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AN ALTERNATIVE TO KIM AND WARDE'S MIXED RANDOMIZED RESPONSE TECHNIQUE

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1. INTRODUCTION

The randomized response technique (RRT) is a survey method especially developed to improve the accuracy of answers to sensitive questions. Socially sensitive questions are thought to be threatening to respondents. When sensitive topics are studied, respondents often react in ways that negatively affect the validity of the data. Such a threat to validity of the results is the respondents' tendency to give socially desirable answers to avoid social embarrassment and to project a positive self – image. Warner (1965) was first to develop a model for estimating the proportion of individuals possessing a sensitive attribute without requiring the individual respondent to report the [1]interviewer whether or not he or she possesses the sensitive attribute. Further modifications in the model and in choice of unrelated questions were suggested by Greenberg *et al.* (1969), Moors (1971), Folsom *et al.* (1973), Bourke (1982), Chaudhuri and Mukerjee (1987,1988), Mangat and Singh (1990), Mahajan *et al.*(1994), Singh and Tracy (1999), Singh and Tarray (2012, 2013, 2014) , Kim and Warde (2005) and others.

To implement the privacy problem with the Moors (1971) model, Mangat *et al.* (1997) and Singh *et al.* (2000) presented various strategies as an alternative to Moors (1971) model, but their models used simple random sampling without replacement (SRSWOR) which led to a high – cost survey compared with the Moors model using simple random sampling with replacement (SRSWR). Keeping the drawbacks with the previous alternative models for the Moors model Kim and Warde (2005) proposed a mixed randomized response model using simple random sampling which modifies the privacy problem. We have also extended the proposed model to stratified sampling.

2. THE PROPOSED MODEL

Here we have proposed two mixed randomized response techniques named as MRRT1 and MRRT2 which are described below:

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MRRT1: A single sample with size n is selected by simple random sampling with replacement from the population. Each respondent from the sample is instructed to answer the direct question, "I am the member of innocuous trait group". If a respondent answers "Yes" to direct question, then she or he is instructed to go to randomization device R_1 consisting of the statements (i) "I am a member of the sensitive trait group" and (ii) "I am a member of the innocuous trait group" with probabilities of selection P_1 and $(1-P_1)$, respectively [see Kim and Warde (2005)]. If a respondent answers "No" to the direct question, then the respondent is instructed to use the randomization device R_2 consisting of the statement (i) "I belong to the sensitive group" with probability P, exactly the same probability as used by Warner (1965) and the statement "Yes" with probability $(1-P)$. The interviewee is instructed to use the device and report "Yes" or "No" for the random outcome of the sensitive statement according to his actual status. Otherwise, he is simply to report the "Yes" statement observed on the randomization device. To protect the respondent's privacy, the respondents should not disclose to the interviewer the question they answered from either R_1 or R_2. The randomization device R_2 is due to Singh (1993) and it is designated as RRT1. Finally, we designate the proposed mixed randomized response technique as MRRT1.

Let n be the sample size confronted with a direct question and n_1 and $n_2(=n-n_1)$ denote the number of "Yes" and "No" answers from the sample. Since all the respondents using a randomization device R_1 already responded "Yes" from the initial direct innocuous question, the proportion 'Y' of getting "Yes" answers from the respondents using randomization device R_1 should be

$$Y = P_1\pi_S + (1-P_1)\pi_1 = P_1\pi_S + (1-P_1) \tag{1}$$

where π_S is the proportion of "Yes" answers from the sensitive trait and π_1 is the proportion of "Yes" answers from the innocuous question.

An unbiased estimator of π_S, in terms of the sample proportion of "Yes" responses $\hat{Y}$, becomes

$$\hat{\pi}_{c1} = \frac{\hat{Y}-(1-P_1)}{P_1} \tag{2}$$

whose variance is given by

$$V(\hat{\pi}_{c1}) = \frac{Y(1-Y)}{n_1P_1^2} = \left[\frac{(1-\pi_S)\pi_S}{n_1} + \frac{(1-\pi_S)\,(1-P_1)]}{n_1P_1}\right] \tag{3}$$

See Kim and Warde (2005).

The proportion of "Yes" answers from the respondents using randomized response technique RRT1 due to Singh (1993) follows:

$$X = P\pi_S + (1-P) \tag{4}$$

where X is the proportion of "Yes" responses.

