NOTE IN MARGINE

SOME REMARKS ON A BOOK ON THE PHILOSOPHY OF PROBABILITY

Samuel Kotz

This note contains some comments on the recently published *A Philosophical Introduction to Probability* by Maria Carla Galavotti.[1] The author, professor of philosophy of science at the University of Bologna, Italy, ought to be congratulated for composing a most valuable book on the history of various philosophical positions and interpretations of probability at a level suitable for an interested layman, a senior undergraduate, or a beginning graduate student in probability theory, mathematical statistics, philosophy, physics, engineering and medical quantitative applications. The book fills in successfully an important gap in the literature on various aspects of probability during the last 100 years.

Galavotti emphasizes that the philosophy of probability is a highly controversial subject and concedes that a "competent theorist" may challenge the views expressed in her book. It is evident that a fully satisfactory theory of probability, which is yet to be found, should do justice to both the variability and the unity of probability concepts.

The book consists of seven well-organized and interconnected chapters introducing the reader to the notion of probability and its interpretations. An extensive and thoughtfully selected bibliography of some 20 pages (covering the period from the middle of the seventeenth century up to the very end of the twentieth century) is included.

Starting with the "obligatory" Chevalier de Méré problem and the Pascal-Fermat correspondence (July 1654) and concluding with the post-de Finetti subjective (Bayesian) approach, the book provides a useful and clearly delineated panorama of the rather tortuous terrain of definitions and properties of probability.

It is a common practice in books on foundations of probability to concentrate solely on two (or at most three) interpretations (typically *frequency* and (*subjective*) Bayesian). In the book under discussion the bold author tackles five *major* interpretations and the same number of secondary ones (which some critics refer to as

[1] Stanford, CA: CSLI, 2005. ISBN: 1-57586-489-4. x+265 pp. $70.00(H). Paperback: ISBN 1-57586-490-8 $25.00.

"Balkanization of Probability"). She also focuses on "the peculiar traits and epistemological implications of the various interpretations of probability." Even the "common sense probability" and the "complex pattern of ordinary uses of probability" require substantial amounts of mental agility and are intrinsically connected with the concepts of human fallibility as well as with the elusive concept of rationality. Actually, according to the exalted Pierre Simon de Laplace, "the most important questions in life are usually those of probability."

Chapter 1 provides a well-organized and lucid description of the history of probability, and is successful in sketching various forerunners of the "modern probability" such as G. W. Leibniz (1646-1716), J. Bernoulli (1634-1705) and other members of the Bernoulli family, Thomas Bayes (1701-1761), Condorcet (1743-1794), and a more contemporary A. Quetelet (1796-1874). Finally the fathers of statistics: F. Galton, K. Pearson, R. A. Fisher, F. Y. Edgeworth and (very briefly) the nineteenth- and twentieth-century physicists (including Maxwell, Heisenberg, and Bohr). The last section, which deals with "Probability and Induction", includes succinct surveys of F. Bacon's, D. Hume's, J. S. Mill's, and Herschel's contributions that the present author found to be illuminating.

A large number of footnotes in this chapter constitute a minor distraction from the continuous exposition. The author does not mention that scholars in the early eighteenth century, as a rule, rejected the notion of randomness (chance), emphasizing "necessity" instead (e.g. John Toland (1704), Anthony Collins (1715)).

A short Chapter 2 contains the standard material of the laws and fundamental properties of probability, including conditional probabilities, Bayes' rule and a brief elementary description of Kolmogorov's axioms. Possibly the material on pp. 42-46 could have been shortened - the avalanche of names may overwhelm an uninitiated reader.

Chapter 3 describes "The Classical Interpretation" of probability and discusses P. S. de Laplace's groundbreaking contribution and his two famous essays: *Théorie analytique des probabilités* (1812) and *Essai philosophique sur les probabilités* (1814). The cornerstones of Laplace's theory are that causality is the overall rule governing the universe. This leads him to the embrace of empiricism, determinism, the principle of insufficient reason, equiprobability assumptions, the rule of succession, and the emphasis that probability is useful in *all* fields of knowledge. All this is presented in an attractive, fluent manner and the problems and objections to Laplace's position (the invalidating Bertrand's paradox) are also delineated. The author refers to Stigler's, Gillies', and P. Suppes' published remarks and comments. Possibly S. Zabell's several contributions could have been mentioned.

