$.
EF2 denotes the relative efficiency of $\hat{\hat{P}}_{II(z_3)}$ with respect to $\hat{P}_{II}$.
EF3 denotes the relative efficiency of $\tilde{\tilde{P}}_{II}$ with respect to $\tilde{P}_{II}$.
EF4 denotes the relative efficiency of $\tilde{\tilde{P}}_{II(z_3)}$ with respect to $\tilde{P}_{II}$.

TABLE 10
Relative efficiencies of the estimators for 'P' under Type I censoring

t_{ox}	t_{oy}	P_0	EF1	EF2	P_0	EF1	EF2	P_0	EF1	EF2
			$P = 0.65$			$P = 0.70$			$P = 0.80$	
0.4	0.4	0.55	2.2583	2.0587	0.60	5.6784	3.3471	0.70	1.9153	1.002
0.4	0.4	0.6	8.9841	8.2891	0.65	22.7294	13.4013	0.75	7.6432	1.0031
0.4	0.4	0.65	30750	89954	0.70	54453	474438	0.80	93031	1.0092
0.4	0.4	0.7	9.1101	8.2176	0.75	22.5662	13.3375	0.85	7.6981	1.0076
0.4	0.4	0.75	2.257	2.0575	0.80	5.6496	3.3353	0.90	1.9157	1.0044
0.4	0.4	0.8	1.0032	0.9146	0.85	2.5118	1.4824	0.95	0.8515	1.002
0.4	0.8	0.55	1.0682	0.7037	0.60	3.602	1.0873	0.70	1.68	1.0015
0.4	0.8	0.6	4.2675	2.8093	0.65	14.4034	4.3632	0.75	6.7473	1.0024
0.4	0.8	0.65	126977	583986	0.70	2253288	200075	0.80	223444	1.0088
0.4	0.8	0.7	4.2687	2.8107	0.75	14.3808	4.3512	0.85	6.7195	1.0071
0.4	0.8	0.75	1.0706	0.7041	0.80	3.5954	1.0888	0.90	1.6813	1.0051
0.4	0.8	0.8	0.4762	0.3131	0.85	1.598	0.484	0.95	0.7474	1.0036
0.8	0.4	0.55	2.2548	9.4685	0.60	4.0636	12.7057	0.70	1.8509	1.0169
0.8	0.4	0.6	9.0065	37.8731	0.65	16.2706	50.8228	0.75	7.3974	1.0187
0.8	0.4	0.65	190833	Inf	0.70	72472	Inf	0.80	339837	1.0247
0.8	0.4	0.7	9.0629	37.8731	0.75	16.1754	50.8228	0.85	7.4247	1.0233
0.8	0.4	0.75	2.2552	9.4683	0.80	4.0485	12.7057	0.90	1.8511	1.0219
0.8	0.4	0.8	1.0004	4.2081	0.85	1.7998	1	0.95	0.8224	1.0204
0.8	0.8	0.55	0.5813	4.8362	0.60	2.2676	7.1884	0.70	0.963	1.0178
0.8	0.8	0.6	2.3223	19.3415	0.65	9.072	28.7529	0.75	3.8587	1.0196
0.8	0.8	0.65	17265458	Inf	0.70	2463709	Inf	0.80	289845	1.0255
0.8	0.8	0.7	2.3197	19.3405	0.75	9.0611	28.7529	0.85	3.8518	1.0242
0.8	0.8	0.75	0.581	4.8351	0.80	2.2654	7.1882	0.90	0.9636	1.0228
0.8	0.8	0.8	0.2584	2.1489	0.85	1.0068	3.1948	0.95	0.4283	1.0213

EF1 denotes the relative efficiency of $\hat{\hat{P}}_I$ with respect to $\hat{P}_I$.
EF2 denotes the relative efficiency of $\tilde{\tilde{P}}_I$ with respect to $\tilde{P}_I$.

ACKNOWLEDGEMENTS

We are thankful to the referee for his valuable comments, which led to considerable improvement in the original version.

REFERENCES

S. Ahmed (2001). *Shrinkage estimation of regression coefficients from censored data with multiple observations.* In *Empirical Bayes and Likelihood Inference*, Springer, pp. 103–120.

M. Ahsanullah (1980). *Linear prediction of record values for the two parameter exponential distribution.* Annals of the Institute of Statistical Mathematics, 32, part A, pp. 363–368.

A. M. Awad, M. K. Gharraf (1986). *Estimation of $P(Y < X)$ in the Burr case: A Comparative Study.* Communications in Statistics-Simulation and Computation, 15, no. 2, pp. 389–403.

A. Baklizi (2003). *Shrinkage estimation of the exponential reliability with censored data.* Focus on Applied Statistics, pp. 195–204.

