STATISTICA, anno LXXV, n. 4, 2015

EXPONENTIAL CHAIN DUAL TO RATIO AND REGRESSION TYPE ESTIMATORS OF POPULATION MEAN IN TWO-PHASE SAMPLING

Garib Nath Singh
Department of Applied mathematics, Indian School of Mines, Dhanbad
Mohd Khalid [1]
Department of Applied mathematics, Indian School of Mines, Dhanbad

1. Introduction

It is customary to utilize the information on auxiliary variables to improve the precision of estimates in sample surveys, if such is readily available or may be made available by incurring some additional cost. Ratio, product and regression methods of estimation are the popular estimation procedures which utilize the information on auxiliary variable in terms of known population parameters at estimation stage. Sometimes, the population parameters of auxiliary variable may not be readily known at the time of generating estimates of population parameters of study variable. Two-phase (double) sampling happens to be a cost effective technique for generating the most reliable estimates of unknown population parameters of auxiliary variable in first phase sample. Chand (1975) introduced a technique of chaining the information of the second auxiliary variable to get a more precise estimate of the unknown population mean of first auxiliary variable in the first-phase sample in form of ratio estimator and which is termed as chain-type ratio estimator. Further, his work was extended by Kiregyera (1980, 1984); Mukerjee *et al.* (1987), Srivnstava *et al.* (1989), Upadhyaya *et al.* (1990), Singh and Singh (1991), Singh *et al.* (1994), Singh and Upadhyaya (1995), Upadhyaya and Singh (2001), Singh (2001), Kadılar and Cıngı (2003); Kadilar and Cingi (2004), Pradhan (2005), Kadılar *et al.* (2007), Gupta and Shabbir (2008), Singh *et al.* (2008), Choudhury and Singh (2012), Ozgul and Cingi (2014), Singh and Majhi (2014), among others. Bahl and Tuteja (1991) suggested exponential methods of estimation of population mean of study variable, which was further followed by Singh and Vishwakarma (2007), who studied the behavior of the exponential method of estimation in two-phase sampling. In follow up of the above works and motivated with the generic nature of exponential method of estimation, we have proposed some new exponential chain-type estimators which are more relevant in producing

[1] Corresponding Author. E-mail: mohdkhalidkhan4870@gmail.com

the precise estimates of the population mean of the study variable. Theoretical and empirical properties of the proposed estimation procedures have been studied and suitable recommendations are put forward to the survey practitioners.

2. Two-phase sampling setup

Let $U = (U_1, U_2, \ldots, U_N)$ be a finite population of size N indexed by triplet characters (y, x, z). Our interest is to estimate the population mean $\bar{Y}$ of study variable y in the presence of two auxiliary variables x and z. Let x and z be the first and second auxiliary variables respectively, such that y is highly correlated with x while in compare to x, it is remotely correlated with z $(\rho_{yx} > \rho_{yz})$. When population mean $\bar{X}$ of the first auxiliary variable x is unknown but information on second auxiliary variable z is available for all the units of the population, we use the two-phase sampling design which is described as follows.
Consider a two-phase sampling design where in the first-phase a large (preliminary) sample $s^{'}(s^{'} \subset U)$of size $n^{'}$ is drawn following simple random sampling without replacement (SRSWOR) scheme and observed for two auxiliary variables x and z to estimate $\bar{X}$, while in the second phase a sub sample $s \subset s^{'}$ of size n is drawn from first-phase sample s by SRSWOR to observe the characteristic y under study.

3. Some estimators in two-phase sampling

In this section, we consider some estimators of population mean in two phase sampling which are frequently used in survey literature's. The estimators along with their respective mean square errors are presented in the following sub- sections.

