

SAMPLING PROPERTIES OF THE DIRECTIONAL MOBILITY INDEX AND THE INCOME OF ITALIAN FAMILIES

Camilla Ferretti ¹

Dept. of Economics and Social Sciences, Univ. Cattolica del Sacro Cuore, Piacenza, Italy

1. INTRODUCTION

In literature, transition matrices have been widely used to describe the dynamics of discrete state processes based on economic variables. In this sense relevant examples are, among others, the analysis of flows of individuals in the labor market (Fougere and Kamionka, 2003), of firms among size classes (Cipollini *et al.*, 2012), or of citizens among income levels (Bourguignon and Morrison, 2002). Any given transition matrix $P = \{p_{ij}\}_{i,j=1,\dots,k}$ provides all the information on the movements of individuals among the states (labeled with $1, \dots, k$), since p_{ij} usually is the conditional probability of moving in j at time t , given the state i at time $t - 1$. Assuming appropriate hypothesis (for example, P is ruling transitions in a Markov Chain), the knowledge of P permits to forecast the evolution of individuals among the k states.

We shortly introduce the theory of mobility indices considering the set of the transition matrices with k states, with $k \in \mathbb{N}_+$:

$$\mathcal{P} = \left\{ P = (p_{ij}) \in \mathbb{R}^{k \times k} \mid p_{ij} \geq 0, \sum_{j=1}^k p_{ij} = 1, \forall i = 1, \dots, k \right\}.$$

A given matrix P can be associated to a certain degree of mobility through a synthetic mobility index $I(P)$. The larger is $I(P)$, the higher is the degree of mobility in the group of individuals ruled by P . Mobility indices for transition matrices are then functions $I(\cdot)$ mapping a given transition matrix $P \in \mathcal{P}$ in the value $I(P) \in \mathbb{R}$. As specified in Ferretti (2012), mobility indices introduce a *total order* " \prec " in \mathcal{P} , that is a rule which permits to establish unequivocally if the mobility of P is lower/higher than the mobility of Q , for every couple $P, Q \in \mathcal{P}$. Basically, chosen the function $I(\cdot)$, we say that $P \prec Q$ (" P is less mobile than Q ") if and only if $I(P) < I(Q)$. It is worth noting that the order in \mathcal{P} depends on the choice of $I(\cdot)$.

¹ Corresponding Author. E-mail: camilla.ferretti@unicatt.it

Famous proposals for the function $I(\cdot)$ are, among others, in Shorrocks (1978), Sommers and Conlinsk (1979), Bartholomew (1982) and Geweke *et al.* (1986). As an important example of $I(\cdot)$ we are going to use in the following, we recall here the *trace index* (Prais, 1955; Shorrocks, 1978):

$$I_{tr}(P) = \frac{k - \sum_{i=1}^k p_{ii}}{k-1} = \frac{\sum_{i=1}^k (1 - p_{ii})}{k-1}.$$

The trace index furnishes an absolute measure of the mobility: in particular it measures the global tendency to leave from the current state, since $1 - p_{ii}$ is the probability to move away from the i -th state.

In Ferretti and Ganugi (2013) we proposed a new mobility measure, the *directional index*:

$$I_{dir}(P) = \sum_i \omega_i \sum_j p_{ij} \cdot \text{sign}(j-i) \cdot v(|j-i|). \quad (1)$$

I_{dir} derives from the necessity to compare different evolutions when the set of states is ordered from the worst (state "1") to the best one (state " k ") as in the case of incomes or firm size. $I_{dir}(P)$ is then a real value which is positive when the prevailing direction of individuals is towards the better states, and negative in the opposite case.

