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DOUBLE LOMAX DISTRIBUTION AND ITS APPLICATIONS

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1. Introduction

Laplace distributions are tractable lifetime models in many areas, including life testing and telecommunications. The properties of the Laplace distribution and applications, were given by Johnson, Kotz and Balakrishnan (1990), Kotz *et al.* (2001). In the present study we deal with the ratio of two independent and identically distributed Laplace distribution. The ratio of two random variables X and Y arises in many statistical studies and have applications in various areas such as economics, nuclear physics, meteorology and genetics. The distribution of X/Y has been studied by several authors in particular when X and Y are independent random variables from the same family, see Marsaglia (1965), Korhonen and Narula (1989) for normal family, Press (1969) for student's t family, Provost (1989) for gamma family, Pharm-Gia (2000) for beta family and so on. Nadaraja and Kotz (2006) considered X and Y respectively normal and Laplace random variables distributed independent of each other.

The present study introduces a new family of distributions called double Lomax distribution. The double Lomax distribution is the ratio of two independent and identically distributed classical Laplace distributions. Also the double Lomax distribution can be obtained by compounding classical Laplace distribution with exponential density (Gamma density with shape parameter $\alpha = 1$ and scale parameter $\beta = 1$). The compound distribution is of interest for the study of production/inventory problems, since it provides a flexible description of the stochastic properties of the system. These distributions play a central role in insurance and other areas of applied probability modelling such as queuing theory, reliability etc.

The Laplace model is an alternative to the normal model in situations where the normality assumption do not hold. In a similar way the double Lomax distribution is an alternative to the Cauchy distribution. This family is a special case of type II compound Laplace distribution (Kotz *et al.*, 2001; Bindu *et al.*, 2012).

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The double Lomax distribution is a natural symmetric extension of Lomax distribution to the real line. A random variable X is said to have a Lomax distribution, Lomax (α, β) with parameters α and β if its probability density function (pdf) is

$$f(x;\alpha,\beta) = \alpha\beta^{\alpha}\left(\beta + x\right)^{-(\alpha+1)},\ x > 0, \alpha, \beta > 0. \tag{1}$$

The Lomax distribution is widely known as the type II Pareto distribution, and is related to the 4-parameter type II generalized beta distribution and the 3-parameter Singh-Maddala distribution, as well as the beta distribution of the second kind.

This article is organized as follows. In section 2, the double Lomax distribution is derived, and various properties explored. In section 3 we describe the maximum likelihood estimation of parameters using the BFGS algorithm of `optim` function (Nash J. C., 1990) in R (R Core Team, 2015). The application of the double Lomax distribution is illustrated in section 4 and we conclude in section 5.

2. Double Lomax distribution

The double Lomax distribution is the ratio of two independent and identically distributed classical Laplace distributions and defined as follows.

Definition 1. *Let X_1 and X_2 be two independent and identically distributed (i.i.d.) standard classical Laplace random variables. Then the corresponding probability distribution of $X = X_1/X_2$ is given by*

$$f(x) = \frac{1}{2\left(1 + |x|\right)^2}, \quad -\infty < x < \infty. \tag{2}$$

The Laplace distribution can also be expressed as the difference of two i.i.d exponentials and hence, $X_i \stackrel{d}{=} I_i W_i$, for $i = 1, 2$ where, $W_i \sim Exp(1)$, for $i = 1,\ 2$ and $I_i's$ are independent of W_i and takes values ± 1 with equal probabilities.
Then the cdf can be given by

$$F(x) = \begin{cases} \frac{1}{2(1-x)}, & \text{for } y \leq 0 \\ \left(1 - \frac{1}{2(1+x)}\right), & \text{for } y > 0. \end{cases} \tag{3}$$

Remark 2. *Let X_1 and X_2 be two independent and identically distributed (i.i.d.) standard classical Laplace random variables. Then the corresponding probability distribution of $Z = |X_1/X_2|$ is given by*

$$f(z) = \frac{1}{\left(1 + z\right)^2}, \quad 0 < z < \infty, \tag{4}$$

which is the pdf of the Lomax distribution. Hence the random variable X having pdf given by Eq. (2) may be referred to as the double Lomax distribution (DLD).