3.1. *Estimators based on one auxiliary variable in two-phase sampling setup*

Ratio and regression methods of estimation require the known population mean of auxiliary variable and in the absence of such information, we go for two-phase sampling setup, where, we estimate the unknown population mean of auxiliary variable in the first- phase sample. For ready-made assessment the ratio and regression estimators respectively in two-phase sampling setup along with their respective mean square errors up to the first order of approximations are reproduced below as:

$$\bar{y}_{rd} = \frac{\bar{y}}{\bar{x}}\bar{x}^{'} \tag{1}$$

$$M(\bar{y}_{rd}) = \bar{Y}^2 \left[f_1 C_y^2 + f_3 \left(C_x^2 - 2\rho_{yx} C_y C_x\right)\right] \tag{2}$$

$$\bar{y}_{lrd} = \bar{y} + b_{yx}(n)(\bar{x}^{'} - \bar{x}) \tag{3}$$

and

$$M(\bar{y}_{lrd}) = \overline{Y}^2 \left[f_1(1-\rho_{yx}^2) + f_2\rho_{yx}^2\right] C_y^2 \tag{4}$$

where $b_{yx}(n)$ is the sample regression coefficient of y on x based on sample s and

$$\bar{y} = \frac{1}{n}\sum_{i\in s} y_i,\quad \bar{x} = \frac{1}{n}\sum_{i\in s} x_i,\quad \bar{x}^{'} = \frac{1}{n^{'}}\sum_{i\in s^{'}} x_i,\quad f_1 = \left(\frac{1}{n} - \frac{1}{N}\right),$$

$$f_2 = \left(\frac{1}{n^{'}} - \frac{1}{N}\right),\ f_3 = \left(\frac{1}{n} - \frac{1}{n^{'}}\right) = (f_1 - f_2)\,,\ C_x = \frac{S_x}{\bar{X}}, C_y = \frac{S_y}{\bar{Y}},$$

$$k = \frac{n'}{N-n'},\quad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2,\quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2,$$

and ρ_{yx} be the correlation coefficient between the variables y and x.

3.2. Estimators based on two auxiliary variables in two-phase sampling setup

Sometimes, information on more than one auxiliary variable is available at the estimation stage in sample surveys. Chand (1975) introduced a chain- type ratio estimator under two-phase sampling setup where he utilized the information on two auxiliary variables x and z under the assumption that the population mean $\bar{X}$ of the auxiliary variable x is unknown and information on auxiliary variable z is available for all the units of the population. The suggested estimator with their respective mean square error up to the first order approximations are reproduced as:

$$\bar{y}_{rc} = \frac{\bar{y}}{\bar{x}}\frac{\bar{x}^{'}}{\bar{z}^{'}}\bar{Z} \tag{5}$$

$$M(\bar{y}_{rc}) = \bar{Y}^2\left[f_1 C_y^2 + f_3(C_x^2 - 2\rho_{yx}C_xC_y) + f_2(C_z^2 - 2\rho_{yz}C_yC_z)\right] \tag{6}$$

where $\bar{Z}$ is the population mean of the auxiliary variable z,

$$\bar{z}^{'} = \frac{1}{n^{'}}\sum_{i^{'}\in s} z_i,\quad C_z = \frac{S_z}{\bar{Z}},\quad S_z^2 = \frac{1}{N-1}\sum_{i=1}^{N}(z_i - \bar{Z})^2$$

and ρ_{yz} be the correlation coefficient between the variables y and z.

Further, Kiregyera (1980, 1984) extended the work of Chand (1975) and suggested chain-type ratio to regression, regression to ratio and regression to regression estimators of the population mean $\bar{Y}$of study variable y in two- phase sampling setup which utilized the information on two auxiliary variables x and z under similar assumptions as considered by Chand (1975). The suggested estimators are given below with their respective mean square errors up to the first order of approximations as:

$$\bar{y}_{k1} = \frac{\bar{y}}{\bar{x}}\left[\bar{x}^{'} + b_{xz}(n^{'})(\bar{Z} - \bar{z}^{'})\right] \tag{7}$$

