EXPOENTIAL CHAIN DUAL TO RATIO AND REGRESSION TYPE ESTIMATORS OF POPULATION MEAN IN TWO-PHASE SAMPLING

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1. Introduction

It is customary to utilize the information on auxiliary variables to improve the precision of estimates in sample surveys, if such is readily available or may be made available by incurring some additional cost. Ratio, product and regression methods of estimation are the popular estimation procedures which utilize the information on auxiliary variable in terms of known population parameters at estimation stage. Sometimes, the population parameters of auxiliary variable may not be readily known at the time of generating estimates of population parameters of study variable. Two-phase (double) sampling happens to be a cost effective technique for generating the most reliable estimates of unknown population parameters of auxiliary variable in first phase sample. Chand (1975) introduced a technique of chaining the information of the second auxiliary variable to get a more precise estimate of the unknown population mean of first auxiliary variable in the first-phase sample in form of ratio estimator and which is termed as chain-type ratio estimator. Further, his work was extended by Kiregyera (1980, 1984); Mukerjee et al. (1987), Srivnstava et al. (1989), Upadhyaya et al. (1990), Singh and Singh (1991), Singh et al. (1994), Singh and Upadhyaya (1995), Upadhyaya and Singh (2001), Singh (2001), Kadilar and Cingi (2003); Kadilar and Cingi (2004), Pradhan (2005), Kadilar et al. (2007), Gupta and Shabbir (2008), Singh et al. (2008), Choudhury and Singh (2012), Ozgul and Cingi (2014), Singh and Majhi (2014), among others. Bahl and Tuteja (1991) suggested exponential methods of estimation of population mean of study variable, which was further followed by Singh and Vishwakarma (2007), who studied the behavior of the exponential method of estimation in two-phase sampling. In follow up of the above works and motivated with the generic nature of exponential method of estimation, we have proposed some new exponential chain-type estimators which are more relevant in producing

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the precise estimates of the population mean of the study variable. Theoretical and empirical properties of the proposed estimation procedures have been studied and suitable recommendations are put forward to the survey practitioners.

2. Two-phase sampling setup

Let \( U = (U_1, U_2, \ldots, U_N) \) be a finite population of size \( N \) indexed by triplet characters \((y, x, z)\). Our interest is to estimate the population mean \( \bar{Y} \) of study variable \( y \) in the presence of two auxiliary variables \( x \) and \( z \). Let \( x \) and \( z \) be the first and second auxiliary variables respectively, such that \( y \) is highly correlated with \( x \) while in compare to \( x \), it is remotely correlated with \( z \) (\( \rho_{yx} > \rho_{yz} \)). When population mean \( X \) of the first auxiliary variable \( x \) is unknown but information on second auxiliary variable \( z \) is available for all the units of the population, we use the two-phase sampling design which is described as follows.

Consider a two-phase sampling design where in the first phase a large (preliminary) sample \( s' (s' \subset U) \) of size \( n' \) is drawn following simple random sampling without replacement (SRSWOR) scheme and observed for two auxiliary variables \( x \) and \( z \) to estimate \( \bar{X} \), while in the second phase a sub sample \( s \subset s' \) of size \( n \) is drawn from first-phase sample \( s \) by SRSWOR to observe the characteristic \( y \) under study.

3. Some estimators in two-phase sampling

In this section, we consider some estimators of population mean in two phase sampling which are frequently used in survey literature’s. The estimators along with their respective mean square errors are presented in the following sub-sections.

3.1. Estimators based on one auxiliary variable in two-phase sampling setup

Ratio and regression methods of estimation require the known population mean of auxiliary variable and in the absence of such information, we go for two-phase sampling setup, where, we estimate the unknown population mean of auxiliary variable in the first-phase sample. For ready-made assessment the ratio and regression estimators respectively in two-phase sampling setup along with their respective mean square errors up to the first order of approximations are reproduced below as:

\[
\bar{y}_{rd} = \frac{\bar{y}'}{\bar{x}'}
\]

\[
M(\bar{y}_{rd}) = \bar{Y}^2 \left[ f_1 C_y^2 + f_3 \left( C_x^2 - 2 \rho_{yx} C_y C_x \right) \right]
\]

\[
\bar{y}_{lrd} = \bar{y} + b_{yx} (n)(\bar{x}' - \bar{x})
\]