The directional index is also provided with some parameters: 1) ω_i , $i = 1, \dots, k$ are weights to be assigned to individuals starting from the i -th state (we assume $\omega_i \geq 0$ for every i and $\sum_{i=1}^k \omega_i = 1$); 2) v is a not-decreasing function such that $v(0) = 0$ and $v > 0$ otherwise. It measures the contribute to the whole mobility of individuals making jumps of magnitude $|j-i|$. In the end, the function sign is defined as follows:

$$\text{sign}(x) = \begin{cases} +1, & \text{if } x > 0; \\ -1, & \text{if } x < 0; \\ 0, & \text{if } x = 0; \end{cases}$$

and gives a positive (resp. negative) sign to jumps which lead to a better (resp. worse) state than the current one. For sake of shortness, we will indicate herein $\text{sign}(j-i) \cdot v(|j-i|)$ with v_{ij} .

In the past works we found that for any $P \in \mathcal{P}$, given $\{\omega_i\}$ and v , I_{dir} assumes values in $[m_1, m_2]$, where $m_1 = \sum_{i=1}^k \omega_i v_{i1} \leq 0$ and $m_2 = \sum_{i=1}^k \omega_i v_{ik} \geq 0$ (Ferretti and Ganugi, 2013, p. 414). In the light of that, we also proposed to use the normalized version I^* of I_{dir} :

$$I^*(P) = \begin{cases} I_{dir}(P)/m_2 & \text{if } I_{dir}(P) \geq 0; \\ -I_{dir}(P)/m_1 & \text{if } I_{dir}(P) < 0; \end{cases} \quad (2)$$

which belongs to $[-1, +1]$ for every choice of ω and v . Note that normalization is always possible. In fact we have $m_1 = 0$ if and only if $\omega = (1, 0, \dots, 0)$. In this case we have $I_{dir} \geq 0$ for every $P \in \mathcal{P}$ and it is normalized dividing by $m_2 = v_{1k} > 0$. If $m_2 = 0$ analogous results hold. From a descriptive point of view, I^* simplifies the comparison of two matrices, since its extreme values are univocally defined, and intermediate values of mobility can be considered as a percentage.

Aim of this paper is to carry on the analysis of the directional index, facing the problem of its statistical properties. In the following we will assume the existence of an underlying theoretical model, observed through a sample of size n . In particular we suppose that the movements among the states are ruled by a Markov Chain with unobservable transition matrix P . Following the ideas proposed in Schluter (1998) and Formby *et al.* (2004), we derive the asymptotic sampling distribution of $I_{dir}(\hat{P})$, where \hat{P} is the estimate of P .

The paper is organized as follows: Sect. 2 recalls some important results about the asymptotic sampling distribution of \hat{P} and proves that $I_{dir}(\hat{P})$ is asymptotically normally distributed; Sect. 2.3 reports the results of a numerical example which support the asymptotic Normality of the sampling directional index; Sect. 3 shows the analysis of mobility of four samples of Italian families provided by Banca d'Italia for the years 2004-2012. The last section contains comments and possible further developments.

2. ASYMPTOTIC SAMPLING DISTRIBUTION

We then consider the case of an unobservable $P \in \mathcal{P}$ which rules a Markov chain among k states. Mobility is measured on samples of size n and the function $I(\cdot)$ is evaluated on the estimated \hat{P} . In literature there exist some results about the sampling distribution of the \hat{p}_{ij} . In particular we will refer to the seminal paper of Anderson and Goodman (1957). Statistical procedures will be applied on \hat{I}_{dir} instead of \hat{I}^* because, being the latter a piecewise linear function of the former, we may run into some problems in the variance calculus, especially if $I^*(P) = 0$. Nevertheless tests results on \hat{I}_{dir} can be extended to \hat{I}^* exploiting the fact that $I^*(P) = 0$ if and only if $I_{dir}(P) = 0$. The normalized directional index will be shown to make easier the descriptive comparison between different samples.