$$M(\bar{y}_{k1}) = \bar{Y}^2\left[f_3(C_x^2 + C_y^2 - 2\rho_{yx}C_xC_y) + f_2C_y^2 + f_2\rho_{xz}C_x(\rho_{xz}C_x - 2\rho_{yz}C_y)\right] \tag{8}$$

$$\bar{y}_{k2} = \bar{y} + b_{yx}(n)(\bar{x}^{'}_{rd} - \bar{x}) \tag{9}$$

where $\bar{x}'_{rd} = \frac{\bar{x}'}{\bar{z}'}\bar{Z}$

$$M(\bar{y}_{k_2}) = \bar{Y}^2 C_y^2 \left[f_1(1-\rho_{yx}^2) + f_2 \left(\rho_{yx}^2 + \rho_{yx}^2 \frac{C_z^2}{C_x^2} - 2\rho_{yx}\rho_{yz}\frac{C_z}{C_x} \right) \right] \tag{10}$$

$$\bar{y}_{k_3} = \bar{y} + b_{yx}(n)(x'_{ld} - \bar{x}) \tag{11}$$

where $\bar{x}'_{ld} = \left[\bar{x}' + b_{xz}(n')(\bar{Z} - \bar{z}') \right]$
and

$$M(\bar{y}_{k_3}) = \bar{Y}^2 C_y^2 \left[f_3(1-\rho_{yx}^2) + f_2(1 + \rho_{xz}^2\rho_{yx}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}) \right] \tag{12}$$

where $b_{xz}(n')$ is the sample regression coefficient of the variable x on z based on the sample s$'$ and ρ_{xz} be correlation coefficient between variables x and z.

4. The proposed exponential chain-type dual to ratio and regression estimators

The intelligible use of auxiliary information for making estimates more precise has been a fascinating area of research in sample surveys. In the presence of auxiliary information Bhal and Tuteja (1991) suggested an exponential type estimator to estimate the population mean of study variable and their work was further extended by Singh and Vishwakarma (2007) in two-phase sampling setup. Motivated by the work related to the proposition of chain-type estimators in two-phase sampling setup, and looking at the nice behavior of the exponential type estimators, we suggest a new exponential chain-type dual to ratio and regression estimators to estimate the population mean $\bar{Y}$under two phase sampling setup with an assumption that the population mean $\bar{X}$ of the auxiliary variable x is unknown. The proposed estimators are formulated as:

$$T_1 = \bar{y}\ \exp\left[\frac{\frac{\bar{x}'}{\bar{Z}}\bar{z}^* - \bar{x}}{\frac{\bar{x}'}{\bar{Z}}\bar{z}^* + \bar{x}} \right] \tag{13}$$

and

$$T_2 = \bar{y}\ \exp\left[\frac{\bar{x}' + b_{xz}(n')(\bar{Z} - \bar{z}') - \bar{x}}{\bar{x}' + b_{xz}(n')(\bar{Z} - \bar{z}') + \bar{x}} \right], \tag{14}$$

where $\bar{z}^* = \frac{N\bar{Z} - n'\bar{z}'}{N - n'}$.

5. The Bias and Mean square errors (MSE) of the proposed estimators T_i(i = 1, 2)

Since the proposed estimators $T_i(i = 1, 2)$ are exponential chain-type dual to ratio and regression estimators, they are biased estimators of the population mean $\bar{Y}$, therefore, bias B (.) and mean square errors M (.) of the estimators T_i are derived up to the first order of approximations and presented in the following theorems:

THEOREM 5.1. *The bias of the estimators T_i(i=1, 2) to the first order of approximations are obtained as*

$$B(T_1) = \bar{Y}\left[f_3(\frac{1}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_yC_x) + f_2(\frac{k^2}{8}C_z^2 - \frac{k}{4}C_z^2 - \frac{k}{2}\rho_{yz}C_yC_z)\right] \qquad (15)$$

and

$$B(T_2) = \bar{Y}\left[\begin{array}{l} f_3(\frac{3}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_yC_x) \\ +f_2(\frac{5}{8}\rho_{xz}^2C_x^2 - \frac{1}{2}\rho_{xz}\frac{C_x}{C_z}\frac{\alpha_{102}}{\bar{Z}S_{xz}} + \frac{1}{2}\rho_{xz}\frac{C_x}{C_z}\frac{\alpha_{003}}{\bar{Z}S_z^2} - \frac{1}{2}\rho_{xz}\rho_{yz}C_xC_y) \end{array}\right] \qquad (16)$$