and

\[
M(\bar{y}_{lrd}) = \bar{Y}^2 \left[ f_1 (1 - \rho_{yx}^2) + f_2 \rho_{yx}^2 \right] C_y^2
\]
where \( b_{yx}(n) \) is the sample regression coefficient of \( y \) on \( x \) based on sample \( s \) and

\[
\hat{y} = \frac{1}{n} \sum_{i \in s} y_i, \quad \bar{x} = \frac{1}{n} \sum_{i \in s} x_i, \quad \hat{x}' = \frac{1}{n} \sum_{i \in s} x_i, \quad f_1 = \left( \frac{1}{n} - \frac{1}{N} \right),
\]

\[
f_2 = \left( \frac{1}{n'} - \frac{1}{N} \right), \quad f_3 = \left( \frac{1}{n'} - \frac{1}{n} \right) = (f_1 - f_2), \quad C_x = \frac{S_x}{\bar{X}}, C_y = \frac{S_y}{\bar{Y}},
\]

\[
k = \frac{n'}{N-n'}, \quad S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{X})^2, \quad S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2,
\]

and \( \rho_{yx} \) be the correlation coefficient between the variables \( y \) and \( x \).

### 3.2. Estimators based on two auxiliary variables in two-phase sampling setup

Sometimes, information on more than one auxiliary variable is available at the estimation stage in sample surveys. Chand (1975) introduced a chain-type ratio estimator under two-phase sampling setup where he utilized the information on two auxiliary variables \( x \) and \( z \) under the assumption that the population mean \( \bar{X} \) of the auxiliary variable \( x \) is unknown and information on auxiliary variable \( z \) is available for all the units of the population. The suggested estimator with their respective mean square error up to the first order approximations are reproduced as:

\[
y_{rc} = \frac{\bar{y}}{\bar{x}} \frac{\hat{x}'}{\bar{z}'} \bar{Z}
\]

\[M(\bar{y}_{rc}) = \bar{Y}^2 \left[ f_1 C_y^2 + f_3 (C_x^2 - 2 \rho_{yx} C_x C_y) + f_2 (C_z^2 - 2 \rho_{xz} C_x C_z) \right]
\]

where \( \bar{Z} \) is the population mean of the auxiliary variable \( z \),

\[
\hat{z}' = \frac{1}{n'} \sum_{i' \in s} z_{i'}, \quad C_z = \frac{S_z}{\bar{Z}}, \quad S_z^2 = \frac{1}{N-1} \sum_{i=1}^{N} (z_i - \bar{Z})^2
\]

and \( \rho_{yz} \) be the correlation coefficient between the variables \( y \) and \( z \).

Further, Kiregyera (1980, 1984) extended the work of Chand (1975) and suggested chain-type ratio to regression, regression to ratio and regression to regression estimators of the population mean \( \bar{Y} \) of study variable \( y \) in two-phase sampling setup which utilized the information on two auxiliary variables \( x \) and \( z \) under similar assumptions as considered by Chand (1975). The suggested estimators are given below with their respective mean square errors up to the first order of approximations as:

\[
y_{k1} = \frac{\hat{y}}{\bar{x}} \left[ \hat{x}' + b_{xz}(n') (\bar{Z} - \bar{z}') \right]
\]

\[M(\bar{y}_{k1}) = \bar{Y}^2 \left[ f_3 (C_x^2 + C_y^2 - 2 \rho_{yx} C_x C_y) + f_2 C_y^2 + f_2 \rho_{xz} C_x (\rho_{xz} C_x - 2 \rho_{yz} C_y) \right]
\]

\[
y_{k2} = \hat{y} + b_{yx}(n) (\hat{x} - \bar{x})
\]

\[M(\bar{y}_{k2}) = \bar{Y}^2 \left[ f_3 (C_x^2 + C_y^2 - 2 \rho_{yx} C_x C_y) + f_2 C_y^2 + f_2 \rho_{xz} C_x (\rho_{xz} C_x - 2 \rho_{yz} C_y) \right]
\]
where $\bar{x}_r = \frac{x'_r \bar{Z}}{x}$

$$M(\bar{y}_{k2}) = \bar{Y}^2 C_y^2 \left[ f_1(1 - \rho_{yz}^2) + f_2 \left( \rho_{yx}^2 + \rho_{yz}^2 \frac{C_z^2}{C_y^2} - 2\rho_{yx}\rho_{yz} \frac{C_z}{C_y} \right) \right]$$  \hspace{1cm} (10)$$

$$\bar{y}_{k3} = \bar{y} + b_{xz}(n)(x'_{ld} - \bar{x})$$  \hspace{1cm} (11)$$

and

$$M(\bar{y}_{k3}) = \bar{Y}^2 C_y^2 \left[ f_3(1 - \rho_{yz}^2) + f_4(1 + \rho_{z}\rho_{yx}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}) \right]$$  \hspace{1cm} (12)$$

where $b_{xz}(n)$ is the sample regression coefficient of the variable $x$ on $z$ based on the sample $s$ and $\rho_{xz}$ be correlation coefficient between variables $x$ and $z$.