The objection to the classical definition is that it can be used only in very simple and comparatively unimportant cases like games of chance. This objection was originally made by G. W. Leibniz in 1704 against J. Bernoulli's views and was stressed by von Mises. (Proponents of this definition claim that it is not the fault of the definition but rather our ignorance of the innermost mechanism that, *apart from chance*, contributes to the materialization or non-materialization of *contingent* events.) Classical probability is described in a scholarly and vivid manner in the book by Lorraine Daston (1988), to which the reader is referred for a more comprehensive discussion.

Chapter 4 embarks on the frequency interpretation of probability and describes the two essays on probability (1849 and 1856) of an early exponent of frequency interpretation - R. L. Ellis (1817-1859), a fellow of Trinity College. Unfortunately, biographical details on Ellis are not provided; but an interested reader is referred to a biographical memoir by H. Goodwin.

Ellis' fundamental principle is: "On a long run of similar trials every possible event tends ultimately to recur in a definite ratio of frequency." He concludes his 1849 essay with the observation: "The principle on which the whole depends is the necessity of recognizing the tendency of a series of trials towards regularity as the basis of the theory of probabilities."

R. L. Ellis remains today a somewhat obscure researcher, despite Boole's (1854) comments: "There is no living mathematician for whose intellectual character I entertain a more sincere respect than I do for that of Mr. Ellis." Also W. Salmon (1981) observes: "Ellis took us to the very threshold of a frequency theory of probability, but it was J. Venn who opened the door and led us in."

Next Galavotti provides a more detailed discussion of John Venn's (1834-1923) contributions. Venn - an outstanding logician in Cambridge - is the author of the first book devoted to the frequency interpretation of probability, *The Logic of Chance*, which went through three editions (1866), (1876), and (1888). The fourth edition (unaltered reprint of the third edition of 1888) was published by Chelsea in 1962. An analysis of the basic points delineated in *The Logic of Chance* is also provided.

According to Venn, probability belongs to "*material* logic" aimed at taking "cognizance of the laws of things" (rather than "conceptual(ist) logic", which deals with "the laws of our minds in thinking about things"). It is ultimately based on empirical knowledge and here our reference is exclusively to facts. By counting the number of repetitions in a series of things or events we obtain proportions (frequencies) that generate order out of disorder. "As we keep on taking more terms of the series we shall find ... that its fluctuations will grow less. The proportion in fact will gradually approach towards some fixed numerical value ... termed its limit." This quote describes Venn's formulation of the frequency theory of probability as the limiting value of frequency in infinite series of events. Venn then specifies the rules of probabilistic inference: the additive rule for mutually exclusive and non-exclusive events and the multiplication rule.

Venn strongly criticizes Laplace's *rule of succession*. His ire was directed against the English mathematician and logician Augustus De Morgan, whose textbook on logic, *Formal Logic* (1847), breaks with tradition by presenting probability as a branch of formal logic. He also opposes the view that probability is a measure of belief. Finally (in the Hulsean Lectures of 1869) he analyzes the religious versus scientific belief, claiming: "Reasons to act and reasons to belief should be kept separate." (See *On Some of the Characteristics of Belief Scientific and Religious* (1870)).

Galavotti then proceeds to a survey of R. von Mises' (1883-1953) classical and influential philosophical book *Probability, Statistics and Truth*, whose original German version was published in 1928 and whose English edition appeared in 1939.

(Some of his more technical contributions and lectures were posthumously republished in 1964.)