2.1. One observed matrix

We suppose to have at disposal observations about two consecutive instants of time t_0 and t_1 . On such basis we build the $k \times k$ empirical matrix X such that x_{ij} is the number of individuals being in state i at time t_0 and in state j at time t_1 . It holds

$$\sum_{j=1}^k x_{ij} = n_i = \text{nr. of individuals starting from } i$$

and

$$\sum_{i,j=1}^k x_{ij} = \sum_{i=1}^k n_i = n = \text{size of the sample.}$$

In this case the MLE estimator \hat{P} of P has elements

$$\hat{p}_{ij} = \frac{x_{ij}}{n_i}, \quad (3)$$

as in Anderson and Goodman (1957).

Let $x_i = (x_{i1}, \dots, x_{ik})$ be the observed vector of individuals moving from i . It is proved that

$$x_i \sim \text{Multinomial}(p_i, n_i),$$

where $p_i = (p_{i1}, \dots, p_{ik})$. Consequently $\hat{p}_i = (\hat{p}_{i1}, \dots, \hat{p}_{ik})$ is asymptotically distributed as a multivariate Normal $\mathcal{N}(p_i, \Sigma)$, where Σ has elements:

$$\Sigma_{jl} = \text{cov}(\hat{p}_{ij}, \hat{p}_{il}) = \begin{cases} \frac{p_{ij}(1-p_{ij})}{n_i} & \text{if } j = l; \\ -\frac{p_{ij}p_{il}}{n_i} & \text{if } j \neq l. \end{cases}$$

(Rows of P are still supposed to be independent one from the each others, and n_i is supposed to be known for every $i = 1, \dots, k$).

The sampling directional index is then asymptotically distributed as a Normal with mean $\mu(I_{dir}(\hat{P}))$ and variance $\sigma^2(I_{dir}(\hat{P}))$. We have that

$$\mu(I_{dir}(\hat{P})) = I_{dir}(P). \quad (4)$$

This condition of unbiased estimator is valid also for finite n , since it is a direct consequence of the linearity of the expected value.

The formula of $\sigma^2(I_{dir}(\hat{P}))$ needs some additional computation. We firstly compute the variance of $\hat{I}_i = I_i(\hat{P})$, where

$$I_i(P) = \sum_j p_{ij} \cdot v_{ij}.$$

$$\begin{aligned} \sigma^2(\hat{I}_i) &= \sigma^2\left(\sum_{j=1}^k \hat{p}_{ij} v_{ij}\right) = \sum_{j=1}^k \left[\sigma^2(\hat{p}_{ij} v_{ij}) + 2 \sum_{l=j+1}^k \text{Cov}(\hat{p}_{ij} v_{ij}, \hat{p}_{il} v_{il}) \right] = \\ &= \sum_{j=1}^k \left[\frac{p_{ij}(1-p_{ij})}{n_i} \cdot v_{ij}^2 - 2 \sum_{l=j+1}^k \frac{p_{ij}p_{il}}{n_i} \cdot v_{ij} \cdot v_{il} \right]. \end{aligned}$$

From the independence between I_i and I_j , the final variance of $I(\hat{P})$ is given by

$$\sigma^2(I_{dir}(\hat{P})) = \sigma^2\left(\sum_i \omega_i I_i(\hat{P})\right) = \sum_i \omega_i^2 \sigma^2(\hat{I}_i). \quad (5)$$

2.2. Two or more observed matrices

In many cases, relevant economic variables are observed in more than two consecutive instants of time: t_0, \dots, t_T . Usually we consider equi-spaced instants of time, indicating them simply with $0, 1, \dots, T$. Coherently with the previous section, let $x_{ij}(t)$ be the number of individuals being in the i -th state at time $t-1$ and in the j -th state at time t , for every $t = 1, \dots, T$. Entries of P are in this case estimated with:

$$\hat{p}_{ij} = \frac{\sum_{t=1}^T x_{ij}(t)}{\sum_{t=1}^T \sum_{j=1}^k x_{ij}(t)}. \quad (6)$$

Note that the time-homogeneity of Markov Chains is here needed, that is the model of the transitions is the same in every step from $t - 1$ to t , $t = 1, \dots, T$.