THEOREM 5.2. *The mean square errors of the estimators T_i(i=1, 2) to the first order of approximations are obtained as*

$$M(T_1) = \bar{Y}^2\left[f_1C_y^2 + f_2(\frac{k^2}{4}C_z^2 - k\rho_{yz}C_yC_z) + \frac{1}{4}f_3(C_x^2 - 4\rho_{yx}C_yC_x)\right] \qquad (17)$$

and

$$M(T_2) = \bar{Y}^2\left[f_1C_y^2 + \frac{1}{4}f_2(\rho_{xz}^2C_x^2 - 4\rho_{yz}\rho_{xz}C_yC_x) + \frac{1}{4}f_3(C_x^2 - 4\rho_{yx}C_yC_x)\right] \qquad (18)$$

6. Comparison of the estimators

In this section, we compare the proposed estimators $T_i(i = 1, 2)$ with respect to the estimators $\bar{y}_{rd}$, $\bar{y}_{lrd}$, $\bar{y}_{rc}$, $\bar{y}_{k_1}$, $\bar{y}_{k_2}$, and $\bar{y}_{k_3}$ in terms of mean square criterions. The conditions for which the proposed estimators T_i will dominate other estimators under mean square criterions are defined as:

(i) The estimators $T_i(i = 1, 2)$ are better than $\bar{y}_{rd}$ if $M(T_i) \leq M(\bar{y}_{rd})$, which gives the conditions

$$\frac{(3C_x^2 - 4\rho_{yx}C_yC_x)}{(k^2C_z^2 - 4k\rho_{yz}C_zC_y)} \geq \frac{f_2}{f_3} \qquad (19)$$

and

$$\frac{(3C_x^2 - 4\rho_{yx}C_yC_x)}{(\rho_{xz}^2C_x^2 - 4\rho_{yz}\rho_{xz}C_xC_y)} \geq \frac{f_2}{f_3} \qquad (20)$$

for the estimators T_1 and T_2 respectively.

(ii) The estimators $T_i(i = 1, 2)$ are more efficient than $\bar{y}_{lrd}$ if $M(T_i) \leq M(\bar{y}_{lrd})$, such that following conditions are satisfied for the estimators $T_1$and T_2 respectively.

$$\frac{(C_x - 2\rho_{yx}C_y)^2}{(4k\rho_{yz}C_yC_z - k^2C_z^2)} \leq \frac{f_2}{f_3} \qquad (21)$$

and

$$\frac{(4\rho_{yz}C_yC_x - 4\rho_{yx}^2C_y^2 - C_x^2)}{(\rho_{xz}^2C_x^2 - 4\rho_{yz}\rho_{xz}C_yC_x)} \geq \frac{f_2}{f_3} \qquad (22)$$

(iii) The estimators $T_i (i = 1, 2)$ will dominate $\bar{y}_{rc}$ if $M(T_i) \leq M(\bar{y}_{rc})$, which gives the results

$$\frac{(C_x^2 - 4\rho_{yx} C_y C_x)}{\left\{(\frac{k^2}{4} - 1)C_z^2 - (k-2)\rho_{yz} C_y C_z)\right\}} \geq \frac{f_2}{f_3} \tag{23}$$

and

$$\frac{(\frac{3}{4} C_x^2 - \rho_{yx} C_y C_x)}{\left\{\frac{1}{4}\rho_{xz} C_x(\rho_{xz} C_x - 4\rho_{yz} C_y) + C_z(2\rho_{yz} C_y - C_z)\right\}} \geq \frac{f_2}{f_3} \tag{24}$$

for the estimators T_1 and T_2 respectively.