R. von Mises, a talented and versatile applied mathematician, with important contributions to meteorology, is the main representative of frequentist probability of the logical empirical school. His "fundamental system" provides the central point of his work in probability. (In fact, the debates around his fundamental system seem to overshadow his other works in probability.) The main requirement for randomness is postulated for infinite sequences of labels, thereafter called *collectives*.[2] Von Mises task was: "through abstraction and idealization to represent the connections and dependencies of well determined observable phenomena." [J. von Kries (1886), Bohlman (1901), Broggi (1907) and Borel (1909) seem to be his predecessors.]

In her book Galavotti presents a quote from the earlier part of the PST volume that gives a meaningful and clear expression of von Mises' definition of probability: "A collective is a mass phenomenon or a repetitive event, or, simply, a long sequence of observations for which there are sufficient reasons to believe that the relative frequency of the observed attribute would tend to a fixed limit. ... This limit will be called the probability of the attribute considered within the given collective" (von Mises 1928). Also, von Mises proposes a rigorous definition of randomness using the notion of *place selection* and defines randomness as insensitivity to *all* place selections (also calling it "the principle of impossibility of a gambling system"). An objection to von Mises' definition was elaborated by Reichenbach (to be mentioned below) who considered the definition to be overly restrictive.

Von Mises pushes the positivistic view of probability to an extreme - embracing indeterminism and denying the principle of causality. Von Mises' hope and strong belief expressed in the last passage of the *PST* volume is that: "Starting from a logically clear concept of probability based on experience ... we can discover truth in wide domains of human interest" (p. 220). Russian probabilists, including Kolmogorov and Khinchin, have paid special attention to von Mises' contributions: his book was translated into Russian in 1930 - some nine years before the first English version.

Next Galavotti devotes (in Section 4.4) ten pages to an analysis and detailed comments on H. Reichenbach's (1891-1951) probabilistic epistemology. We shall dwell here on these achievements, although he can be viewed as an isolated figure. Reichenbach completed his doctoral dissertation on the applicability of mathematical probability to the physical world in Berlin in 1916, at the age of 25. In 1926 (due to Einstein's influence) he became professor of Philosophy of Physics in Berlin where he remained till 1933. Having been dismissed from Berlin University, he emigrated to Turkey to become the head of the Philosophy Department at Istanbul University. During his stay in Turkey, he published (in Germany) the *Theory of Probability* (1935). In 1938, he moved to the United States, be-

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[2] This theory of random events is nowadays viewed by some as a "crank semimathematical theory" serving as a warning of the state of probability before the "measure-theoretic revolution" of A. N. Kolmogorov.

coming professor at UCLA, where he remained till his death in 1953. Both von Mises and Reichenbach were in Istanbul in the thirties of the twentieth century but apparently did not collaborate.

In 1938 he returned to the issue of probability with a full-scale attack on logical positivism. Reichenbach embraced a frequency theory of probability and the empiricist view but resented being categorized as "confirmer" of von Mises. He wrote to B. Russell (1949): "My mathematical theory is more comprehensive than Mises' theory and not restricted to random sequences." Unlike von Mises he allows for *single case* probabilities and develops a theory of induction.

He introduces the concept of psychological randomness and points out that random sequences represent merely a special type of probability sequences (tractable by means of statistical methodology).

He stresses the *inductive inference* stated by means of the *rule of induction*, asserting that an inductive step is required to define probability as a limiting frequency. For Reichenbach any probability attribution is "a *posit*, a statement with which we deal as true although the truth value is unknown." The importance of posits is that they "represent the bridge between probability of a sequence and the compulsion to make a decision in a single case." He distinguishes between posits made in a situation of primitive or advanced knowledge. Scientific hypotheses are confirmed within the framework of advanced knowledge and are relying on the Bayesian method. This makes Reichenbach an "objective Bayesian", to use the modern terminology.