As before, we consider the number of individuals being in the i -th state at time t , for $t = 0, \dots, T$:

$$n_i(t) = \sum_{j=1}^k x_{ij}(t),$$

and the number of individuals passing from i before time T :

$$n_i^* = \sum_{t=0}^{T-1} n_i(t).$$

Following again Anderson and Goodman (1957), we prove that

$$\hat{p}_i \stackrel{asym.}{\sim} \mathcal{N}(p_i, \Sigma^*),$$

where Σ^* has elements:

$$\Sigma_{jl}^* = \text{cov}(\hat{p}_{ij}, \hat{p}_{il}) = \begin{cases} \frac{p_{ij}(1-p_{ij})}{n_i^*} & \text{if } j = l; \\ -\frac{p_{ij}p_{il}}{n_i^*} & \text{if } j \neq l. \end{cases}$$

Consequently, $I_{dir}(\hat{P})$ is asymptotically distributed as a Normal with expected values equal to $I_{dir}(P)$ and variance obtained by Eq. (5) simply substituting n_i with n_i^* .

2.3. Simulation

The purpose of the following numerical example is to support the Normality of the asymptotic distribution. We consider the randomly chosen theoretical matrix

$$P = \begin{bmatrix} .2 & .3 & .1 & .4 \\ .3 & .5 & .1 & .1 \\ .2 & .4 & .4 & 0 \\ .1 & .2 & .6 & .1 \end{bmatrix}.$$

Our data consist in 5000 simulated realizations of the corresponding homogeneous Markov chain. For any simulation we set $n = 500$ (the number of individuals), with the given starting frequencies $p_0 = (.2, .1, .3, .4)$. In the first case we consider $T = 1$, in the second case $T = 5$. Consequently \hat{P} is estimated as in Eqs. (3) and (6).

We evaluate $I_{dir}(\hat{P})$ setting $v(|j - i|) = |j - i|$, which gives a linearly increasing weight to jumps from i to j , as specified in the following section. We also analyze two different cases for the weights ω_i : on one hand we suppose that the starting state gives no information about the mobility, that is $\omega = (.25, .25, .25, .25)$; on the other hand we choose $\omega = (.1, .2, .3, .4)$, which means that individuals in the 4-th

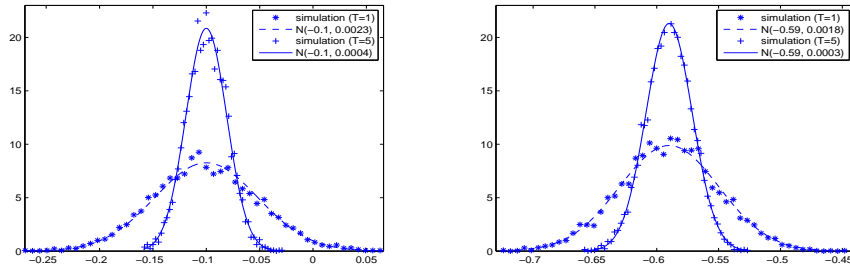


Figure 1 – Simulated versus theoretical density distribution of $I^*(\hat{P})$, with $\omega = (.25, .25, .25, .25)$ (left) and $\omega = (.1, .2, .3, .4)$ (right).

state carry more weight than others in the global mobility measure². We obtain

$$I_{dir}(P) = -0.1,$$

when $\omega = (.25, .25, .25, .25)$, and

$$I_{dir}(P) = -0.59,$$

when $\omega = (.1, .2, .3, .4)$. In both the cases we find that the mobility is negative, that is individuals mainly tend to move towards left (i.e. to make their condition worse, as for example families evolving among income classes and moving towards the smaller classes). If we would compare the two measures without statistical procedures, a better choice is the evaluation of their normalized versions. Respectively we find $I^* = -0.067$ (-6.7%) and $I^* = -0.295$ (-29.5%).