(iv) The estimators $T_i (i = 1, 2)$ are better than $\bar{y}_{k_1}$ if $M(T_i) \leq M(\bar{y}_{k_1})$, and subsequently, we get the following conditions

$$\frac{3(C_x^2 - 4\rho_{yx} C_y C_x)}{4\left\{(\rho_{yz} C_y - \frac{k}{2} C_z)^2 - (\rho_{xz} C_x - \rho_{yz} C_y)^2\right\}} \geq \frac{f_2}{f_3} \tag{25}$$

and

$$\frac{(C_x^2 - \frac{4}{3}\rho_{yx} C_y C_x)}{(\frac{4}{3}\rho_{xz}\rho_{yz} C_y C_x - \rho_{xz}^2 C_x^2)} \geq \frac{f_2}{f_3} \tag{26}$$

for the estimators T_1 and T_2 respectively.

(v) The estimators $T_i (i = 1, 2)$ are preferable over $\bar{y}_{k_2}$ if $M(T_i) \leq M(\bar{y}_{k_2})$, which leads to the conditions

$$\frac{(\frac{1}{2} C_x - \rho_{yx} C_y)^2}{\left\{C_y^2(2\rho_{yx}^2 - \frac{\rho_{yz}^2}{C_y^2}) + (\rho_{yx}\frac{C_z}{C_x} - \rho_{yz})^2 - C_z^2(\frac{k^2}{4} - k\rho_{yz}\frac{C_y}{C_z})\right\}} \leq \frac{f_2}{f_3} \tag{27}$$

and

$$\frac{(C_x - 2\rho_{yx} C_y)^2}{\left\{4\rho_{yx}\frac{C_z}{C_x} C_y^2(\rho_{yx}\frac{C_z}{C_x} - 2\rho_{yz}) + \rho_{xz} C_x\left(\rho_{xz} C_x - 4\rho_{yz} C_y\right)\right\}} \leq \frac{f_2}{f_3} \tag{28}$$

for the estimators T_1 and T_2 respectively.

(vi) The estimator $T_i (i = 1, 2)$ will influence over $\bar{y}_{k_3}$if $M(T_i) \leq M(\bar{y}_{k_3})$, which gives the conditions

$$\frac{(\frac{1}{2} C_x - \rho_{yx} C_y)^2}{\left\{(\rho_{yx}^2\rho_{xz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}) - (\frac{k^2}{4} C_z^2 - k\rho_{yz} C_y C_z)\right\}} \leq \frac{f_2}{f_3} \tag{29}$$

and

$$\frac{(C_x - 2\rho_{yx} C_y)^2}{\left\{4\rho_{yx}\rho_{xz} C_y^2(\rho_{yx}\rho_{xz} - 2\rho_{yz}) - \rho_{xz} c_x\left(\rho_{xz} C_x - 4\rho_{yz} C_y\right)\right\}} \leq \frac{f_2}{f_3} \tag{30}$$

for the estimators T_1 and T_2 respectively.

7. Cost aspect

The principle of optimization is an important criterion in sample surveys and to make estimators more precise for given cost is an integral part of it. Since, two-phase sampling setup is more prone to cost, therefore, it is essential to formulate a strategy which reduces the survey cost. Considering this fact, we derive the expressions for optimum sample sizes in primarily (first-phase) and final (second-phase) samples respectively by considering the suitable cost function so that the mean square errors of the estimators are minimized. In most of the practical situations total cost is the linear function of samples selected at first and second phases. Keeping this point in views, in this section, we define the following cost function which is given below:

$$C = nC_1 + n^{'}(C_2 + C_3), \tag{31}$$

where C is the total cost of the survey, C_1 is the cost per unit associated with the second phase sample of size n for collecting the information on study variable y and C_2, C_3 be the cost per unit associated with the first phase sample of size $n^{'}$ for collecting the information on auxiliary variable x and z respectively.