The double Lomax distribution, which is the extension of the Lomax distribution over the real line is defined as follows.

DEFINITION 3. *A random variable X is said to have a double Lomax distribution with parameters μ and σ, $DLD(\mu,\sigma)$ if its probability distribution function is,*

$$F_X(x) = \begin{cases} \frac{1}{2\left(1+\frac{(\mu-x)}{\sigma}\right)}, & x \leq \mu \\ 1 - \frac{1}{2\left(1+\frac{(x-\mu)}{\sigma}\right)}, & x > \mu, \end{cases} \tag{5}$$

and the probability density function is,

$$f_X(x) = \frac{1}{2\,\sigma\left(1+|\frac{(x-\mu)}{\sigma}|\right)^2}, -\infty < x < \infty. \tag{6}$$

If $X \sim DLD(0,1)$ then it is referred as the *standard double Lomax distribution.* The corresponding probability distribution and density functions are given by Eq. (2) and Eq. (3), respectively.

If X has the standard double Lomax distribution and $q = F(x)$, then $x = F^{-1}(q)$ and quantiles ξ_q can be written explicitly as follow

$$\xi_q = \begin{cases} \left(1 - \frac{1}{2q}\right), & \text{for } 0 < q \leq \frac{1}{2} \\ \left(\frac{1}{2(1-q)} - 1\right), & \text{for } \frac{1}{2} \leq q < 1. \end{cases} \tag{7}$$

Thus if X has the double Lomax distribution with cdf given by Eq. (5) then quantiles $\xi_q^{'}$ is obtained as $\xi_q^{'} = \mu + \sigma\xi_q$. In particular, the first and the third quartiles are given by

$$Q_1 = \xi_{1/4}^{'} = \mu - \sigma, \quad Q_2 = \xi_{1/2}^{'} = \mu, \quad Q_3 = \xi_{3/4}^{'} = \mu + \sigma.$$

Hence, μ and σ can be estimated using corresponding sample quantities so that an estimator of μ, $\hat{\mu} = \hat{Q}_2$, and an estimator of σ, $\hat{\sigma} = \frac{\hat{Q_3}-\hat{Q_1}}{2}$, the sample quartile deviation.

PROPOSITION 4. *Let P_1, P_2, P_3 and P_4 be i.i.d. Pareto Type I random variables with density $1/x^2$, $x \geq 1$. Then $X_1 = log(P_1/P_2)$ and $X_2 = log(P_3/P_4)$ are i.i.d. standard classical Laplace random variable (Kotz* et al., *2001) and hence $X = X_1/X_2$ has the standard double Lomax density.*

PROPOSITION 5. *Let P_1, P_2, P_3 and P_4 be i.i.d. standard power function distribution with parameter p, having the density*

$$f(x) = px^{p-1},\ p > 0,\ x \in (0,1).$$

Then the random variables $X_1 = p\ log(P_1/P_2)$ and $X_2 = p\ log(P_3/P_4)$ are i.i.d. standard classical Laplace (Kotz et al., *2001) and hence $X = X_1/X_2$ has the standard double Lomax density.*

2.1. Properties

The following properties of the probability density function Eq. (2) hold. For most of them the proof is immediate.

- The standard double Lomax distribution is symmetric around the location parameter $\mu = 0$.

- Like the Cauchy distribution, the standard double Lomax distribution has infinite mean and variance. But for this distribution the fractional moments $E|X|^{\alpha}$ exit for $0 < \alpha < 1$ and more robust estimates for location parameter and variance, such as median and Median Absolute Deviation (MAD) exist.