Reichenbach attaches a probabilistic meaning to the notion of causality. He defines causal relevance on the basis of statistical relevance (with a restriction, since causal relations are asymmetrical). He claims that the causal structure of the universe can be comprehended with the help of the concept of *probable determination* alone. He has some difficulties with the notion of probabilities of hypotheses. Indeed, how can probabilities of hypotheses be constructed in terms of relative frequencies? Bayes' theorem seems to be able to handle this problem, but Reichenbach does not provide a clear explanation. Reichenbach's writings span many subjects; but he is best known for his work in induction and probability and the philosophy of space and time. His posthumously published work *The Direction of Time* (1956) is a pioneering work on the concept of probabilistic causality. Reichenbach's theory of probability is very insightful, but his frequentism did not attract the attention of statisticians and scientists. W. Salmon, however, was influenced by Reichenbach in developing his theory of scientific explanation.

Galavotti provides a scholarly and lucid summary of Reichenbach's contributions to the philosophy of probability (which are to some extent neglected in the literature). At present, an excellent biography of H. Reichenbach by K. Gemer (1977) is available in German.

The final section of Chapter 4 (Section 4.5) discusses E. Nagel's (1901-1985) *truth frequency theory*. Born in Slovakia, he received a Ph.D. from Columbia University in 1931 for a dissertation on the logic of measurement. He stayed at Columbia until his retirement in 1970, being a proponent of *contextual naturalism* (which involves distrust of reductionists' claims that are *not* outcomes of scientific inquiries). His most extensive contributions are focused on the "Logic of Science". In his classical monograph, *The Principles of the Theory of Probability* (1939), he tends to favour the limit of relative frequency interpretation of Venn and von Mises (but opposes Keynes', Carnap's, and Reichenbach's efforts to develop the notions of degree of confirmation and evidential support). Nagel's truth-frequency interpretation is defined with an explicit reference to the experimental context. Galavotti asserts that Nagel's theory "undeniably harbours some interesting and original traits" (p. 104).

Chapter 5 is devoted to the *propensity interpretation* of probability. The author starts with Charles Sanders Peirce (1839-1914), whom she rightfully considers to be "anticipating" the propensity theory. In his *Notes on the Doctrine of Chance* (1910), Peirce remarks: "[The statement ']The probability that ... a die ... will turn up a number divisible by three ... is one-third['] means that the die has a certain 'would be'; and to say that a die has a 'would be' is to say that it has a property, analogous to any habit that a man might have."

A limitation of Peirce's view is that he regards the dispositional property of probability pertaining to objects while propensity interpretation ascribes probability to the set of conditions surrounding occurrences and events. Peirce is the founder of pragmatism (*The Fixation of Belief* (1877)) and originator of the objective "idealism".

He was a son of Benjamin Peirce - a famous American mathematician of the 19th century. Charles' remarkable intellectual abilities were recognizable early. He lectured at Harvard between 1865 and 1869 and in 1879 at the Johns Hopkins University.

His professional life fell apart in the mid 1880s. Due to his "personal irregularities" and difficult personality, he was terminated both at Johns Hopkins and his permanent job at the Coast Survey. He retreated to rural Pennsylvania and lived there with his second wife till his death in 1914, occasionally venturing to Cambridge, Mass., to give a series of lectures sponsored by his friend William James.

Next, some 10 pages in Chapter 5 are devoted to a (careful and informative) explanation of K. R. Popper's propensity interpretation - which, unfortunately, cannot be discussed here, in view of lack of space. This is then followed by overviews of the contributions of his successors and collaborators: Giere, Mellor, Miller, Fetzer, and Donald Gillies (who is particularly scrutinized). P. Suppes' modern idea that the notion of propensity is a useful ingredient to describe chance phenomena (and is not necessarily connected with probability) is examined. (He is also briefly discussed in Chapter 7.) Galavotti also mentions A. Shimony's (a philosopher of physics) contributions related to physical probabilities, noting that the propensity interpretation represents physical probabilities *better* than the frequency one.

The last section (5.4) provides an overview of the notion of chance, detailing H. Poincaré's (1854-1912) views on chance and randomness, and also noting R. von Mises', M. G. Kendall's, and A. Kolmogorov's contributions. It is evident that Galavotti is fascinated with these concepts (as are many other serious probabilists and statisticians).