4. The Proposed Exponential Chain-Type Dual to Ratio and Regression Estimators

The intelligible use of auxiliary information for making estimates more precise has been a fascinating area of research in sample surveys. In the presence of auxiliary information Bhal and Tuteja (1991) suggested an exponential type estimator to estimate the population mean of study variable and their work was further extended by Singh and Vishwakarma (2007) in two-phase sampling setup. Motivated by the work related to the proposition of chain-type estimators in two-phase sampling setup, and looking at the nice behavior of the exponential type estimators, we suggest a new exponential chain-type dual to ratio and regression estimators to estimate the population mean $\bar{Y}$ under two phase sampling setup with an assumption that the population mean $\bar{X}$ of the auxiliary variable $x$ is unknown. The proposed estimators are formulated as:

$$T_1 = \bar{y} \exp \left[ \frac{x'_r \bar{Z} - \bar{x}}{x'_r \bar{Z} + \bar{x}} \right]$$  \hspace{1cm} (13)$$

and

$$T_2 = \bar{y} \exp \left[ \frac{x' + b_{xz}(n)(\bar{Z} - \bar{Z}')} - \bar{x}}{x' + b_{xz}(n)(\bar{Z} - \bar{Z}')} + \bar{x} \right],$$  \hspace{1cm} (14)$$

where $\bar{z}^* = \frac{N - n^*}{N - n} \bar{z}$.  \hspace{1cm} (14)$$

5. The Bias and Mean Square Errors (MSE) of the Proposed Estimators $T_i (i = 1, 2)$

Since the proposed estimators $T_i (i = 1, 2)$ are exponential chain-type dual to ratio and regression estimators, they are biased estimators of the population mean $\bar{Y}$, therefore, bias $B(\cdot)$ and mean square errors $M(\cdot)$ of the estimators $T_i$ are derived up to the first order of approximations and presented in the following theorems:
Theorem 5.1. The bias of the estimators $T_i(i=1, 2)$ to the first order of approximations are obtained as

$$B(T_1) = \bar{Y} \left[ f_3 \left( \frac{1}{8} C_z^2 - \frac{1}{2} \rho_{yz} C_y C_z \right) + f_2 \left( \frac{k^2}{8} C_z^2 - \frac{k}{4} C_z^2 - \frac{k}{2} \rho_{yz} C_y C_z \right) \right]$$

and

$$B(T_2) = \bar{Y} \left[ f_3 \left( \frac{1}{8} C_z^2 - \frac{1}{2} \rho_{yz} C_y C_z \right) + f_2 \left( \frac{k^2}{8} C_z^2 - \frac{k}{4} C_z^2 - \frac{k}{2} \rho_{yz} C_y C_z \right) \right]$$

Theorem 5.2. The mean square errors of the estimators $T_i(i=1, 2)$ to the first order of approximations are obtained as

$$M(T_1) = \bar{Y}^2 \left[ f_1 C_y^2 + f_2 \left( \frac{k^2}{4} C_z^2 - k \rho_{yz} C_y C_z \right) + \frac{1}{4} f_3 \left( C_z^2 - 4 \rho_{yz} C_y C_z \right) \right]$$

and

$$M(T_2) = \bar{Y}^2 \left[ f_1 C_y^2 + f_2 \left( \rho_{xz}^2 C_z^2 - k \rho_{yz} C_y C_z \right) + \frac{1}{4} f_3 \left( C_z^2 - 4 \rho_{yz} C_y C_z \right) \right]$$

6. Comparison of the estimators

In this section, we compare the proposed estimators $T_i(i = 1, 2)$ with respect to the estimators $\bar{y}_{rd}$, $\bar{y}_{lrd}$, $\bar{y}_{rc}$, $\bar{y}_{k}$, and $\bar{y}_{kz}$ in terms of mean square criterions. The conditions for which the proposed estimators $T_i$ will dominate other estimators under mean square criterions are defined as:

(i) The estimators $T_i(i = 1, 2)$ are better than $\bar{y}_{rd}$ if $M(T_i) \leq M(\bar{y}_{rd})$, which gives the conditions

$$\frac{(3C_z^2 - 4 \rho_{yz} C_y C_z)}{(k^2 C_z^2 - 4k \rho_{yz} C_z C_y)} \geq \frac{f_2}{f_3}$$

and

$$\frac{(3C_z^2 - 4 \rho_{yz} C_y C_z)}{(\rho_{xz}^2 C_z^2 - 4 \rho_{yz} \rho_{xz} C_y C_z)} \geq \frac{f_2}{f_3}$$

for the estimators $T_1$ and $T_2$ respectively.

(ii) The estimators $T_i(i = 1, 2)$ are more efficient than $\bar{y}_{lrd}$ if $M(T_i) \leq M(\bar{y}_{lrd})$, such that following conditions are satisfied for the estimators $T_1$ and $T_2$ respectively.

$$\frac{(C_z - 2 \rho_{yz} C_y)^2}{(4k \rho_{yz} C_y C_z - k^2 C_z^2)} \leq \frac{f_2}{f_3}$$

and

$$\frac{(4 \rho_{yz} C_y C_z - 4 \rho_{xz}^2 C_y^2 - C_z^2)}{(\rho_{xz}^2 C_z^2 - 4 \rho_{yz} \rho_{xz} C_y C_z)} \geq \frac{f_2}{f_3}$$
(iii) The estimators \( T_i(i = 1, 2) \) will dominate \( \bar{y}_{rc} \) if \( M(T_i) \leq M(\bar{y}_{rc}) \), which gives the results

\[
\frac{(C_x^2 - 4\rho_{xy}C_yC_z)}{(\{\frac{k^2}{4} - 1\}C_z^2 - (k - 2)\rho_{yz}C_yC_z)} \geq \frac{f_2}{f_3} \tag{23}
\]

and

\[
\frac{(\frac{3}{4}C_x^2 - \rho_{yz}C_yC_z)}{\{\frac{1}{4}\rho_{xz}C_x(\rho_{xz}C_x - 4\rho_{yz}C_y) + C_z(2\rho_{yz}C_y - C_z)\}} \geq \frac{f_2}{f_3} \tag{24}
\]

for the estimators \( T_1 \) and \( T_2 \) respectively.

(iv) The estimators \( T_i(i = 1, 2) \) are better than \( \bar{y}_{k_1} \) if \( M(T_i) \leq M(\bar{y}_{k_1}) \), and subsequently, we get the following conditions

\[
\frac{3(C_x^2 - 4\rho_{xy}C_yC_z)}{4\{(\rho_{yx}C_y - \frac{k}{2}C_z)^2 - (\rho_{xz}C_x - \rho_{yz}C_y)^2\}} \geq \frac{f_2}{f_3} \tag{25}
\]

and

\[
\frac{(C_x^2 - \frac{4}{3}\rho_{xy}C_yC_z)}{\{\frac{4}{3}\rho_{xz}\rho_{yz}C_yC_x - \rho_{xz}C_z^2\}} \geq \frac{f_2}{f_3} \tag{26}
\]

for the estimators \( T_1 \) and \( T_2 \) respectively.

(v) The estimators \( T_i(i = 1, 2) \) are preferable over \( \bar{y}_{k_2} \) if \( M(T_i) \leq M(\bar{y}_{k_2}) \), which leads to the conditions

\[
\frac{(\frac{1}{2}C_x - \rho_{yz}C_y)^2}{\{\frac{2}{3}\rho_{yx}^{2} - \frac{\rho_{yz}^2}{C_y^2} + (\rho_{yx}\frac{C_y}{C_z} - \rho_{yz})^2 - C_z^2(\frac{k^2}{4} - k\rho_{yz}\frac{C_y}{C_z})\}} \leq \frac{f_2}{f_3} \tag{27}
\]

and

\[
\frac{(C_x - 2\rho_{yz}C_y)^2}{4\rho_{yx}\frac{C_y}{C_z}C_y^2(\rho_{yx}\frac{C_y}{C_z} - 2\rho_{yz}) + \rho_{xz}C_x(\rho_{xz}C_x - 4\rho_{yz}C_y))} \leq \frac{f_2}{f_3} \tag{28}
\]

for the estimators \( T_1 \) and \( T_2 \) respectively.