For large population, ignoring finite population correction (f.p.c), the mean square errors of the proposed estimators $T_i(i = 1, 2)$ can be expressed as

$$MSE(T_i) = \frac{1}{n}V_{1i} + \frac{1}{n^{'}}V_{2i}; \;\; (i = 1, 2), \tag{32}$$

where

$$V_{11} = \bar{Y}^2(C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_xC_y), V_{21} = \bar{Y}^2(\frac{k^2}{4}C_z^2 - \frac{C_x^2}{4} - k\rho_{yz}C_zC_y + \rho_{yx}C_yC_x),$$

$$V_{12} = \bar{Y}^2(C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_xC_y), V_{22} = \bar{Y}^2(\frac{\rho_{xz}^2C_x^2}{4} - \rho_{yz}\rho_{xz}C_xC_y - \frac{C_x^2}{4} + \rho_{yx}C_yC_x).$$

Therefore, fixed for cost C_0 the objective function is defined as:

$$\phi_i = M(T_i) + \lambda_i(nC_1 + n^{'}(C_2 + C_3) - C_0); \qquad (i = 1, 2), \tag{33}$$

where λ_i is the Lagrange's multipliers.

It is assumed that $C_1 > C_2 > C_3$. Utilizing the function of Lagrange's multipliers, the optimum values of n and $n^{'}$ for fixed cost C_0 which minimize the mean square errors of the estimators $T_i(i = 1, 2)$ in equation (32) are derived as:

$$n_{opt} = \left\{ \frac{C_0\sqrt{V_{1i}}}{C_1\sqrt{V_{1i}} + \sqrt{V_{2i}C_1(C_2 + C_3)}} \right\}; \quad (i = 1, 2), \tag{34}$$

and

$$n^{'}_{opt} = \left\{ \frac{C_o\sqrt{V_{2i}}}{\sqrt{(C_2 + C_3)}\{C_1\sqrt{C_1V_{1i}} + \sqrt{V_{2i}(C_2 + C_3)}\}} \right\}; \; (i = 1, 2). \tag{35}$$

Substituting, the optimum values of n and $n^{'}$ derived in equation (34) and (35) in equation (32), we get the optimum mean square errors of the estimators $T_i(\, i = 1, 2)$, as

$$MSE(T_i)_{opt} = \frac{1}{C_0}\left(C_1\sqrt{V_{1i}} + \sqrt{V_{2i}(C_2 + C_3)}\right)^2; \;\; (i = 1, 2). \tag{36}$$

8. Empirical Study

To analyze the performances of the proposed estimators $T_i(i = 1, 2)$ with respect to the other estimators, we have computed percent relative efficiency of the estimators T_i with respect to the estimators $\bar{y}_{rd}$, $\bar{y}_{lrd}$, $\bar{y}_{rc}$, $\bar{y}_{k_1}$, $\bar{y}_{k_2}$, and $\bar{y}_{k_3}$ using three natural populations. The PRE of the estimators $T_i(i = 1, 2)$ is calculated by using the following formula.

$$PRE = \left[\frac{MSE(\delta)}{MSE(T_i)}\right] \times 100 \;; \quad (i = 1, 2)\,,$$

where δ is some estimator of population mean $\bar{Y}$.

Population source-I: Cochran (1977)

Y=Number of "placebo" children.
X=Number of paralytic polio cases in the placebo group.
Z=Number of paralytic polio cases in the 'not inoculated' group.

$N = 34$, $n = 10$, $n^{'} = 15$, $\bar{Y} = 4.92$, $\bar{X} = 2.59$, $\bar{Z} = 2.91$, $C_y^2 = 1.0248$, $C_z^2 = 1.1492$, $C_x^2 = 1.5175$, $\rho_{yx} = 0.7326$, $\rho_{yz} = 0.6430$, $\rho_{xz} = 0.6837$.