A. Baklizi, W. Abu Dayyeh (2003). *Shrinkage estimation of $P(Y < X)$ in the exponential case.* Communications in Statistics-Simulation and Computation, 32, no. 1, pp. 31–42.

D. J. Bartholomew (1957). *A problem in life testing.* Journal of the American Statistical Association, 52, pp. 350–355.

D. J. Bartholomew (1963). *The sampling distribution of an estimate arising in life testing.* Technometrics, 5, pp. 361–374.

A. Basu (1964). *Estimates of reliability for some distributions useful in life testing.* Technometrics, 6, pp. 215–219.

R. C. Blattberg, E. I. George (1991). *Shrinkage estimation of price and promotional elasticities: Seemingly unrelated equations.* Journal of the American Statistical Association, 86, no. 414, pp. 304–315.

I. W. Burr (1942). *Cumulative frequency functions.* The Annals of mathematical statistics, 13, pp. 215–232.

I. W. Burr, P. J. Cislak (1968). *On a general system of distributions i. its curve-shape characteristics ii. the sample median.* Journal of the American Statistical Association, 63, pp. 627–635.

M. Carreras, W. Brannath (2013). *Shrinkage estimation in two-stage adaptive designs with midtrial treatment selection.* Statistics in Medicine, 32, no. 10, pp. 1677–1690.

A. Chao (1982). *On comparing estimators of Pr* $\{Y < X\}$ *in the exponential case*. Reliability, IEEE Transactions, R-26, pp. 389–392.

A. Chaturvedi, K. Chauhan, M. W. Alam (2009). *Estimation of the reliability function for a family of lifetime distributions under type i and type ii censorings*. Journal of Reliability and Statistical Studies, 2, no. 2, pp. 11–30.

A. Chaturvedi, U. Rani (1997). *Estimation procedures for a family of density functions representing various life-testing models*. Metrika, 46, pp. 213–219.

A. Chaturvedi, U. Rani (1998). *Classical and bayesian reliability estimation of the generalized maxwell failure distribution*. Jour. Statist. Res, 32, pp. 113–120.

A. Chaturvedi, K. Surinder (1999). *Further remarks on estimating the reliability function of exponential distribution under type i and type ii censorings*. Brazilian Journal of Probability and Statistics, 13, pp. 29–39.

Y. Chen, A. Wiesel, A. O. Hero III (2009). *Shrinkage estimation of high dimensional covariance matrices*. In *IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, pp. 2937–2940.

Y. Chen, A. Wiesel, A. O. Hero III (2010). *Robust shrinkage estimation of high-dimensional covariance matrices*. Sensor Array and Multichannel Signal Processing Workshop (SAM), 59, no. 9, pp. 4097–4107.

Z. Chen (2000). *A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function*. Statistics & Probability Letters, 49, pp. 155–161.

M. Clyde, G. Parmigiani, B. Vidakovic (1998). *Multiple shrinkage and subset selection in wavelets*. Biometrika, 85, no. 2, pp. 391–401.

K. Constantine, S.-K. Tse, M. Karson (1986). *Estimation of* $p(y < x)$ *in the gamma case*. Communications in Statistics-Simulation and Computation, 15, no. 2, pp. 365–388.

N. Ebrahimi, B. Hosmane (1987). *On shrinkage estimation of the exponential location parameter*. Communications in Statistics-Theory and Methods, 16, no. 9.

A. Erdelyi (1954). *Tables of Integral Transformations*, vol. 1. McGraw-Hill.

D. Fourdrinier, W. E. Strawderman, M. T. Wells (2003). *Robust shrinkage estimation for elliptically symmetric distributions with unknown covariance matrix*. Journal of multivariate analysis, 85, no. 1, pp. 24–39.

George (1986). *Minimax multiple shrinkage estimation*. The Annals of Statistics, 14, no. 1, pp. 188–205.

M. Ghosh, D. M. Nickerson, P. K. Sen (1987). *Sequential shrinkage estimation*. The Annals of Statistics, 15, no. 2, pp. 817–829.

N. L. Johnson (1975). *Letter to the editor.* Technometrics, 17, p. 393.

N. L. Johnson, S. Kotz, N. Balakrishnan (1994). *Continuous univariate distributions, vol. 1-2.*

G. D. Kelley, J. A. Kelley, W. Schucany (1976). *Efficient estimation of $P(Y < X)$ in the exponential case.* Technometrics, 18, pp. 359–360.

R. Khan, B. Zia (2009). *Recurrence relations for single and product moments of record values from gompertz distribution and a characterization.* World Applied Sciences Journal, 7, no. 10, pp. 1331–1334.