(vi) The estimator \( T_i(i = 1, 2) \) will influence over \( \bar{y}_{k_3} \) if \( M(T_i) \leq M(\bar{y}_{k_3}) \), which gives the conditions

\[
\frac{(\frac{1}{4}C_x - \rho_{yz}C_y)^2}{\{(\rho_{yx}\rho_{xz} - 2\rho_{yx}\rho_{xz}\rho_{zx}) - (\frac{k^2}{4}C_z^2 - k\rho_{yz}C_yC_z)\}} \leq \frac{f_2}{f_3} \tag{29}
\]

and

\[
\frac{(C_x - 2\rho_{yz}C_y)^2}{4\rho_{yx}\rho_{xz}C_y^2(\rho_{yx}\rho_{xz} - 2\rho_{yz}) - \rho_{xz}C_x(\rho_{xz}C_x - 4\rho_{yz}C_y))} \leq \frac{f_2}{f_3} \tag{30}
\]

for the estimators \( T_1 \) and \( T_2 \) respectively.
7. Cost aspect

The principle of optimization is an important criterion in sample surveys and to make estimators more precise for given cost is an integral part of it. Since, two-phase sampling setup is more prone to cost, therefore, it is essential to formulate a strategy which reduces the survey cost. Considering this fact, we derive the expressions for optimum sample sizes in primarily (first-phase) and final (second-phase) samples respectively by considering the suitable cost function so that the mean square errors of the estimators are minimized. In most of the practical situations total cost is the linear function of samples selected at first and second phases. Keeping this point in views, in this section, we define the following cost function which is given below:

\[ C = nC_1 + n'(C_2 + C_3), \]  

(31)

where \( C \) is the total cost of the survey, \( C_1 \) is the cost per unit associated with the second phase sample of size \( n \) for collecting the information on study variable \( y \) and \( C_2, C_3 \) be the cost per unit associated with the first phase sample of size \( n' \) for collecting the information on auxiliary variable \( x \) and \( z \) respectively.

For large population, ignoring finite population correction (f.p.c), the mean square errors of the proposed estimators \( T_i \) \((i = 1, 2)\) can be expressed as

\[ \text{MSE}(T_i) = \frac{1}{n} V_{1i} + \frac{1}{n'} V_{2i}; \quad (i = 1, 2), \]  

(32)

where

\[ V_{11} = \bar{Y}^2(C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_xC_y), \quad V_{21} = \bar{Y}^2(\frac{k^2}{4}C_z^2 - \frac{C_y^2}{4} - k\rho_{yz}C_xC_y + \rho_{yx}C_yC_x), \]

\[ V_{12} = \bar{Y}^2(C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_xC_y), \quad V_{22} = \bar{Y}^2(\frac{\rho_{xz}^2C_z^2}{4} - \rho_{yz}\rho_{xz}C_xC_y - \frac{C_y^2}{4} + \rho_{yx}C_yC_x). \]

Therefore, fixed for cost \( C_0 \) the objective function is defined as:

\[ \phi_i = M(T_i) + \lambda_i(nC_1 + n'(C_2 + C_3) - C_0); \quad (i = 1, 2), \]  

(33)

where \( \lambda_i \) is the Lagrange’s multipliers.

It is assumed that \( C_1 > C_2 > C_3 \). Utilizing the function of Lagrange’s multipliers, the optimum values of \( n \) and \( n' \) for fixed cost \( C_0 \) which minimize the mean square errors of the estimators \( T_i \) \((i = 1, 2)\) in equation (32) are derived as:

\[ n_{opt} = \left\{ \frac{C_0 \sqrt{V_{11}}}{C_1 \sqrt{V_{11}} + \sqrt{V_{21}}C_1(C_2 + C_3)} \right\}; \quad (i = 1, 2), \]  

(34)

and

\[ n'_{opt} = \left\{ \frac{C_0 \sqrt{V_{2i}}}{\sqrt{(C_2 + C_3)}(C_1 \sqrt{V_{11}} + \sqrt{V_{2i}}(C_2 + C_3))} \right\}; \quad (i = 1, 2). \]  

(35)
Substituting, the optimum values of \( n \) and \( n' \) derived in equation (34) and (35) in equation (32), we get the optimum mean square errors of the estimators \( T_i (i = 1, 2) \), as

\[
MSE(T_{i\text{opt}}) = \frac{1}{C_0} \left( C_1 \sqrt{V_{1i}} + \sqrt{V_{2i}(C_2 + C_3)} \right)^2 ; \quad (i = 1, 2).
\]  

