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ESTIMATION AND PREDICTION BASED ON K-RECORD VALUES FROM NORMAL DISTRIBUTION

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1. INTRODUCTION

Let $\{X_i, i \geq 1\}$ be a sequence of independent and identically distributed (iid) random variables having an absolutely continuous cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$. An observation X_j is called an upper record if its value exceeds that of all previous observations. Thus, X_j is an upper record if $X_j > X_i$ for every $i < j$. An analogous definition deals with lower record values.

Record data arise in a wide variety of practical situations, such as in destructive stress testing, meteorological analysis, hydrology, seismology, sporting and athletic events and oil and mining surveys. Interest in records has increased steadily over the years since Chandler (1952) first formulated the theory of records. The problem of estimation of parameters and prediction of future record values have been discussed by several authors including Balakrishnan and Chan (1998), Sultan et al. (2002), Sultan (2000), Raqab et al. (2007) and Raqab (2002). For more details about record values and their applications, one may refer to Arnold et al. (1998) and Ahsanullah (1995).

Serious difficulties for statistical inference based on records arise due to the fact that the occurrences of record data are very rare in practical situations and the expected waiting time is infinite for every record after the first. These problems are avoided if we consider the model of k-record statistics introduced by Dziubdziela and Kopocinski (1976).

For a positive integer k, the upper k-record times $T_{n(k)}$ and the upper k-record values $R_{n(k)}$ are defined as follows:

$$T_{0(k)} = k, \quad with\ probability\ 1$$

and, for $n \geq 1$

$$T_{n(k)} = min\{j : j > T_{n-1(k)}, X_j > X_{T_{n-1(k)}-k+1:T_{n-1(k)}}\}$$

where $X_{i:m}$ denote the *i*-th order statistic in a sample of size *m*. The sequence of upper k-records are then defined by

$$R_{n(k)} = X_{T_{n(k)}-k+1:T_{n(k)}}\ , n \geq 0, k \geq 1.$$

In an analogous way, one can define the lower k-record statistics. Since the ordinary record values are contained in the k-records, by putting $k=1$, the results for usual records can be obtained as special case. Statistical inference problems based on k-records have been considered by several authors, see, Malinowska and Szynal (2004), Ahmadi et al. (2005), Ahmadi and Doostparast (2008) and Shy Mary and Chacko (2010).

The pdf of $R_{n(k)}$, for $n \geq 0$ is given by (see, Arnold et al., 1998)

$$f_{n(k)}(x) = \frac{k^{n+1}}{n!}\left[-log(1-F(x))\right]^n \left[1-F(x)\right]^{k-1} f(x), -\infty < x < \infty \tag{1}$$

and the joint pdf of mth and nth k-record values for $m < n$ is given by,

$$f_{m,n(k)}(x,y) = \frac{k^{n+1}}{m!(n-m-1)!}\left[-log(1-F(x))\right]^m$$

$$\times\left[-log(1-F(y)) + log(1-F(x))\right]^{n-m-1}$$

$$\times\frac{[1-F(y)]^{k-1}}{1-F(x)} f(x)f(y), \quad x < y. \tag{2}$$

In this paper we consider k-record values arising from normal distribution. In section 2, we compute the means, variances and covariances of k-record values arising from normal distribution. In section 3, we determine the BLUEs of the mean μ and standard deviation σ of normal distribution based on k-record values. These BLUEs are then used to predict the future k-record values. In section 4 we illustrate the inference procedures developed in this paper based on k-record values from normal distribution using real data.

2. MOMENTS OF THE K-RECORD VALUES

Let $R^*_{0(k)}, R^*_{1(k)}, \ldots, R^*_{n(k)}$ be the first $(n+1)$ upper k-record values arising from a sequence of iid standard normal random variables. If we denote $\Phi(\cdot)$ as the cdf and $\varphi(\cdot)$ as the pdf of a standard normal random variable, then by using (1), the pdf of nth upper k-record value $R^*_{n(k)}$ is given by

$$f^*_{n(k)}(x) = \frac{k^{n+1}}{n!}\left[-log(1-\Phi(x))\right]^n \left[1-\Phi(x)\right]^{k-1} \varphi(x), -\infty < x < \infty, n \geq 0. \tag{3}$$