- The α^{th} moment of X, $E(X^{\alpha})$ exist for $0 < \alpha < 1$ and is given as follows.

$$E(X^{\alpha}) = \Gamma(1+\alpha)\Gamma(1-\alpha).$$

- If X is distributed as the standard double Lomax then $1/X$ also has the standard double Lomax distribution. That is, the standard double Lomax distribution is log symmetric. Also,

$$X^{+} \stackrel{d}{=} 1/X^{+},$$

  where X^{+} follows the half standard double Lomax distribution, or the Lomax distribution, and

$$X^{-} \stackrel{d}{=} 1/X^{-},$$

  where X^{-} has the standard double Lomax density truncated below zero.

- The standard double Lomax distribution is heavy tailed than Laplace distribution and has more area concentrated towards the center (mode). Note that the tail probability of double Lomax density is $\bar{F} \sim cx^{-\alpha}, \alpha = 1 \;\; as \;\; x \to \pm\infty$, where $\bar{F}$ is the survival function. The heavy tail characteristic makes this density appropriate for modelling network delays, signals and noise, financial risk or microarray gene expression or interference which are impulsive in nature.

- The standard Double Lomax distribution is unimodal mode $x = 0$ since the distribution is monotonically increasing and convex on $(-\infty, 0)$ and monotonically decreasing and concave on $(0, \infty)$.

- The standard double Lomax distribution is a scale-mixture of the Laplace distribution. Note that every symmetric density on $(-\infty, \infty)$ which is completely monotone on $(0, \infty)$ is a scale mixture of Laplace distribution (Dreier, 1999).

- The half standard double Lomax density (Lomax density)is a scale-mixture of exponential densities. Note that every completely monotone density on $(0, \infty)$ is a scale mixture of exponential densities on $(0, \infty)$ (Steutel, 1970).

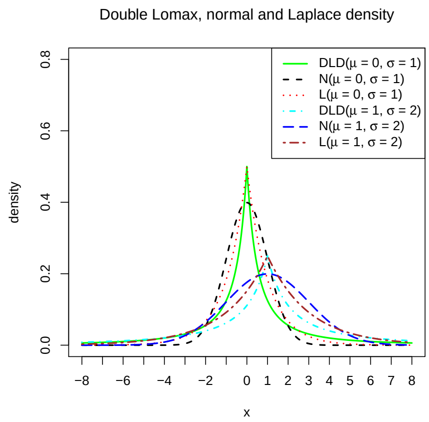

Figure 1 – Double Lomax density functions for various values of parameters.

- If $X \sim DLD(\mu, \sigma)$, then $Y = aX + b \sim DLD(b + a\mu, a\sigma)$ where $a \in \mathcal{R}$, $b \in \mathcal{R}$ and $a \neq 0$. Hence, the distribution of a linear combination of a random variable with $DLD(\mu, \sigma)$ distribution is also DLD. If $X \sim DLD(\mu, \sigma)$, then $Y = (X - \mu)/\sigma \sim DLD(0, 1)$, which can be called as the standard double Lomax distribution.

PROPOSITION 6. *Let f be a symmetric (about zero) probability density on* $(-\infty, \infty)$ *which is completely monotone on* $(0, \infty)$ *(Kotz* et al.*, 2001). Then there exists a distribution G on* $(0, \infty)$ *such that*

$$f(x) = \int_0^\infty \frac{1}{2} y e^{-y|x|} \text{ dG(y); } \text{ x} \neq 0,$$

while the characteristic function corresponding to f is

$$\Psi(t) = \int_0^\infty \frac{1}{1 + t^2/y^2} \text{dG(y)}, \quad -\infty < \text{t} < \infty.$$

REMARK 7. *The converse of the proposition Eq. (6) holds well. Thus every density* $f(x)$ *of the form given above with some cdf G on* $(0, \infty)$ *is a symmetric density on* $(-\infty, \infty)$ *which is completely monotone on* $(0, \infty)$ *(Kotz* et al.*, 2001).*

The shape of the density Eq. (6) is given in Figure 1 along with the normal and Laplace density functions. From Figure 1 we can see that for double Lomax density (DLD) more area is concentrated towards the center and has heavier tails than normal and Laplace distribution.