An unbiased estimator of π_S, in terms of the sample proportion of "Yes" responses $\hat{X}$, becomes

$$\hat{\pi}_{c2} = \frac{\hat{X} - (1-P)}{P} \tag{5}$$

Its variance is

$$V(\hat{\pi}_{c2}) = \frac{1}{n_2}\left[\pi_S(1-\pi_S) + \frac{(1-P)(1-\pi_S)}{P}\right] \tag{6}$$

Now we shall pool the two estimators using weight, to formulate an estimator for π_S as

$$\hat{\pi}_c = \frac{n_1}{n}\hat{\pi}_{c1} + \frac{(n-n_1)}{n}\hat{\pi}_{c2}\ ,\ for\ 0 < \frac{n_1}{n} < 1 \tag{7}$$

As both $\hat{\pi}_{c1}$ and $\hat{\pi}_{c2}$ are unbiased estimators of π_S, therefore the expected value of $\hat{\pi}_c$ is

$$E(\hat{\pi}_c) = \frac{n_1}{n}\pi_S + \frac{(n-n_1)}{n}\pi_S = \pi_S$$

Since the estimators $\hat{\pi}_{c1}$ and $\hat{\pi}_{c2}$ are unbiased independent, therefore the variance of $\hat{\pi}_c$ is given by

$$V(\hat{\pi}_c) = \frac{n_1}{n^2}\left[\pi_S(1-\pi_S) + \frac{(1-\pi_S)(1-P_1)}{P_1}\right] + \frac{n_2}{n^2}\left[\pi_S(1-\pi_S) + \frac{(1-\pi_S)(1-P)}{P}\right] \tag{8}$$

Horvitz et al. (1967) presented Simmon's method, which has two cases (known and unknown π_1). Under the situation that the Warner (1965) model and Simmon's method (known π_1) are equally confidential to respondents, Lanke (1976) derived a unique value of P as

$$P = \frac{1}{2} + \frac{P_1}{2P_1 + 4(1-P_1)\pi_1} \quad \text{for every } P_1 \text{ and every } \pi_1$$

Since the proposed mixed RR model also uses Simmon's method when $\pi_1 = 1$, we can apply Lanke's idea to our suggested model. Thus, we can establish the following equality:

$$P = \frac{1}{2} + \frac{P_1}{2P_1 + 4(1-P_1)} = \frac{1}{2-P_1}$$

Putting $P=(2-P_1)^{-1}$ in (8) we get the variance of $\hat{\pi}_c$ and is given in the following theorem.

Theorem 2.1 The variance of $\hat{\pi}_c$ is given by

$$V(\hat{\pi}_c)=\frac{\pi_S(1-\pi_S)}{n}+\frac{(1-\pi_S)(1-P_1)\{\lambda+(1-\lambda)P_1\}}{nP_1}, \textit{for } n=n_1+n_2 \textit{ and } \lambda=\frac{n_1}{n}. \tag{9}$$

MRRT2: This method is exactly like MRRT1 except for a change in probabilities on the randomization device R_2 in MRRT1 i.e., the probabilities for the 'sensitive' statement and "Yes" statement have been interchanged. Then the proportion of "Yes" answers from the respondents using randomized response technique MRRT2 due to Singh (1993) is

$$Z=(1-P)\pi_S+P \tag{10}$$

where Z is the proportion of "Yes" responses.

An unbiased estimator of π_S, in terms of the sample proportion of "Yes" responses $\hat{Z}$, becomes

$$\hat{\pi}_{c3}=\frac{(\hat{Z}-P)}{1-P} \tag{11}$$

Its variance is

$$V(\hat{\pi}_{c3})=\frac{1}{n_2}\left[\pi_S(1-\pi_S)+\frac{P(1-\pi_S)}{(1-P)}\right] \tag{12}$$

Now pooling the two estimators $\hat{\pi}_{c1}$ and $\hat{\pi}_{c3}$ we get an estimator for π_S under MRRT2 as

$$\hat{\pi}_c^*=\frac{n_1}{n}\hat{\pi}_{c1}+\frac{(n-n_1)}{n}\hat{\pi}_{c3}\text{ , } \textit{for } 0<\frac{n_1}{n}<1 \tag{13}$$

As both $\hat{\pi}_{c1}$ and $\hat{\pi}_{c3}$ are unbiased estimators of π_S, therefore the expected value of $\hat{\pi}_c^*$ is

$$E(\hat{\pi}_c^*)=\frac{n_1}{n}\pi_S+\frac{(n-n_1)}{n}\pi_S=\pi_S$$

Thus the pooled estimator $\hat{\pi}_c^*$ is an unbiased estimator of π_S.

Since the two estimators $\hat{\pi}_{c1}$ and $\hat{\pi}_{c3}$ are independent unbiased estimator of π_S, therefore the variance of both $\hat{\pi}_c^*$ is given by

$$V(\hat{\pi}_c^*)=\frac{n_1^2}{n^2}\left[\frac{\pi_S(1-\pi_S)}{n_1}+\frac{(1-\pi_S)(1-P_1)}{n_1P_1}\right]+\frac{n_2^2}{n^2}\left[\frac{\pi_S(1-\pi_S)}{n_2}+\frac{(1-\pi_S)P}{(1-P)}\right] \tag{14}$$

Inserting $P = (2 - P_1)^{-1}$ [see Lanke (1976) and Kim and Warde (2005)] in (14) we get the variance $\hat{\pi}_c^*$ and is given in the following theorem.