From (2), the joint pdf of mth and nth upper k-record values, $R^*_{m(k)}$ and $R^*_{n(k)}$ for $m < n$ is given by

$$f^{*}_{m,n(k)}(x,y)=\frac{k^{n+1}}{m!(n-m-1)!}\left[-\log\left(1-\Phi(x)\right)\right]^{m}$$
$$\times\left[-\log\left(1-\Phi(y)\right)+\log\left(1-\Phi(x)\right)\right]^{n-m-1} \quad 4)$$
$$\times\frac{[1-\Phi(y)]^{k-1}}{1-\Phi(x)}\varphi(x)\varphi(y),\ \infty<x<y<\infty.$$

Let us denote $E\left(R^{*}_{n(k)}\right)$ by $\alpha_{n(k)}$, $Var\left(R^{*}_{n(k)}\right)$ by $\beta_{n,n(k)}$, $E\left(R^{*}_{m(k)}R^{*}_{n(k)}\right)$ by $\alpha_{m,n(k)}$ and $Cov\left(R^{*}_{m(k)},R^{*}_{n(k)}\right)$ by $\beta_{m,n(k)}$. Then, the single moments of the *n*th upper k-record value for $n\geq 0$ is given by

$$\alpha_{n(k)}=\int_{-\infty}^{\infty}xf^{*}_{n(k)}(x)\,dx,$$

and the product moments of the *m*th and *n*th upper k-record values for $m<n$ is given by

$$\alpha_{m,n(k)}=\int_{-\infty}^{\infty}\int_{x}^{\infty}xyf^{*}_{m,n(k)}(x,y)\,dy\,dx,$$

where $f^{*}_{n(k)}(\cdot)$ and $f^{*}_{m,n(k)}(\cdot,\cdot)$ are given, respectively, in (3) and (4).

We have evaluated numerically the values of $\alpha_{n(k)}$ and $\alpha_{m,n(k)}$ for sample size up to 10. The values of means, $\alpha_{n(k)}$ for $n=0(1)9$ and $k=2(1)5$ are given in table 1. By using the values of $\alpha_{n(k)}$ and $\alpha_{m,n(k)}$ we have computed the values of variances and covariances, $\beta_{m,n(k)}$ for $0\leq m\leq n\leq 9$ and $k=2(1)5$ and are given in table 2.

REMARK 1. Suppose $\left\{R'_{n(k)}, n\geq 0\right\}$ *is the sequence of lower k-record values arising from a sequence* $\{X_i\}$ *of iid standard normal variables. Then, due to the symmetry of the standard normal distribution, one can observe that*

$$R'_{n(k)}\overset{d}{=}-R^{*}_{n(k)}\ \ \text{and}\ \ \left(R'_{m(k)},R'_{n(k)}\right)\overset{d}{=}\left(-R^{*}_{m(k)},-R^{*}_{n(k)}\right) \quad (5)$$

and hence,

$$E\left(R'_{n(k)}\right)=\alpha_{n(k)},\ \ Var\left(R'_{n(k)}\right)=\beta_{n,n(k)}\ \ \text{and}\ \ Cov\left(R'_{m(k)},R'_{n(k)}\right)=\beta_{m,n(k)}$$

As a result, the entries in tables 1 and 2 also yield the means, variances and covariances of the lower k-record values from the standard normal distribution.

3. BEST LINEAR UNBIASED ESTIMATION

Let $R_{0(k)}, R_{1(k)}, \dots, R_{n(k)}$ be the first $(n+1)$ upper k-record values from a normal distribution with mean μ and variance σ^2. Then we have

$$E\left(R_{n(k)}\right) = \mu + \sigma\alpha_{n(k)},$$

$$\mathrm{V}ar\left(R_{n(k)}\right) = \sigma^2\beta_{n,n(k)}$$

and for $m < n$

$$\mathrm{C}ov\left(R_{m(k)}, R_{n(k)}\right) = \sigma^2\beta_{m,n(k)}.$$

Suppose $\boldsymbol{R}_{n(k)} = \left(R_{0(k)}, R_{1(k)}, \dots, R_{n(k)}\right)^T$ denote the vector of upper k-record values. Then

$$E\left(\boldsymbol{R}_{n(k)}\right) = \mu 1 + \sigma\boldsymbol{\alpha}, .$$

where $\boldsymbol{\alpha} = \left(\alpha_{0(k)}, \alpha_{1(k)}, \dots, \alpha_{n(k)}\right)^T$ and **1** is a column vector of *(n+1)* ones. The variance-covariance matrix of $\boldsymbol{R}_{n(k)}$ is given by