Population-II: Fisher (1936)

Consisting of measurements on three variables, namely sepal width(y), sepal length(x), and petal length (z) for 50 Iris flowers (versicolor) such that:

$N = 50$, $n = 9$, $n^{'} = 20$, $\bar{Y} = 2.770$, $C_y^2 = 0.012566$,
$C_z^2 = 0.011924$, $C_x^2 = 0.007343$, $\rho_{yx} = 0.5605$, $\rho_{yz} = 0.5259$, $\rho_{xz} = 0.7540$.

Population-III: Shukla (1966)

Y= Fiber.
X= Height.
Z= Base Diameter.

$N = 50$, $n = 8$, $n^{'} = 15$, $\bar{Y} = 4.92$, $\bar{X} = 2.5840$, $C_y^2 = 0.0866$,
$C_z^2 = 0.0170$, $C_x^2 = 0.1163$, $\rho_{yx} = 0.4800$, $\rho_{yz} = 0.3700$, $\rho_{xz} = 0.7300$.

TABLE 1
The percent relative efficiency (PRE) based on population I, II and III of the proposed estimators $T_i (i = 1, 2)$ with respect to the estimators $\bar{y}_{rd}, \bar{y}_{lrd}, \bar{y}_{rc}, \bar{y}_{k_1}, \bar{y}_{k_2}$ and $\bar{y}_{k_3}$.

Proposed Estimators	Known Estimators					
	$\bar{y}_{rd}$	$\bar{y}_{lrd}$	$\bar{y}_{rc}$	$\bar{y}_{k_1}$	$\bar{y}_{k_2}$	$\bar{y}_{k_3}$
PRE for population I						
T_1	152.4514	132.7579	129.8864	116.9996	93.9573*	95.8502*
T_2	152.3208	132.6442	129.7751	116.8993	93.8768*	95.7680*
PRE for population II						
T_1	111.5045	107.6983	108.1047	100.0192	96.8661*	95.7515*
T_2	110.7073	106.9283	107.3319	98.4838*	96.17366*	95.0670*
PRE for population III						
T_1	132.6666	102.5553	125.7897	137.3524	97.2172*	95.4195*
T_2	137.9183	106.6151	130.7693	142.7897	101.0657	99.1968*

* show the proposed estimators are not preferable.

9. Conclusions

From Table-1, it is visible that the proposed estimators $T_i (i = 1, 2)$ are dominating over the estimators $\bar{y}_{rd}, \bar{y}_{lrd}, \bar{y}_{rc}$ and $\bar{y}_{k_1}$ for all population. The proposed estimators will also be preferable over the estimators $\bar{y}_{k_2}$ and $\bar{y}_{k_3}$ for the populations, which satisfy the conditions derived in equations (27)-(28) and (29)-(30) respectively. Looking on the nice behavior of the proposed estimator, this may be recommended to the survey practitioners for their practical applications.

Acknowledgements

Authors are thankful to the reviewer for his valuable suggestion. Authors are also thankful to the Indian School of Mines, Dhanbad, for providing financial assistance and necessary infrastructure to carry out the present research work.

References

S. Bahl, R. Tuteja (1991). *Ratio and product type exponential estimators.* Journal of Information and Optimization Sciences, 12, no. 1, pp. 159–164.

L. Chand (1975). *Some ratio-type estimators based on two or more auxiliary variables.*

S. Choudhury, B. Singh (2012). *A class of chain ratio-cum-dual to ratio type estimator with two auxiliary characters under double sampling in sample surveys.* Statistics in Transition-New Series, 13, no. 3, pp. 519–536.

W. G. Cochran (1977). *Sampling Techniques.* New York: JohnWiley.

R. A. Fisher (1936). *The use of multiple measurements in taxonomic problems.* Annals of eugenics, 7, no. 2, pp. 179–188.

S. Gupta, J. Shabbir (2008). *On improvement in estimating the population mean in simple random sampling.* Journal of Applied Statistics, 35, no. 5, pp. 559–566.