3. LIKELIHOOD AND ESTIMATION

In this section we study the problem of estimating unknown parameters, $\Theta = (\mu, \sigma)$, of the double distribution. To estimate the parameter μ we use the quantile estimation. The quantile estimate of μ is the $\hat{\mu} =$ sample median.

Let $X = (X_1, \cdots, X_n)$ be independent and identically distributed samples from a double Lomax distribution with parameters Θ. The log-likelihood function takes the form

$$logL(\mu, \sigma; X) = -nlog2 - nlog\sigma - 2\sum_{i=1}^{n} log\left(1 + |(x_i - \mu)/\sigma|\right).$$

The MLE of σ for given $\mu = \hat{\mu}$ is obtained the following score equation.

$$\hat{\sigma} = \frac{2}{n}\sum_{i=1}^{n} \frac{|x_i - \hat{\mu}|}{\left[1 + \frac{|x_i - \hat{\mu}|}{\hat{\sigma}}\right]}.$$

Maximum likelihood estimates (MLE) of the parameter σ can be obtained by solving the score equation. Numerical methods are needed to solve this score equation. In our illustration, the maximization of the likelihood is implemented using the BFGS algorithm of `optim` function (Nash J. C., 1990) in R (R Core Team, 2015). Estimates of the standard errors were obtained by inverting the numerically differentiated information matrix at the maximum likelihood estimates.

3.1. Simulation

In this section we use the simulation study of the double Lomax distribution to validate the estimation algorithm developed in R. Since we can express the distribution function of the double Lomax distribution as well as its inverse in closed form, the inversion method of simulation is straightforward to implement. We performed simulation studies for various choices of parameters to evaluate the performance of the estimation procedure. We generated 1000 samples, each of size $n = 1000$ from the double Lomax distribution with parameters (μ, σ) by inverting the distribution function Eq. (5) in R and then applied the algorithm to obtain the MLEs of the parameters. The results from the 1000 replications are presented in Table 1. It is clear from Table 1 that the estimation algorithm works satisfactorily for various choices of parameters and the asymptotic standard errors of the maximum likelihood estimators agree well with the sample standard deviations over the replications.

4. Application

In this section we apply the double Lomax distribution to model the distribution of a cDNA dual dye microarray gene expression data set. The dataset consists of self-self hybridizations of 19 different cell lines, as well as the Stratagene universal reference RNA (Yang *et al.*, 2002). The self-self arrays were normalized using (Lowess) locally weighted linear regression method (Cleveland and Delvin, 1988)(Cleveland and Delvin, 1988). This method is capable of removing intensity dependence in $log_2(R_i/G_i)$ values and it has been successfully applied to microarray data (Yang *et al.*, 2002), where R_i is the red dye intensity and G_i is the green dye intensity for the i^{th} gene. For example, see Figure 2 for the box plots of intensities from $NT2\_3$ before and after Lowess normalization. After normalisation,

TABLE 1
Simulation study - Maximum likelihood estimates of (μ, σ) for various choices of parameters. Standard deviations over 1000 replications of dataset of size $n = 1000$. SE stands for the asymptotical standard errors of the maximum likelihood estimates.

μ	σ	$\hat{\mu}$	$\hat{\sigma}$	SD($\hat{\mu}$)	SE($\hat{\mu}$)	SD($\hat{\sigma}$)	SE($\hat{\sigma}$)
-0.05	0.5	-0.049	0.508	0.002	0.001	0.010	0.009
	1	- 0.051	0.978	0.001	0.002	0.051	0.070
	1.5	- 0.051	1.464	0.003	0.001	0.025	0.093
-0.01	0.5	- 0.011	0.461	0.003	0.001	0.021	0.028
	1	-0.009	1.154	0.002	0.002	0.104	0.046
	1.5	-0.009	1.548	0.002	0.003	0.112	0.067
0.01	0.5	0.009	0.515	0.001	0.005	0.022	0.041
	1	0.011	1.052	0.002	0.002	0.041	0.065
	1.5	0.011	1.451	0.002	0.002	0.072	0.089
0.05	0.5	0.049	0.527	0.001	0.003	0.036	0.046
	1	0.049	1.029	0.001	0.002	0.053	0.099
	1.5	0.051	1.544	0.002	0.003	0.086	0.121

TABLE 2
Application - maximum likelihood estimates and their asymptotical standard deviations for DLD, Laplace and normal.