Theorem 2.2 The variance of $\hat{\pi}_c^*$ is given by

$$V(\hat{\pi}_c^*) = \frac{\pi_S(1-\pi_S)}{n} + \frac{(1-\pi_S)}{n}\left\{\frac{\lambda(1-P_1)}{P_1} + \frac{(1-\lambda)}{(1-P_1)}\right\} \tag{15}$$

3. EFFICIENCY COMPARISONS

An efficiency comparison of the proposed models, under completely truthful reporting case, has been done with Kim and Warde's (2005) model.

From Kim and Warde's (2005, Eq. (2.10), p. 213), the variance of the Kim and Warde (2005) estimator $\hat{\pi}_{kw}$ based on mixed randomized response model is given by

$$V(\hat{\pi}_{kw}) = \frac{\pi_S(1-\pi_S)}{n} + \frac{\lambda(1-P_1)(1-\pi_S)}{nP_1} + \frac{(1-P_1)(1-\lambda)}{nP_1^2} \tag{16}$$

From (9), (15) and (16) we have

$$[V(\hat{\pi}_{kw}) - V(\hat{\pi}_c)] = \frac{(1-\lambda)(1-P_1)}{n}\left[\frac{1}{P_1} - 1 + \pi_S\right] > 0 \ \textit{if} \ P_1 < 1 \tag{17}$$

$$[V(\hat{\pi}_{kw}) - V(\hat{\pi}_c^*)] = \frac{(1-\lambda)}{n(1-P_1)}\left[\frac{(1-P_1)^2}{P_1^2} - 1 + \pi_S\right] > 0 \ \textit{if} \ P_1 < 1/2 \tag{18}$$

Thus it follows from (17) and (18) that the condition $P_1 < 1/2$ is sufficient condition for the proposed models MRRT1 and MRRT2 to be better than the Kim and Warde (2005) mixed randomized response model.

Further from (9) and (15) we have

$$[V(\hat{\pi}_c^*) - V(\hat{\pi}_c)] = \frac{(1-\lambda)(1-\pi_S)}{n(1-P_1)}\left[1-(1-P_1)^2\right]$$

which is always positive.

Thus the proposed mixed randomized response model MRRT1 is better than the proposed model MRRT2.

To have tangible idea about the performance of the proposed model we have computed the percent relative efficiency (PRE) of $(\hat{\pi}_c, \hat{\pi}_c^*)$ with respect to Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$ and also the PRE of $\hat{\pi}_c$ with respect to $\hat{\pi}_c^*$ by using the formulae:

$(i)\ PRE\ (\hat{\pi}_c, \hat{\pi}_{kw}) = \dfrac{V(\hat{\pi}_{kw})}{V(\hat{\pi}_c)} \times 100$; $(ii)\ PRE\ (\hat{\pi}_c^*, \hat{\pi}_{kw}) = \dfrac{V(\hat{\pi}_{kw})}{V(\hat{\pi}_c^*)} \times 100$

$(iii)\ PRE\ (\hat{\pi}_c, \hat{\pi}_c^*) = \dfrac{V(\hat{\pi}_c^*)}{V(\hat{\pi}_c)} \times 100$, for different values of P_1, n and n_1.

We have obtained the values of the percent relative efficiencies $PRE\ (\hat{\pi}_c, \hat{\pi}_{kw})$, $PRE\ (\hat{\pi}_c^*, \hat{\pi}_{kw})$ and $PRE\ (\hat{\pi}_c^*, \hat{\pi}_c)$ for λ = (0.2, 0.4, 0.6, 0.8) and for different cases of π_S , n, n_1 and P_1.Findings are shown in Table 1, 2, 3 and diagrammatic representations are also given in Fig. 1, 2, and 3 respectively.

It is observed from Table 1 and Fig. 1 that:

The values of percent relative efficiencies $PRE(\hat{\pi}_c , \hat{\pi}_{kw})$ is more than 100. We can say that the envisaged estimator $\hat{\pi}_c$ is more efficient than the Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$. Fig. 1 show results for π_S = (0.1 and 0.4), T = (0.1, 0.3,), λ = (0.2, 0.4, 0.6, 0.8) and different values of P_1 , n, n_1 .

We note from Table 1 that the values of the percent relative efficiencies $PRE(\hat{\pi}_c , \hat{\pi}_{kw})$ decrease as the value of P_1 increases. Also the values of the percent relative efficiencies $PRE(\hat{\pi}_c , \hat{\pi}_{kw})$ increase as the value of λ decreases for fixed values of P_1. We further note from the results of Fig. 1 that there is large gain in efficiency by using the suggested estimator $\hat{\pi}_c$ over the Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$ when the proportion of stigmatizing attribute is moderately large.