$$D\left(\boldsymbol{R}_{n(k)}\right) = B\sigma^2,$$

where $B = ((\beta_{i,j(k)}, 0 \le i, j \le n))$. Following the generalized least-squares approach, the BLUEs of μ and σ are given, respectively, by (see, Balakrishnan and Cohen, 1991, pp. 80-81)

$$\begin{aligned} \mu^* &= \frac{\boldsymbol{\alpha}^T B^{-1}\boldsymbol{\alpha} 1^{\mathrm{T}} B^{-1} - \boldsymbol{\alpha}^T B^{-1} 1 \boldsymbol{\alpha}^{\mathrm{T}} B^{-1}}{(\boldsymbol{\alpha}^T B^{-1}\boldsymbol{\alpha})(1^T B^{-1} 1) - (\boldsymbol{\alpha}^T B^{-1} 1)^2} \boldsymbol{R}_{n(k)} \\ &= \sum_{i=0}^{n} a_i R_{i(k)} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \boldsymbol{\alpha}^* &= \frac{1^T B^{-1} 1 \boldsymbol{\alpha}^T B^{-1} - 1^T B^{-1}\boldsymbol{\alpha} 1^T B^{-1}}{(\boldsymbol{\alpha}^T B^{-1}\boldsymbol{\alpha})(1^T B^{-1} 1) - (\boldsymbol{\alpha}^T B^{-1} 1)^2} \boldsymbol{R}_{n(k)} \\ &= \sum_{i=0}^{n} b_i R_{i(k)} \, . \end{aligned} \tag{7}$$

Furthermore, the variances and covariance of the above estimators are given by (see, Balakrishnan and Cohen, 1991, pp. 80-81)

$$Var(\mu^*) = \sigma^2 \left(\frac{\boldsymbol{\alpha}^T B^{-1} \boldsymbol{\alpha}}{(\boldsymbol{\alpha}^T B^{-1} \boldsymbol{\alpha})(1^T B^{-1} 1) - (\boldsymbol{\alpha}^T B^{-1} 1)^2} \right), \tag{8}$$

$$Var(\sigma^*) = \sigma^2 \left(\frac{1^T B^{-1} 1}{(\boldsymbol{\alpha}^T B^{-1} \boldsymbol{\alpha})(1^T B^{-1} 1) - (\boldsymbol{\alpha}^T B^{-1} 1)^2} \right) \tag{9}$$

and

$$Cov(\mu^*, \sigma^*) = \sigma^2 \left(\frac{-\boldsymbol{\alpha}^T B^{-1} 1}{(\boldsymbol{\alpha}^T B^{-1} \boldsymbol{\alpha})(1^T B^{-1} 1) - (\boldsymbol{\alpha}^T B^{-1} 1)^2} \right). \tag{10}$$

By making use of the values of means, variances and covariances presented in tables 1 and 2, we have calculated the coefficients a_i and $b_i, i = 0,1,\ldots,n$ of BLUEs of μ and σ for $n = 1(1)9$; $k = 2(1)5$ and are given in tables *3* and *4*. The values of $\frac{Var(\mu^*)}{\sigma^2}$ *and* $\frac{Var(\sigma^*)}{\sigma^2}$ for $n = 1(1)9$; $k = 2(1)5$ are also computed and are incorporated in tables *3* and *4*. From tables *3* and *4*, we see that as *n* increases the variances of μ^* and σ^* decreases.

4. PREDICTION OF FUTURE K-RECORD VALUES

Prediction of future events on the basis of the past and present knowledge is a fundamental problem of statistics, arising in many contexts and producing varied solutions. Similarly prediction of future records is also a problem of great interest in many areas such as sports, weather etc. For example, while studying the record rainfalls or snowfalls, having observed the record values until the present time, we will be naturally interested in predicting the amount of rainfall or snowfall that is to be expected when the present record is broken for the first time in future. Based on the usual records *(k=1)*, the problem of predicting future records has been studied by several authors. See for example, Ahsanullah (1980), Berred (1998), Sultan et al. (2008) and Raqab and Balakrishnan (2008). Ahmadi and Balakrishnan (2010) discuss the problem of predicting future order statistics based on observed record values and similarly, the prediction of future records based on observed order statistics. Ahmadi et al. (2012) study the problem of predicting future k-records based on k-record data from a general class of distributions under balanced type loss functions.