	DLD	Laplace	Normal
$\hat{\mu}$	-0.025 (0.009)	-0.025 (0.003)	0.011 (0.002)
$\hat{\sigma}$	0.066 (0.003)	0.327 (0.002)	0.283 (0.001)

each distribution of the gene expression has a similar shape and exhibits heavier tails compared to a Gaussian distribution. We fitted the double Lomax distribution, Laplace and normal to the normalized microarray intensities. The maximum likelihood estimates of the parameters and standard errors (SE) are reported in Table 2.

Figure 3 depicts a histogram of the gene expression data, the fitted DLD, Laplace and normal density functions evaluated at the MLEs. Heavy-tailed distributions are proper distribution to accommodate outliers in the data; in case of microarray gene expression the number of genes differently expressed is usually a very small proportion of the whole dataset. Hence we believe that statistical model presented in this paper will be very useful in estimation and detection problems involving gene expression data.

We used Akaike's Information Criterion (AIC) (Akaike, 1973; Burnham and Anderson, 1998) and Bayesian Information Criterion (BIC) (Schwarz, 1978) to assess the appropriateness of DLD over the Laplace and normal. The AIC and BIC are given by

$$AIC = -2logL + 2K \textit{ and } BIC = -2logL + K\log(n),$$

where $logL = \log(L_f(\hat{\theta}|x_1, \cdots, x_n))$ is the log-likelihood of the data $x_1, \cdots, x_n$ under the probability distribution f, K is the number of parameters being es-

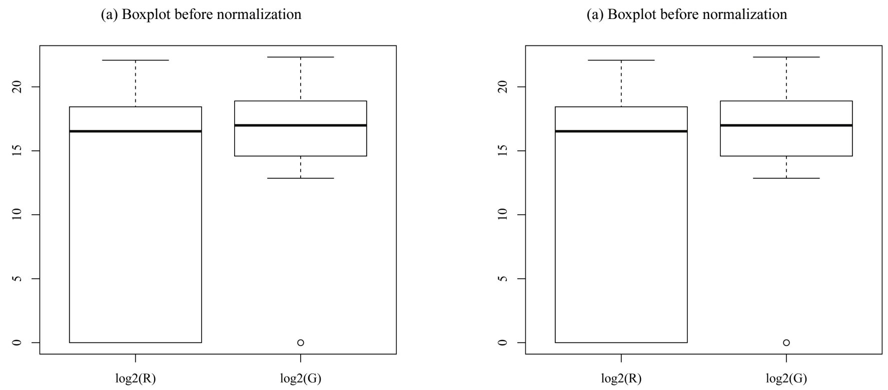

Figure 2 – Box plots of intensities from microarray Experiment $NT2\_3$ (a) Before normalization, (b) After loess normalization.

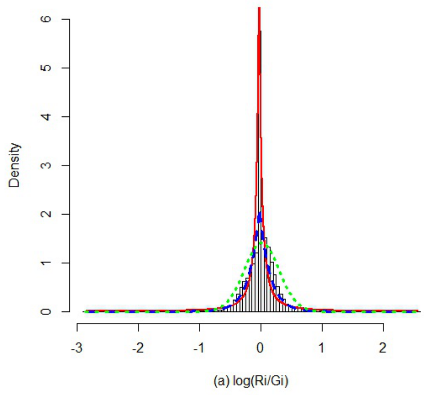

Figure 3 – Fitted double Lomax probability density function (red line), Laplace density function (blue dash) and normal density function (green dot) to the microarray gene expression data.