Figure 1 shows the results of our simulation for $T = 1$ and $T = 5$. We compare the simulated density distribution of the index (built up using 50 class intervals) and the Normal density distribution with expected value and variance calculated as in Eqs. (4) and (5). The graphical comparison supports the Normality of the sampling index. Table 1 instead contains the theoretical and simulated values for the mean and the variance of $I_{dir}(\hat{P})$. We display also the p-value obtained with the Shapiro-Wilk test of normality, which still confirms our results.

3. AN EMPIRICAL ILLUSTRATION

As an empirical illustration of the previous results, we propose the analysis of the mobility of Italian families subdivided according with their income. We aim to rigorously determine if mobility among income classes before and after the crisis is negative, positive, or equal to zero, and if it has significantly changed in

² As explained in Ferretti and Ganugi (2013), we may need to make distinction among different states. For example, in the empirical application it is useful to give more weight to states with more individuals, by setting ω equal to the starting frequency distribution $n_1(0)/n, \dots, n_k(0)/n$.

TABLE 1
Comparison between simulated and theoretical mean and variance, and Shapiro-Wilk test's p-value for Normality.

	$\mu(\hat{I}_{dir})$	simulated mean	$\sigma^2(\hat{I}_{dir})$	simulated variance	p-value
$\omega = (.25, .25, .25, .25)$					
$T = 1$	-0.1	-0.0998	0.0024	0.0023	0.1787
$T = 5$	-0.1	-0.0997	0.0004	0.0004	0.5074
$\omega = (.1, .2, .3, .4)$					
$T = 1$	-0.59	-0.5909	0.0018	0.0016	0.6756
$T = 5$	-0.59	-0.5901	0.0004	0.0003	0.5027

TABLE 2
Income classes for Italian families (Euros).

q_1	q_2	q_3	q_4	q_5
[0; 17097)	[17097; 24453)	[24453; 34036)	[34036; 48762)	[48762; +∞)

the time. For our proposals we use the data provided every two years by Banca d'Italia, focusing on the surveys about the two-years periods 2004-2006, 2006-2008, 2008-2010 ad 2010-2012 (Banca d'Italia, 2008, 2010, 2012, 2014) and we build the sampling transition matrices using five classes q_1, \dots, q_5 delimited by the 2004-quintiles and displayed in Table 2. To avoid the bias caused by the inflation we convert the observed incomes into the 2013 values using the ISTAT revalorization's coefficients (<http://www.istat.it/it/archivio/30440>).

The estimated matrices are obtained as in Eq. (3), and they are shown in Table 3³.

As a first descriptive step we evaluate the trace index (Table 4). We see that the "turbulence" tends to decrease until 2010, that is Italian families undergo a decline of their capacity to move from one class to the others. In 2010-2012 we observe instead an increase in the mobility. At the same time we note the lack of information about the prevailing direction of families.

In the second step we make statistical inference about the directional mobility of Italian families. To evaluate $I_{dir}(\hat{P})$ we set ω equal to the starting distribution of every two-years span of time (last column of Table 3). The parameter v is instead needed to give more weight to larger jumps. For example, if $v \equiv 1$, individuals moving from the first to the second class play the same part in the mobility than individuals moving from the first to the third class. If we had geometrically increasing classes, an exponential function of $|j - i|$ would be a suitable choice. Since income intervals do not have a recognizable trend in their size, we choose the simplest way to assign more weight to larger jumps by setting $v(|j - i|) = |j - i|$.

³ The 2004 distribution is not equal to (0.2, 0.2, 0.2, 0.2, 0.2) because quintiles in Table 2 are evaluated on the whole 2004's sample (8012 families), and matrices are estimated on the 3957 families which appear in both the 2004's and the 2006's samples.

TABLE 3
 Estimated income transition matrices for the years 2004-2006, 2006-2008, 2008-2010,
 2010-2012 (values are expressed as percentages).