(36)

8. Empirical Study

To analyze the performances of the proposed estimators \( T_i (i = 1, 2) \) with respect to the other estimators, we have computed percent relative efficiency of the estimators \( T_i \) with respect to the estimators \( \bar{y}_{rd}, \bar{y}_{rlc}, \bar{y}_{kc}, \bar{y}_{k_1}, \bar{y}_{k_2}, \) and \( \bar{y}_{k_3} \) using three natural populations. The PRE of the estimators \( T_i (i = 1, 2) \) is calculated by using the following formula.

\[
PRE = \left[ \frac{MSE(\delta)}{MSE(T_i)} \right] \times 100 ; \quad (i = 1, 2),
\]

where \( \delta \) is some estimator of population mean \( \bar{Y} \).

Population source-I: Cochran (1977)

Y=Number of “placebo” children.
X=Number of paralytic polio cases in the placebo group.
Z=Number of paralytic polio cases in the ‘not inoculated’ group.
\( N = 34, \quad n = 10, \quad n' = 15, \quad \bar{Y} = 4.92, \quad \bar{X} = 2.59, \quad \bar{Z} = 2.91, \quad C_y^2 = 1.0248, \)
\( C_z^2 = 1.1492, \quad C_x^2 = 1.5175, \quad \rho_{yx} = 0.7326, \quad \rho_{yz} = 0.6430, \quad \rho_{xz} = 0.6837. \)

Population-II: Fisher (1936)

Consisting of measurements on three variables, namely sepal width(\( y \)), sepal length(\( x \)), and petal length (\( z \)) for 50 Iris flowers (versicolor) such that:
\( N = 50, \quad n = 9, \quad n' = 20, \quad \bar{Y} = 2.770, \quad C_y^2 = 0.012566, \)
\( C_z^2 = 0.011924, \quad C_x^2 = 0.007343, \quad \rho_{yx} = 0.5605, \quad \rho_{yz} = 0.5259, \quad \rho_{xz} = 0.7540. \)

Population-III: Shukla (1966)

Y=Fiber.
X=Height.
Z=Base Diameter.
\( N = 50, \quad n = 8, \quad n' = 15, \quad \bar{Y} = 4.92, \quad \bar{X} = 2.5840, \quad C_y^2 = 0.0866, \)
\( C_z^2 = 0.0170, \quad C_x^2 = 0.1163, \quad \rho_{yx} = 0.4800, \quad \rho_{yz} = 0.3700, \quad \rho_{xz} = 0.7300. \)
TABLE 1
The percent relative efficiency (PRE) based on population I, II and III of the proposed estimators $T_i (i = 1, 2)$ with respect to the estimators $\bar{y}_{rd}, \bar{y}_{lrd}, \bar{y}_{rc}, \bar{y}_{k_1}, \bar{y}_{k_2}$ and $\bar{y}_{k_3}$.

<table>
<thead>
<tr>
<th>Proposed Estimators</th>
<th>Known Estimators</th>
</tr>
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<tbody>
<tr>
<td>$T_1$</td>
<td>$\bar{y}_{rd}$</td>
</tr>
<tr>
<td></td>
<td>PRE for population I</td>
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<tr>
<td>$T_2$</td>
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<td>PRE for population II</td>
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<tr>
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<td>PRE for population III</td>
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<td>132.6666</td>
</tr>
<tr>
<td>$T_2$</td>
<td></td>
</tr>
</tbody>
</table>

* show the proposed estimators are not preferable.

9. Conclusions

From Table-1, it is visible that the proposed estimators $T_i (i = 1, 2)$ are dominating over the estimators $\bar{y}_{rd}, \bar{y}_{lrd}, \bar{y}_{rc}$ and $\bar{y}_{k_i}$ for all population. The proposed estimators will also be preferable over the estimators $\bar{y}_{k_2}$ and $\bar{y}_{k_3}$ for the populations, which satisfy the conditions derived in equations (27)-(28) and (29)-(30) respectively. Looking on the nice behavior of the proposed estimator, this may be recommended to the survey practitioners for their practical applications.

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Summary

The present work suggests some estimation procedures of population mean in two phase sampling. Chain-type exponential dual to ratio and regression estimators have been proposed and their properties are studied. Cost aspects of the suggested estimation procedures have been examined and recommendations are made to the survey practitioners for their practical applications.

Keywords: Two-phase chain-type; auxiliary information; bias; mean square error; cost.