Assume that we have the first *(n+1)* upper k-record values from normal distribution. The problem of interest then is to predict the value of the next upper k-record $R_{n+1(k)}$, or, more generally, the value of the *m*th upper k-record $R_{m(k)}$ for some $m > n$. For a location-scale family with location parameter μ and scale parameter σ, the best linear unbiased predictor (BLUP) of *m*th record value was considered by Balakrishnan and Chan (1998). For k-record values, the BLUP of $R_{m(k)}$ for some $m > n$ can be written as

$$\tilde{R}_{m(k)} = \mu^* + \alpha_{m(k)} \sigma^*, \tag{11}$$

where μ^* and σ^* are the BLUEs of μ and σ based on the first $(n+1)$ upper k-record values and $\alpha_{m(k)}$ is the mean of *m*th k-record value from the standard distribution.

5. ILLUSTRATIVE EXAMPLE

In order to illustrate the usefulness of the prediction procedure described in section *3*, we use the following data which represent the records of the total annual rainfall (in inches) at Oxford, England, for the years 1858-1903; (see, Arnold et al., 1998, p. 173). The upper 2-records extracted from these data are as follows:

20.77, 25.3, 26.72, 28.07, 30.17, 30.41, 31.77, 31.94

A simple plot of these eight upper 2-record values against the expected values in table *1* indicate a very strong correlation (correlation coefficient as high as *0.9905*). Hence, the assumption that these 2-record values have come from a normal $N(\mu,\sigma^2)$ distribution is quite reasonable. Using the first four upper 2-record values we predict the $6th$ upper 2-record value. From tables *3* and *4*, we determine the BLUEs of μ and σ to be

$$\mu^* = (0.4768\times 20.77)+(0.2360\times 25.23)+(0.1629\times 26.72)+(0.1243\times 28.07)$$
$$= 23.7162$$

and

$$\sigma^* = -(0.4944\times 20.77)-(0.2103\times 25.3)-(0.1378\times 26.72)+(0.8424\times 28.07)$$
$$= 4.3774$$

Then by using (11), the BLUP of $6th$ value of 2-record is obtained as

$$\tilde{R}_{5(2)} = 30.60,$$

while the actual value of the $6th$ 2-record is 30.41.

REMARK 2. Due to the symmetry of the normal distribution and hence the relation in (5), all the inference procedures based on tables 3 and 4 can be suitably changed to handle the case when the lower k-record values are given instead of the upper k-record values.

6. CONCLUSION

Record values and the associated statistics are of interest and important in many real life applications. Serious difficulties for statistical inference based on records arise due to the fact that the occurrences of record data are very rare in practical situations and the expected waiting time is infinite for every record after the first. These problems are avoided if we consider the model of k-record statistics. In this paper, we have obtained the BLUEs of the location and scale parameters of normal distribution based on k-record values. We have also obtained the BLUP of future k-record value for the normal distribution. A real data set is used to illustrate the inferential procedure developed in this paper.

TABLE 1
Means of the upper k-record values from N(0,1)

k	n									
	0	1	2	3	4	5	6	7	8	9
2	-0.564	0.170	0.639	1.003	1.307	1.573	1.812	2.029	2.231	2.419
3	-0.846	-0.187	0.227	0.544	0.807	1.036	1.240	1.426	1.598	1.758
4	-1.029	-0.415	-0.035	0.255	0.494	0.701	0.885	1.052	1.206	1.349
5	-1.163	-0.580	-0.222	0.049	0.271	0.463	0.634	0.789	0.930	1.062

TABLE 2
Variances and covariances of the upper k-record values from N(0,1)