C. Kadilar, H. Cingi (2003). *A study on the chain ratio-type estimator.* Hacettepe Journal of Mathematics and Statistics, 32, pp. 105–108.

C. Kadilar, H. Cingi (2004). *Ratio estimators in simple random sampling.* Applied Mathematics and Computation, 151, no. 3, pp. 893–902.

C. Kadilar, H. Çingi, *et al.* (2007). *Ratio estimators using robust regression.* Hacettepe Journal of Mathematics and Statistics, 36, no. 2.

B. Kiregyera (1980). *A chain ratio-type estimator in finite population double sampling using two auxiliary variables.* Metrika, 27, no. 1, pp. 217–223.

B. Kiregyera (1984). *Regression-type estimators using two auxiliary variables and the model of double sampling from finite populations.* Metrika, 31, no. 1, pp. 215–226.

R. Mukerjee, T. Rao, K. Vijayan (1987). *Regression type estimators using multiple auxiliary information.* Australian Journal of Statistics, 29, no. 3, pp. 244–254.

N. Ozgul, H. Cingi (2014). *A new class of exponential regression cum ratio estimator in two phase sampling.* Hacettepe Journal of Mathematics and Statistics, 43, no. 1.

B. Pradhan (2005). *A chain regression estimator in two phase sampling using multi-auxiliary information.* Bull. Malays. Math. Sci. Soc, 2, no. 28, p. 1.

G. Shukla (1966). *An alternative multivariate ratio estimate for finite population.* Calcutta Statistical Association Bulletin, 15, no. 58-5, p. 125.

G. Singh (2001). *On the use of transformed auxiliary variable in the estimation of population mean in two-phase sampling.* Statistics in Transition, 5, no. 3, pp. 405–416.

G. Singh, D. Majhi (2014). *Some chain-type exponential estimators of population mean in two-phase sampling.* STATISTICS, 221.

G. Singh, L. Upadhyaya (1995). *A class of modified chain type estimators using two auxiliary variables in two-phase sampling.* Metron, 53, no. 3-4, pp. 117–125.

H. P. Singh, G. K. Vishwakarma (2007). *Modified exponential ratio and product estimators for finite population mean in double sampling.* Austrian Journal of Statistics, 36, no. 3, pp. 217–225.

R. Singh, P. Chauhan, N. Sawan, F. Smarandache (2008). *Improvement in estimating population mean using two auxiliary variables in two-phase sampling.* arXiv preprint arXiv:0810.2094.

V. Singh, G. Singh (1991). *Chain type regression estimators with two auxiliary variables under double sampling scheme.* Metron, 49, no. 1-4, pp. 279–289.

V. K. Singh, H. P. Singh, H. P. Singh, D. Shukla (1994). *A general class of chain estimators for ratio and product of two means of a finite population.* Communications in Statistics-Theory and Methods, 23, no. 5, pp. 1341–1355.

R. S. Srivnstava, S. Srivastava, B. Khare (1989). *Chain ratio type estimator for ratio of two population m3ans using auxiliary characters.* Communications in Statistics-Theory and Methods, 18, no. 10, pp. 3917–3926.

L. Upadhyaya, K. Kushwaha, H. Singh (1990). *A modified chain ratio-type estimator in two-phase sampling using multi auxiliary information.* Metron, 48, no. 1-4, pp. 381–393.

L. Upadhyaya, G. Singh (2001). *Chain-type estimators using transformed auxiliary variable in two-phase sampling.* Advances in Modelling and Analysis-A-, 38, no. 1/2, pp. 1a–10a.

Summary

The present work suggests some estimation procedures of population mean in two phase sampling. Chain-type exponential dual to ratio and regression estimators have been proposed and their properties are studied. Cost aspects of the suggested estimation procedures have been examined and a recommendations are made to the survey practitioners for their practical applications.

Keywords: Two- phase chain- type; auxiliary information; bias; mean square error; cost.