These concepts have attracted contributions from numerous researchers. In his *Calcul des Probabilités* (1912), H. Poincaré defines random events from a deterministic position. To paraphrase Galavotti: A typical case obtains when very small causes, or very small differences in the initial conditions of a phenomenon produce macroscopic differences in the final result. In such cases, prediction of the final happening becomes impossible, and a fortuitous phenomenon arises. In addition to the "random events" Poincaré also considers events whose randomness is due to *complexity* and a large number of causes (such as events in the kinetic theory of gases, random distribution of rain drops on a surface). Distribution of cards in a properly shuffled deck also belongs to this category.

Finally, Galavotti describes Humphreys' paradox (1985) (stating that conditional propensities are not correctly represented by the standard theory of conditional probability) based on D. Gillies's discussion (2000).

Humphreys' paradox, originally described in his (1985) paper in *Philosophical Review*, was later discussed by a number of authors. Galavotti also refers to the work of W. Salmon (1979) and (1984), who takes propensities to be causal properties rather than probabilities.

In the long sixth chapter, Ga1avotti skilfully introduces a somewhat heterogeneous group of top scholars from the nineteenth and twentieth centuries whose contributions contain philosophical aspects of probability (although they all have devoted considerable attention to other topics). The players appearing in these encounters (some examined briefly, some more extensively) include: G. W. Leibniz (1646-1716); B. Bolzano (1781-1848); A. De Morgan (1806-1871); G. Boole (1815-1864); W. S. Jevons (1835-1882); J. M. Keynes (1883-1946); W. E. Johnson (1858-1931); L. Wittgenstein (1889-1951); F. Waismann (1896-1959); R. Carnap (1891-1970) and H. Jeffreys (1891-1989).

We shall provide here a survey of some basic ideas and concepts developed by *several* of these pioneers of the logical interpretation. The starting point is the observation that while deductive conclusions provide certain (sure) knowledge, induction gives us only *probable* outcomes. Hence it is necessary to determine the meaning of probability when we are dealing with the connection between *hypotheses* and *conclusions*. Here probability reflects the relation between propositions rather than between real objects or processes. This probability is therefore called *logical* (to distinguish it from "statistical", which is also referred to as "empirical"). Sometimes logical probability is interpreted subjectively as a measure of the subject's belief in the likelihood of some assertions. These measures differ from subject to subject and change with time. This is possibly why the subjective interpretation of logical probability does not have so far many adherents.

The famous British economist who has revolutionized this post-WWI economic theory and practice, J. M. Keynes, is one of the earlier proponents of logical probability, emphasizing that "probability is not subjective, but is in a sense a matter of human 'caprice'." According to Keynes: "As soon as the facts are available which determine our knowledge, one fixes objectively what should be considered probable *irrespective of our opinion*." Thus Keynes views probability theory as a branch of logic.

An unresolved problem (even at present) is how one should numerically assess logical probability. Many experts are of the opinion that when using logical probability one should employ only comparative estimates, i.e. assess probability using only notions such as larger, smaller, or equal.

R. Carnap (one of the most influential philosophers of the twentieth century) and his disciples require a numerical assessment of probabilistic assertions. In his *Logical Foundations of Probability* (1962), Carnap tried to construct a logical theory of confirmation. According to him, "[d]eductive logic may be regarded as the theory of the relation of logical consequence, and inductive logic as the theory of another concept that is likewise objective and logical, viz., ... *degree of confirmation*" (Carnap 1950, 1962). Confirmation, he argued, can be a logical relation if it is considered to be a quantitative generalization of the logical (or deductive) notion of entailment.

Some profound comments of Carnap's degree of confirmation are given by K. R. Popper in publications during the last forty years (1963, 1965, 1969, 1972, 1989, 1994, 2002).

Those who adhere to the frequency concept of probability (e.g. H. Reichenbach) attempt to incorporate the logical probability into the frequency framework. Inductive probability, however, can be also assigned to individual events that do not possess frequency and hence from the point of view of the frequency approach do not possess probabilistic logical relation (which is valid between two propositions). As von Mises, Keynes does not believe that all probabilities have a numerical value.