Income in the initial year	Income in the final year					p_0
	q_1	q_2	q_3	q_4	q_5	
2004-2006 ($n = 3957$)						
q_1	64.10	24.35	7.57	3.44	0.55	18.37
q_2	17.10	43.21	27.42	9.27	3.00	19.36
q_3	6.35	17.79	41.68	26.18	8.01	19.89
q_4	2.42	6.40	18.48	49.15	23.55	20.92
q_5	0.71	2.47	6.01	20.26	70.55	21.46
2006-2008 ($n = 4345$)						
q_1	69.39	22.46	5.21	2.14	0.80	17.22
q_2	18.87	52.33	20.38	6.79	1.64	18.30
q_3	7.05	21.27	48.90	18.38	4.39	19.91
q_4	1.46	5.61	20.27	55.51	17.15	22.14
q_5	0.31	1.44	5.74	19.69	72.82	22.44
2008-2010 ($n = 4621$)						
q_1	73.52	19.09	4.56	1.60	1.23	17.57
q_2	19.27	53.82	20.64	5.59	0.68	18.98
q_3	5.71	19.21	54.00	17.76	3.32	20.84
q_4	1.78	4.95	18.69	56.28	18.30	21.88
q_5	0.63	1.36	4.38	19.42	74.22	20.73
2010-2012 ($n = 4611$)						
q_1	78.75	16.09	3.24	1.68	0.24	18.07
q_2	27.21	53.10	15.99	3.10	0.60	18.17
q_3	10.44	26.52	45.26	15.34	2.45	20.36
q_4	3.05	8.66	25.05	47.86	15.38	21.30
q_5	0.88	2.75	6.67	23.75	65.95	22.10

Source: processing of microdata from Banca d'Italia, *Indagine sui bilanci delle famiglie italiane*.

TABLE 4
 Sampling trace indices.

Years	2004-2006	2006-2008	2008-2010	2010-2012
\hat{I}_{tr}	0.5783	0.5026	0.4704	0.5227

TABLE 5
 Sampling directional indices and p-values for the one-sample test ($H_0: I_{dir}(P) = 0$).

Years	2004-2006	2006-2008	2008-2010	2010-2012
\hat{I}_{dir}	0.0695	-0.0304	-0.0249	-0.1945
$\hat{s}e$	0.0144	0.0124	0.0117	0.0123
p-value	1.30E-06	0.0143	0.0329	0.0000
\hat{I}^*	3.61%	-1.42%	-1.19%	-9.2%

TABLE 6
 \hat{I}_{dir} values evaluated on families moving from different income classes.

Starting class	2004-2006	2006-2008	2008-2010	2010-2012
q_1	0.5199	0.4251	0.3793	0.2857
\hat{se}	0.0304	0.0277	0.0265	0.0219
q_2	0.3786	0.2000	0.1460	-0.0322
\hat{se}	0.0351	0.0311	0.0275	0.0270
q_3	0.1169	-0.0821	-0.0623	-0.2716
\hat{se}	0.0357	0.0313	0.0275	0.0303
q_4	-0.1498	-0.1871	-0.1563	-0.3615
\hat{se}	0.0324	0.0269	0.0264	0.0302
q_5	-0.4252	-0.3672	-0.3476	-0.4887
\hat{se}	0.0265	0.0220	0.0222	0.0253

TABLE 7
Difference-of-means test's p-values on 2006-2008 versus 2008-2010.

Class	q_1	q_2	q_3	q_4	q_5
p-value	0.2318	0.1933	0.6350	0.4130	0.5304

Table 5 shows the values of $I_{dir}(\hat{P})$, the estimated standard error and the normalized values $I^*(\hat{P})$. Mobility is positive in 2004-2006 and assumes negative values in the years over the crisis. Still more relevant is the negative sign joined with an increased absolute value of the index in the last period 2010-2012 (-9.2%). Italian families have then apparently experienced a slowing down in their mobility over the crisis (confirmed by the trace index, too), followed by a stronger worsening of the mean conditions in the last two years. We also apply a difference-of-mean test based on two samples to test " H_0 : the mobility has not significantly changed in the time". H_0 is always rejected, except for the couple 2006-2008 and 2008-2010. In this case the corresponding p-value is equal to 0.747. We conclude that from 2006 to 2010 families have undergone a sort of stagnation in their mobility.