timated, $\hat{\theta}$ is the maximum likelihood estimate of the parameters of f and n is the sample size. Note that AIC does not explicitly take into account the sample size as BIC does. However, in most cases AIC and BIC are of similar nature and give consistent results for model selection. Here AIC and BIC values coincide. A smaller value of AIC or BIC indicates a better fit. We calculated AIC and BIC for the $DLD(\mu, \sigma)$, $L(\mu, \sigma)$ and $N(\mu, \sigma)$ distributions for the dataset examined. $AIC_{DLD} - AIC_L = -1216 < 0$, $AIC_{DLD} - AIC_N = -2364 < 0$, $BIC_{DLD} - BIC_L = -1216 < 0$ and $BIC_{DLD} - BIC_N = -2364 < 0$. Hence $DLD(\mu, \sigma)$ distribution had a lower AIC, and BIC for the dataset compared to Laplace and normal. A smaller value of AIC and BIC indicates a better fit, and hence, DLD seems to fit the data better than Laplace and normal.

5. Conclusion

The standard double Lomax random variable is the ratio of two i.i.d. classical Laplace variables. The standard double Lomax distribution (DLD) is a flexible distribution that can take heavy tails into account. These are some of the common features of the data related to financial modelling and microarray modelling. The standard double Lomax distribution is log symmetric and heavy tailed like Cauchy distribution. Hence the double Lomax distribution (DLD) introduced in this paper is useful in analysing datasets that are symmetric, leptokurtic, and deviate considerably from the classical symmetric distributions such as normal, Laplace, logistic etc. among others.

We illustrated the application of DLD to a microarray gene expression data set of Yang *et al.* (2002). From Figure 3, it is clear that the tail behaviour and peakdness of the data is captured better in DLD than in normal. The reduction in AIC or BIC for DLD compared to Laplace and normal is small for the dataset considered in the earlier section. As our distribution is a heavy-tailed, it is a proper distribution to accommodate outliers in the data. In case of microarray gene expression data, the number of genes differently expressed is usually a very small proportion of the entire collection of genes studied. Hence, the probability distribution presented in this paper will be very useful in estimation and detection problems involving gene expression data.

The ratio of two i.i.d. classical Laplace variables is log symmetric and heavy tailed like Cauchy distribution. The Laplace model is an alternative to the normal model in situations where the normality assumption do not hold. In a similar way the ratio of independent and identically distributed Laplace distribution is an alternative to the Cauchy distribution. Hence this distribution can be useful in analyzing data sets which exhibits heavy tail. The heavy tail characteristic makes this density appropriate for modeling network delays, signals and noise, financial risk or interference which are impulsive in nature.

Laplace distributions and its generalizations have recently got more importance in various fields including financial modeling, image processing, communication engineering, modeling currency exchange rate, interest rates, stock price changes etc. Hsu (1979) used Laplace distribution to model the position errors observed in large navigation systems. DLD could be applied to modeling error distribution in cases where the distribution deviates from normal. The DLD introduced in this paper can be useful in analyzing data sets which exhibits heavy tails and peakedness. We found that DLD is suitable for modeling microarray gene expression data, since it is having thick tails and sharp peak in the middle. Another application is in modeling of the size distribution of diamonds from a large mining area. Our model also found applications in many fields including modeling of the shapes of long bright gamma-ray bursts discussed by Norris *et al.* (1996); behavioural systems; random fluctuations of response data.

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References

H. Akaike (1973). *Information theory and an extension of the maximum likelihood principle.* In Kotz and Johnson (eds.), *Breakthroughs in Statistics*, Vol I. Springer Verlag, New York, pp. 610–624.

P. P. Bindu, K. Sangita, G. Sebastian (2012). *Asymmetric type II compound Laplace distribution and its application to microarray gene expression.* Computational Statistics and Data analysis, 56, pp. 1396–1404.