E. D. Kolaczyk (1999). *Wavelet shrinkage estimation of certain poisson intensity signals using corrected thresholds.* Statistica Sinica, pp. 119–135.

T. Kubokawa (1998). *Double shrinkage estimation of common coefficients in two regression equations with heteroscedasticity.* Journal of Multivariate Analysis, 67, no. 2, pp. 169–189.

C. Lai, M. Xie, D. Murthy (2003). *A modified Weibull model.* Reliability, IEEE Transactions on, 52, no. 1, pp. 33–37.

O. Ledoit, M. Wolf (2012). *Nonlinear shrinkage estimation of large-dimensional covariance matrices.* The Annals of Statistics, 40, no. 2, pp. 1024–1060.

Z. Liao (2013). *Adaptive gmm shrinkage estimation with consistent moment selection.* Econometric Theory, 29, no. 05, pp. 857–904.

M. Ljubo (1965). *Curves and concentration indices for certain generalized pareto distributions.* Statistical Review, 15, pp. 257–260.

K. Lomax (1954). *Business failures: Another example of the analysis of failure data.* Journal of the American Statistical Association, 49, no. 268, pp. 847–852.

N. T. Longford (1999). *Multivariate shrinkage estimation of small area means and proportions.* Journal of the Royal Statistical Society: Series A (Statistics in Society), 162, no. 2, pp. 227–245.

M. Mahmoud, H. S. Al-Nagar (2009). *On generalized order statistics from linear exponential distribution and its characterization.* Statistical Papers, 50, no. 2, pp. 407–418.

M. Nikulin, F. Haghighi (2006). *A chi-squared test for the genralized power weibull family for the head-and-neck cancer censored data.* Journal of Mathematical Sciences, 133, no. 3, pp. 1333–1341.

B. Pandey (1983). *Shrinkage estimation of the exponential scale parameter.* IEEE Transactions on Reliability, 2, pp. 203–205.

C. Philipson (1963). *A note on moments of a poisson probability distribution.* Scandinavian Actuarial Journal, 1963, no. 3-4, pp. 243–244.

A. C. Pope, I. Szapudi (2008). *Shrinkage estimation of the power spectrum covariance matrix.* Monthly Notices of the Royal Astronomical Society, 389, no. 2, pp. 766–774.

G. Prakash, D. Singh (2008). *Shrinkage estimation in exponential type-ii censored data under linex loss.* Journal of the Korean Statistical Society, 37, no. 1, pp. 53–61.

E. L. Pugh (1963). *The best estimate of reliability in the exponential case.* Operations research, 11, no. 1, pp. 57–61.

J. A. Royle, W. A. Link (2002). *Random effects and shrinkage estimation in capture-recapture models.* Journal of Applied Statistics, 29, no. 1-4, pp. 329–351.

Y. Sathe, S. Shah (1981). *On estimating $P(X < Y)$ for the exponential distribution.* Communications in Statistics-Theory and Methods, 10, no. 1, pp. 39–47.

L. Şendur, I. W. Selesnick (2002). *Bivariate shrinkage with local variance estimation.* Signal Processing Letters, IEEE, 9, no. 12, pp. 438–441.

S. K. Sinha (1986). *Reliability and life testing.* Wiley Eastern Limited, New Delhi.

T. Siu-Keung, T. Geoffrey (1996). *Shrinkage estimation of reliability for exponentially distributed lifetimes.* Communications in Statistics-Simulation and Computation, 25, no. 2, pp. 415–430.

P. R. Tadikamalla (1980). *A look at the burr and related distributions.* International Statistical Review/Revue Internationale de Statistique, pp. 337–344.

J. R. Thompson (1968). *Some shrinkage techniques for estimating the mean.* Journal of the American Statistical Association, 63, no. 321, pp. 113–122.

H. Tong (1974). *A Note on the Estimation of $P(Y < X)$ in the Exponential Case.* Technometrics, 16, no. 4, pp. 625–625.

H. Tong (1975). *Letter to the editor.* Technometrics, 17, p. 393.

R. Tyagi, S. Bhattacharya (1989). *A note on the mvu estimation of reliability for the maxwell failure distribution.* Estadistica, 41, no. 137, pp. 73–79.

G. N. Watson (1952). *Treatise on the theory of Bessell Functions (2nd ed.* Cambridge University Press, London.

Summary

A family of distributions is considered, which covers many lifetime distributions as specific cases. Two measures of reliability are considered, $R(t) = P(X > t)$ and $P = P(X > Y)$. Shrinkage estimators are considered for the powers of parameter, R(t) and 'P' under type I and type II censorings. Simulation study is conducted to judge the performance of estimators.

Keywords: Family of lifetime distributions; shrinkage estimation; type I and type II censorings; p-value