To get numerical probabilities Keynes advances the *Principle of Indifference* (also known as the *Principle of Non-sufficient Reason*). It states that if there is no *known* reason for predicating of a subject, one rather than another, of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability. This principle leads to "grave contradictions" which are discussed by Keynes (who makes a suggestion on how to construct a modified principle of indifference). Von Mises in the (1928) edition of *Probability, Statistics and Truth* observes that Keynes makes *every* effort "to avoid the dangerous consequences of the subjective theory."

At present there seem to be three basic approaches to the logical (inductive) probability:

1) The first involves an axiomatic description of the properties of logical probability. However, any system of axioms may admit various interpretations and we should, naturally, apply an inductive interpretation of the axiom system. These axioms reflect the formal properties of logical probability - which are common to many other objects - thus the specifics of logical probability are not revealed here.

2) The second approach - called semantic - is based on the analogy between deduction and induction. As already mentioned, in a deductive approach the result *necessarily* follows from the assumptions; in inductive arguments we can view it only as a probabilistic conclusion. Hence the logical probability is a degree of confirmation of one assertion by means of the other. This approach was developed by R. Carnap in *Logical Foundations of Probability* (1950, 1962). Here the basic

concept is the *confirmation degree* to which a metrical value is assigned. However, the approach seems to be inapplicable to physics and some other sciences where the notion of probability is quite important.

3) The third approach, alluded to above, is the frequency interpretation of logical probability (Reichenbach 1949). Here the logical interpretation appears as a particular case in the frequency definitions.

Before proceeding to the final Chapter 7 on "The Subjective Interpretation", which is one of the fields of Galavotti's expertise, she discusses the legendary Harold Jeffreys' (1891-1989) contributions, classifying him to be between "logicism and subjectivism". Jeffreys was a remarkable scholar who during his long productive life spent 75 years at Cambridge, being a pioneer in geophysics. He also contributed substantially to seismology and meteorology. Regrettably, the interaction between H. Jeffreys and R. A. Fisher is barely mentioned and the collaboration between him and the young mathematician Dorothy Wrinch is somewhat underplayed in the book. Jeffreys' *Theory of Probability* (1939; 1948; 1961; and 1963) is a masterpiece and a landmark achievement that was perhaps not sufficiently appreciated by his contemporaries. He defines probability in a pure logical way; where it comes *before* the notions of objectivity, reality, and external world.[3] He emphasizes that probability is *the* most fundamental and general guiding principle of the whole science.

It should be pointed out that the Cambridge scholars J. M. Keynes and H. Jeffreys have referred to probability as a "degree of *rational belief*". This approach led von Mises and some other scholars to suppose that they were subjectivists. (The adjective "rational" did not sufficiently counter the subjective meaning of "belief").

The last Chapter 7 - which, in my opinion, can be compared to Maria Callas singing Carmen - provides a skilful exposition of the subtle, not always intuitive, subjective approaches that by now are becoming the dominant interpretations of probability theory.

The author discusses in some detail the contributions of: W. F. Donkin (1814-1869); É. Borel (1871-1956); F. P. Ramsey (and the connection between him and Keynes and Wittgenstein); B. de Finetti (1906-1985) (who is unquestionably the central player); Richard Jeffrey (1926-2002); and Patrick Suppes (1922-). These highly talented, profound philosophers and mathematicians belong to the élite of modern philosophy of probability. Taken together they are, no doubt, larger than life. It is of course impossible here to dwell on each scholar individually. However, I will briefly discuss these seven personalities and their contributions to the foundations of probability theory.

William Donkin - a gifted astronomer and mathematician, and an expert on Greek music - seems to be the one who, in addition to influencing F. P. Ramsey, addresses the issue of *belief conditioning* in a modern way, anticipating R. Jeffrey's contributions some one hundred years later.