k	*n*	*m*									
		0	1	2	3	4	5	6	7	8	9
2	0	0.682									
	1	0.387	0.490								
	2	0.288	0.366	0.424							
	3	0.236	0.301	0.349	0.389						
	4	0.204	0.260	0.302	0.336	0.367					
	5	0.182	0.231	0.269	0.300	0.327	0.352				
	6	0.165	0.210	0.244	0.272	0.297	0.320	0.340			
	7	0.152	0.194	0.225	0.251	0.274	0.295	0.314	0.332		
	8	0.141	0.180	0.209	0.234	0.255	0.274	0.292	0.309	0.325	
	9	0.133	0.169	0.197	0.220	0.240	0.258	0.275	0.290	0.305	0.319
3	0	0.559									
	1	0.309	0.384								
	2	0.227	0.283	0.324							
	3	0.184	0.230	0.264	0.293						
	4	0.158	0.198	0.227	0.251	0.273					
	5	0.140	0.175	0.201	0.223	0.242	0.259				
	6	0.127	0.159	0.182	0.202	0.219	0.234	0.249			
	7	0.116	0.146	0.167	0.185	0.201	0.215	0.229	0.241		
	8	0.108	0.135	0.155	0.172	0.187	0.200	0.212	0.224	0.235	
	9	0.101	0.127	0.146	0.161	0.175	0.188	0.199	0.210	0.220	0.229
4	0	0.492									
	1	0.267	0.327								
	2	0.194	0.238	0.271							
	3	0.157	0.193	0.220	0.242						
	4	0.134	0.165	0.188	0.207	0.224					
	5	0.118	0.146	0.166	0.183	0.198	0.211				
	6	0.106	0.131	0.150	0.165	0.178	0.190	0.201			
	7	0.097	0.120	0.137	0.151	0.163	0.174	0.185	0.194		
	8	0.090	0.112	0.127	0.140	0.151	0.162	0.171	0.180	0.188	
	9	0.084	0.104	0.119	0.131	0.142	0.151	0.160	0.168	0.176	0.183
5	0	0.448									
	1	0.239	0.291								
	2	0.172	0.210	0.238							
	3	0.139	0.169	0.192	0.210						
	4	0.118	0.144	0.163	0.179	0.193					
	5	0.104	0.127	0.144	0.158	0.170	0.181				
	6	0.093	0.114	0.129	0.142	0.153	0.163	0.172			
	7	0.085	0.104	0.118	0.130	0.140	0.149	0.157	0.165		
	8	0.079	0.097	0.110	0.120	0.130	0.138	0.146	0.153	0.160	
	9	0.074	0.090	0.102	0.112	0.121	0.129	0.136	0.143	0.149	0.155

TABLE 3

Coefficients for the BLUE of μ and the values of $\frac{\mathrm{Var}(\mu^*)}{\sigma^2}$ based on the first (n+1) upper k-record values from $N(\mu,\sigma^2)$.

k	n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	$\frac{\mathrm{Var}(\mu^*)}{\sigma^2}$
2	1	0.232	0.768									0.464
	2	0.444	0.224	0.332								0.422
	3	0.477	0.236	0.163	0.124							0.419
	4	0.476	0.236	0.163	0.129	-0.003						0.419
	5	0.467	0.232	0.160	0.128	0.103	-0.090					0.419
	6	0.456	0.228	0.157	0.125	0.102	0.081	-0.147				0.417
	7	0.444	0.224	0.153	0.123	0.100	0.076	0.078	-0.198			0.415
	8	0.433	0.219	0.149	0.124	0.100	0.071	0.078	0.045	-0.217		0.413
	9	0.422	0.213	0.148	0.119	0.098	0.070	0.083	0.038	0.083	-0.274	0.410
3	1	-0.284	1.284									0.453
	2	0.168	0.113	0.719								0.301
	3	0.280	0.161	0.118	0.441							0.274
	4	0.320	0.178	0.125	0.112	0.266						0.268
	5	0.335	0.185	0.130	0.112	0.086	0.152					0.267
	6	0.339	0.187	0.132	0.113	0.086	0.084	0.060				0.267
	7	0.339	0.187	0.132	0.113	0.086	0.084	0.051	0.008			0.267
	8	0.337	0.185	0.131	0.113	0.086	0.085	0.047	0.071	-0.055		0.267
	9	0.333	0.183	0.131	0.111	0.083	0.087	0.048	0.071	0.060	-0.106	0.266
4	1	-0.676	1.676									0.538
	2	-0.044	0.023	1.021								0.277
	3	0.131	0.098	0.074	0.697							0.222
	4	0.201	0.128	0.099	0.081	0.491						0.206
	5	0.236	0.142	0.111	0.087	0.070	0.354					0.199
	6	0.253	0.152	0.114	0.089	0.079	0.058	0.255				0.197
	7	0.262	0.156	0.118	0.091	0.080	0.064	0.061	0.169			0.196
	8	0.266	0.157	0.120	0.092	0.083	0.059	0.068	0.064	0.091		0.196
	9	0.268	0.158	0.120	0.093	0.080	0.062	0.066	0.067	0.035	0.050	0.196
5	1	-0.995	1.995									0.653
	2	-0.212	-0.063	1.275								0.288
	3	0.012	0.045	0.025	0.919							0.205
	4	0.108	0.082	0.070	0.056	0.684						0.176
	5	0.157	0.106	0.084	0.064	0.067	0.521					0.165
	6	0.185	0.118	0.098	0.070	0.072	0.043	0.414				0.159
	7	0.201	0.127	0.103	0.074	0.075	0.044	0.069	0.307			0.156
	8	0.210	0.132	0.102	0.083	0.072	0.052	0.061	0.070	0.217		0.155
	9	0.216	0.135	0.107	0.081	0.078	0.047	0.069	0.062	0.022	0.182	0.155