In the last step we analyze the mobility restricted to families starting from the i -th class, obtained by setting $\omega = e_i$ (the i -th canonical vector in \mathbb{R}^k), $i = 1, \dots, 5$.

Table 6 contains the sampling directional index \hat{I}_{dir} restricted to every class, and the corresponding standard error. From such results we derive the following issues:

1. Mobility is generally significantly different from zero (p-values are not displayed for shortness). The restricted directional indices confirm the stagnation from 2006 to 2010 (Table 7).
2. Families starting from the first/fifth class have necessarily a resp. posi-

TABLE 8
Normalized directional index for income classes.

	2004-2006	2006-2008	2008-2010	2010-2012
q_1	13%	10.63%	9.48%	7.14%
q_2	12.62%	6.67%	4.87%	-3.22%
q_3	5.84%	-4.10%	-3.12%	-13.58%
q_4	-4.99%	-6.24%	-5.21%	-12.05%
q_5	-10.63%	-9.18%	-8.69%	-12.22%

tive/negative mobility. It is worth noting that the index values are monotonically decreasing in time.

- Families in q_2 exhibit a positive mobility except for the 2010-2012 period, whose index is statistically equal to zero (p-value equal to 0.116). Then families in the second class show a tendency to improve their income level. The same tendency is slowing down in the time as for families in q_1 .
- Families in q_3 and q_4 have the worst results. Mobility is always negative except for the third class in the first period. We note the negative minimum reached in both the classes in the last two years, indicating that families are still suffering the economic crisis. We also note that mobility values show a slight trend reversal in 2008-2010 with respect to 2006-2008, maybe caused by the partial economic recovery happened in 2010.

Lastly, we show the normalized index for families starting from every income class (Table 8). Normalized values of mobility still confirm the previous results. It is interesting to note that, in 2010-2012, families starting from q_3 , q_4 and q_5 reach similar levels of mobility (around -13%).

4. CONCLUSIONS AND FURTHER RESEARCH

In this paper we develop an analysis of the sampling distribution for directional mobility index proposed in Ferretti and Ganugi (2013). The index results to be asymptotically Normally distributed, with explicit expression for the expected value and the variance.

The asymptotic distribution of the index allows us to rigorously analyze movements of Italian families among income classes, exploiting the sampling microdata provided by Banca d'Italia in the years 2004-2006, 2006-2008, 2008-2010 and 2010-2012. Families are subdivided according with the 2004 income quintiles.

Results show a prevailing tendency to move towards the lowest income classes. Only families starting from the second class (having income comprised between 17097 and 24453 Euros) show a positive behavior, also if their mobility is decreasing in the time. A relevant result regards the 2010-2012 mobility: in this span of time families in the last three classes (that is having income greater than 24435 Euros) show a negative minimum in their mobility value (around -13%).

Further researches will regard the update of the analysis using more recent data. From a theoretical point of view we aim to extend the theory of directional indices, and the related statistical inference, to more refined models, such as mixtures of Markov Chains and continuous-in-time models.

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SUMMARY

We propose statistical inference techniques for testing the directional income mobility. We measure the directional index on income transition matrices about Italian families from 2004 to 2012, and we derive its asymptotic distribution to rigorously determine if mobility has changed. Starting from 2006, results show a prevailing negative mobility, that is the tendency of Italian families to move towards lower income classes. In particular, we observe negative peaks in the mobility values in the period 2010-2012, for middle/high income classes, indicating that Italian families are still suffering the 2008 downturn.

Keywords: Directional mobility index; income transition matrix; sampling distribution