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[3] On Jeffreys see A. Zellner, "Is Jeffreys a 'Necessarist'?", *The American Statistician*, 36 (1982), pp. 28-30.

Émile Borel was a leading mathematician of the 20th century who gave a substantial contribution to Analysis and Probability. Galavotti first discusses Borel's review of Keynes' *Treatise on Probability* (1921) that appeared in 1924. Borel objects to Keynes' concentration on probabilities of judgments and his overlooking of applications of probabilities to sciences. Borel emphasizes that probability has a different value in sciences and everyday life. He states that due to the theory of probability modern physics can now explain "the ... properties of energy and matter, ... biologists succeed in penetrating the ... laws of heredity, permitting agriculturists to improve the stock of their animals and plants." Typically the meaning of probability within science is more objective since here the assessment is grounded on substantial information. Borel calls them *objective probabilities,* to be distinguished from subjective ones, which can have effectively different values for different individuals. This brings Borel to the camp of subjective probabilists, following the route charted by Ramsey and de Finetti. (De Finetti (1939) provides assessment and criticism of Borel's conception).

Next we are treated to a discussion of F. P. Ramsey's brilliant contributions to probability during his short life of 26 years (he passed away a month before his twenty-seventh birthday). Ramsey - a tall, portly, and cheerful person - became a lecturer in Mathematics at Cambridge in 1924, at age twenty. He vigorously interacted with his contemporaries Keynes, Russell and Wittgenstein (translating into English Wittgenstein's *Tractatus*). He was a member of the famous discussion group at Cambridge known as the "Apostles", and read to them seven papers (during 1921-25). The lunches at which Ramsey, P. Sraffa, and Wittgenstein discussed probability remain a piece of Cambridge folklore.

For Ramsey, probability is a degree of belief and the theory of probability is a logic of partial belief. He has shown that the laws of probability are necessarily true for any consistent set of degrees of belief. The link between probability and degrees of belief provided by *coherence* is indeed the cornerstone of subjective probability. Nevertheless, Ramsey is careful to note that: "The degree of belief in p given q is not the same as the degree to which [a subject] would believe p, if he believed q for certain; for knowledge of q might for psychological reasons profoundly alter his whole system of beliefs" (1926, 1931).

Ramsey's theory is in open contrast with that of Keynes. He refuses to attach a definite meaning to the logical relations which constitute the basis of Keynes' approach. But the relationship between degrees of belief and frequency is an open problem from Ramsey's perspective.

Ramsey devotes substantial attention to the concept of chance. He maintains that the definition of chance involves reference to scientific theory and criticizes frequency-based views of chance such as those advocated by the well-known physicist Norman R. Campbell in 1920. For Ramsey chances are degrees of belief within a certain system of beliefs; not those of any particular person, but in a simplified system that consists of natural laws that in this system are believed to be for certain (sure), although people are not really quite certain of them. The chance must be referred to a system containing laws. He accepts the notion of an objective chance. Chances can be said to be objective "in that everyone agrees about them as opposed to, for example, odds on horses." Ramsey's essay *Truth and Probability* (1926, 1931) represents the culmination of a mental process and extensive reading. His idea that within the framework of subjective probability one can accept an objective physical probability goes against B. de Finetti's subjectivism.

Section 7.3 deals with Bruno de Finetti and exchangeability. It is based on two (relatively) recent papers written by Galavotti about "the Master" (who was also an innovator in education and training of mathematics teachers): the first bearing the title "Anti-realism in the Philosophy of Probability: Bruno de Finetti's Subjectivism[4] and the second "Subjectivism, Objectivisim and Objectivity in Bruno de Finetti's Bayesianism".[5] The present author recommends that at least the second paper cited above be consulted to obtain a more comprehensive picture of de Finetti's "radical probabilism".

De Finetti's passion for mathematics during all his academic life is vividly reflected in a letter to his mother after he had switched from Engineering to Mathematics, written in 1925 in Milan when he was nineteen years old: "Mathematics ... is a lively and vital creature, I love it and I wish to devote my life to it". A few years later he writes: "Every word, every formula in the work I have done is blood of my blood, is the fruit of strong-willed inebriation and deep and creative pain" (cited by C. Rossi (2001) from the Fulvia de Finetti's paper (2000)).