K. Burnham, D. Anderson (1998). *Model selection and Inference*, Springer, New York.

W. S. Cleveland, S. J. Delvin (1988). *Locally weighted regression: an approach to regression analysis by local fitting.* Journal of American Statistical Association, 83(403), pp. 596–610.

I. Dreier (1999). *Inequalities for real characteristic functions and their moments.* Ph.D. Dissertation, Technical University of Dresden, Germany.

C. Fernandez, M. F. J. Steel (1998). *On Bayesian modelling of fat tails and skewness.* Journal of American Statistical Association, 93, pp. 359–371.

D. A. Hsu (1979). *Long-tailed distributions for position errors in navigation*, Applied Statistics, 28, pp. 62–72.

Johnson, Kotz, Balakrishnan (1990). *Distributions in Statistics.* Volume I: Continous univariate distributions, Boston, Houghton Miffin.

P. J. Korhonen, S. C. Narula (1989). *The probability distribution of the ratio of the absolute values of two normal variables.* Journal of Statistics Computation and Simulation, 33, pp. 173–182.

S. Kotz, T. J. Kozubowski, K. Podgorski (2001). *The Laplace Distribution and Generalizations: A Revisit with Applications to Communications.* Economics, Engineering and Finance. Birkhäuser, Boston.

G. Marsaglia (1965). *Ratios of normal variables and ratios of sums of uniform variables.* Journal of American Statistical Association, 60, pp. 193–204.

Nadaraja, Kotz (2006). *A note on the ratio of normal and Laplace random variables.* S M A, 15, pp. 151–158.

J. C. Nash (1990). *Compact Numerical Methods for Computers. Linear Algebra and Function Minimisation.* Adam Hilger.

J. P. Norris, R. J. Nemiroff, J. T. Bonnell, J. D. Scargle, C. Kouveliotou, W. S. Paciesas, C. A. Meegan, G. J. Fishman (1996). *Attributes of pulses in long bright gamma-ray bursts*, Astrophysical Journal, 459, pp. 393–412.

T. Pharm-Gia (2000). *Distributions of the ratios of independent beta variables and applications.* Communications in Statistics Theory and Methods, 29, pp. 2693–2715.

S. J. Press (1969). *The t ratio distribution.* Journal of American Statistical Association, 64, pp. 242–252.

S. B. Provost (1989). *On the distribution of the ratio of powers of gamma random variables.* Pakistan Journal of Statistics, 5, pp. 157–174.

R Core Team (2015). *R: A language and environment for statistical computing.* R Foundation for Statistical Computing, Vienna, Austria. `http://www.R-project.org/`.

G. Schwarz (1978). *Estimating the dimension of a model.* Annals of Statistics, 6, pp. 461–464.

F. W. Steutel (1970). *Preservation of Infinite Divisibility Under Mixing and Related Topics.* Mathematisch Centrum, Amsterdam.

Yang, Yee Hwa, Dudoit, Sandrine, Luu, Percy, Lin, M. David, Peng, Vivian, Ngai, John, Speed, P. Terence (2002). *Normalization for cDNA microarray data: a robust composite method addressing single and multiple slide systematic variation.* Nucl. Acids Res., 30(4), e15.

Summary

The Laplace distribution and its generalizations found applications in a variety of disciplines that range from image and speech recognition (input distributions) and ocean engineering to finance. In the present paper, our major goal is to study the family of distribution called the double Lomax distribution which is the ratio of two independent and identically distributed classical Laplace distributions. Also the double Lomax distribution can be obtained by compounding classical Laplace distribution with exponential density. The important statistical properties of double Lomax distribution are explored. Also the relationships with other families of distributions are established. Maximum likelihood estimation procedure is employed to estimate the parameters of the proposed distribution and an algorithm in R package is developed to carry out the estimation. A simulation study is conducted to validate the algorithm. Finally, the application of our model is illustrated. We have used a microarray data set for illustration.

Keywords: Double Lomax distribution; Laplace distribution; Lomax distribution; microarray gene expression.