TABLE 4

Coefficients for the BLUE of σ *and the values of* $\frac{\mathrm{Var}(\sigma^*)}{\sigma^2}$ *based on the first (n+1) upper k-record values from* $N(\mu,\sigma^2)$.

k	n	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	$\frac{\mathrm{Var}(\sigma^*)}{\sigma^2}$
	1	-1.362	1.362									0.739
	2	-0.717	-0.294	1.011								0.354
	3	-0.494	-0.210	-0.138	0.842							0.229
	4	-0.382	-0.164	-0.115	-0.074	0.735						0.168
	5	-0.314	-0.133	-0.094	-0.072	-0.051	0.663					0.132
	6	-0.266	-0.117	-0.080	-0.058	-0.045	-0.048	0.615				0.108
	7	-0.232	-0.105	-0.069	-0.053	-0.042	-0.035	-0.029	0.565			0.092
	8	-0.207	-0.094	-0.059	-0.054	-0.040	-0.024	-0.027	-0.017	0.522		0.079
	9	-0.187	-0.083	-0.058	-0.045	-0.037	-0.022	-0.037	-0.005	-0.032	0.505	0.070
3	1	-1.518	1.518									0.748
	2	-0.794	-0.357	1.151								0.358
	3	-0.551	-0.252	-0.155	0.959							0.234
	4	-0.423	-0.201	-0.131	-0.097	0.852						0.171
	5	-0.347	-0.164	-0.105	-0.094	-0.070	0.780					0.135
	6	-0.294	-0.144	-0.088	-0.084	-0.064	-0.034	0.708				0.110
	7	-0.255	-0.128	-0.081	-0.068	-0.057	-0.032	-0.046	0.667			0.094
	8	-0.228	-0.111	-0.073	-0.062	-0.053	-0.045	-0.009	-0.044	0.626		0.081
	9	-0.203	-0.101	-0.073	-0.051	-0.037	-0.055	-0.010	-0.042	-0.038	0.611	0.071
4	1	-1.629	1.629									0.756
	2	-0.856	-0.393	1.249								0.365
	3	-0.587	-0.278	-0.208	1.073							0.236
	4	-0.452	-0.220	-0.161	-0.108	0.942						0.174
	5	-0.367	-0.187	-0.130	-0.094	-0.092	0.870					0.136
	6	-0.313	-0.154	-0.122	-0.087	-0.066	-0.051	0.791				0.112
	7	-0.272	-0.139	-0.102	-0.078	-0.061	-0.024	-0.086	0.761			0.095
	8	-0.239	-0.129	-0.088	-0.072	-0.038	-0.064	-0.029	-0.062	0.721		0.082
	9	-0.217	-0.111	-0.086	-0.055	-0.067	-0.022	-0.052	-0.026	-0.041	0.677	0.072
5	1	-1.715	1.715									0.768
	2	-0.898	-0.433	1.331								0.370
	3	-0.616	-0.298	-0.242	1.157							0.238
	4	-0.474	-0.243	-0.175	-0.122	1.014						0.176
	5	-0.386	-0.199	-0.149	-0.108	-0.099	0.940					0.138
	6	-0.327	-0.174	-0.122	-0.095	-0.088	-0.056	0.861				0.114
	7	-0.285	-0.150	-0.108	-0.086	-0.080	-0.053	-0.045	0.807			0.096
	8	-0.251	-0.135	-0.111	-0.052	-0.091	-0.024	-0.071	-0.040	0.775		0.083
	9	-0.228	-0.121	-0.092	-0.060	-0.066	-0.045	-0.039	-0.070	-0.001	0.723	0.073

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SUMMARY

Estimation and prediction based on k-record values from normal distribution

In this paper, we introduce the k-record values arising from normal distribution. After computing the means, variances and covariances of the k-record values, we determine the best linear unbiased estimators for the location and scale parameters of normal distribution based on k-record values. The best linear unbiased predictor of future k-record values is also determined. Finally, a real data is given to illustrate the inference procedures developed in this paper.

Keywords: k-record values; normal distribution; best linear unbiased estimation; best linear unbiased prediction