According to de Finetti probability simply "means degree of belief (as actually held by someone on the ground of his/her whole knowledge)." His theory of probability is by now well known and accepted by numerous statisticians, computer experts, and engineers. He started his preaching in the early thirties of the twentieth century in Italian and received worldwide attention in the mid fifties only after being "discovered" by L. J. Savage and D. V. Lindley. His two-volume book in English on *Theory of Probability*, published in 1975, resulted in wide acceptance of his ideas amongst Anglo-American probabilists and statisticians[6]. At present we are witnessing a cult of de Finetti's personality that is being sustained, in part, by the well-organized and informative Valencia meetings on Bayesian Statistics every few years and the recent celebration of the 100th anniversary of his birth.

Section 7.4 "Some Recent Trends" discusses the contributions of R. Jeffrey (1926-2002) and Patrick Suppes (1922-) who, at the time of this writing, is very much active in his research. Galavotti has enjoyed close scientific relations with R. Jeffrey (to whose memory the book is dedicated). This is reflected in her very lucid (and sympathetic) portrait and assessment of his influence and contributions. R. Jeffrey - while being in tune with de Finetti's subjectivism - tried to extend it to the objective probability. In this connection, he disagreed with the

[4] In *Erkenntnis*, 31 (1989), pp. 239-261.

[5] In *Foundation of Bayesianism*, ed. by D. Corfield and J. Williamson, Dordrecht and Boston: Kluwer, 2001, pp.161-174.

[6] The books are translations of the Italian original version (first published in 1970), ably rendered in English by A. Machi (Rome) and A. Smith (Oxford), with a Foreword by D. V. Lindley.

"radicalism" of de Finetti who rejected such basic notions as chance, objective probability, and randomness, making room for a "non-frequentist objectivism"[7].

Patrick Suppes' disagreements with de Finetti concerning objective probability that led to Suppes' pluralistic perspective are being analyzed. However, Galavotti does not discuss upper and lower probabilities, introduced by Suppes and Zanotti (1996). The author also does not mention the Dempster-Shafer theory of belief functions (in probabilistic combination of evidence) that is a generalization of the Bayesian theory. Originally, the upper and lower probabilities were discussed by A. P. Dempster as early as 1967.

The advantage of the Dempster-Shafer theory is that it can describe in a natural manner the state of total ignorance. In this connection G. Shafer's thoughtful paper (1993) is also relevant, as is F. Hampel's recent paper in German, "On the Discussion of Fundamental Principles in Statistics". In his paper Hampel introduces "programmatic" Bayesians such as G. E. P. Box who are content with just approximations. He also mentions works of the extremely productive I. J. Good[8] who includes upper and lower probabilities but continues to call himself a Bayesian. Hampel introduces the concepts of a "successful bet" - an extension of the fair bets - (for which the expected value of "my win" is ≥ 0). This allows a continuous transition from total ignorance to asymptotically complete knowledge of the underlying prior probability distribution.

The short "Closing Remarks" provide a compact summary of the numerous "seemingly irreconcilable perspectives" successfully described in this book. No doubt these remarks will increase our appreciation of the leading players and awesome and overwhelming subject matter (so closely associated with both science and everyday life).

The George Washington University SAMUEL KOTZ
School of Engineering and Applied Sciences

ACKNOWLEDGEMENTS

Thanks to Dr. Jane Harvill and Ms. Jutta Raithel, The Catholic University of America, for their valuable assistance.

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[7] Galavotti also mentions briefly D. Lewis' "A Subjectivist's Guide to Objective Chance" and his postulate of "Principal Principle" that provoked a debate in the literature, spearheaded by Colin Howson, with several papers published in the last twenty years in the *British Journal for the Philosophy of Science*.

[8] At present of Virginia Polytechnic, celebrating his 90th birthday.

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