Table 2 – shows that the values of the percent relative efficiency are greater than 100 for all parametric values tabled. This shows the superiority of the suggested estimator $\hat{\pi}_c^*$ over Kim and Warde (2005) estimator $\hat{\pi}_{kw}$. It is observed from Table 2 that the values of the percent relative efficiencies $PRE(\hat{\pi}_c^* , \hat{\pi}_{kw})$ decrease as the value of P_1 increases. Also the values of the percent relative efficiencies $PRE(\hat{\pi}_c^* , \hat{\pi}_{kw})$ increase as the value of λ decreases for fixed values of P_1. We further note from the results of Fig. 2 that there is large gain in efficiency by using the suggested estimator $\hat{\pi}_c^*$ over the Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$ when the proportion of stigmatizing attribute is moderately large.

Table 3 - exhibits that the percent relative efficiency of the proposed estimator $\hat{\pi}_c$ with respect to the suggested estimator $\hat{\pi}_c^*$ decreases as sample size and value of P_1 increase. Larger gain in efficiency is observed for small as well as moderately large sample sizes. However, the percent relative efficiency is more than 100 % for all parametric values considered here; therefore the proposed estimator $\hat{\pi}_c$ is better than the suggested estimator $\hat{\pi}_c^*$. Fig. 3 demonstrates that there is large gain in efficiency by using the suggested estimator $\hat{\pi}_c$ over the proposed estimator $\hat{\pi}_c^*$ when the proportion of stigmatizing attribute is moderately large.

Thus the proposed model MRRT1 is to be preferred over the suggested model MRRT2 and the Kim and Wardes (2005) mixed randomized response model.

TABLE 1
Percent relative efficiency of the proposed estimator $\hat{\pi}_c$ with respect to Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$.

π_{S1}	π_{S2}	π	w_1	w_2	λ	P_1						
						$P_1 \rightarrow$ 0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.08	0.13	0.1	0.6	0.4	0.2	3125.95	1819.52	1212.84	874.35	663.92	523.28	424.24
0.08	0.13	0.1	0.6	0.4	0.4	1502.35	957.77	689.60	531.42	427.87	355.29	301.88
0.08	0.13	0.1	0.6	0.4	0.6	776.45	528.38	403.86	328.78	278.47	242.32	215.05
0.08	0.13	0.1	0.6	0.4	0.8	364.97	271.23	223.82	194.96	175.39	161.14	150.23
0.18	0.23	0.2	0.6	0.4	0.2	3382.35	1942.60	1280.48	914.28	688.40	538.51	433.67
0.18	0.23	0.2	0.6	0.4	0.4	1642.85	1034.71	736.84	562.16	448.49	369.31	311.42
0.18	0.23	0.2	0.6	0.4	0.6	848.99	570.78	431.50	347.82	291.96	252.00	222.01
0.18	0.23	0.2	0.6	0.4	0.8	394.45	289.14	235.95	203.63	181.77	165.89	153.78
0.28	0.33	0.3	0.6	0.4	0.2	3721.88	2110.78	1376.84	974.28	727.80	565.33	452.38
0.28	0.33	0.3	0.6	0.4	0.4	1825.29	1135.89	800.12	604.39	477.82	390.19	326.53
0.28	0.33	0.3	0.6	0.4	0.6	942.71	625.92	467.83	373.21	310.32	265.56	232.14
0.28	0.33	0.3	0.6	0.4	0.8	432.47	312.33	251.75	215.03	190.26	172.35	158.73
0.38	0.43	0.4	0.6	0.4	0.2	4184.92	2345.04	1514.49	1062.50	787.64	607.56	483.05
0.38	0.43	0.4	0.6	0.4	0.4	2070.48	1273.17	887.09	663.41	519.64	420.71	349.26
0.38	0.43	0.4	0.6	0.4	0.6	1068.18	700.12	517.09	408.00	335.82	284.71	246.75
0.38	0.43	0.4	0.6	0.4	0.8	483.29	343.42	273.04	230.50	201.90	181.29	165.69

TABLE 2

Percent relative efficiency of the proposed estimator $\hat{\pi}_c^*$ with respect to Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$.

π_{S1}	π_{S2}	π_S	w_1	w_2	λ	P_1						
						$P_1 \rightarrow$ 0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.08	0.13	0.1	0.6	0.4	0.2	2936.65	1600.95	983.04	643.39	437.56	304.78	215.38
0.08	0.13	0.1	0.6	0.4	0.4	1458.77	896.70	613.60	442.85	328.93	248.04	188.23
0.08	0.13	0.1	0.6	0.4	0.6	765.42	511.01	379.62	297.26	239.29	195.25	160.00
0.08	0.13	0.1	0.6	0.4	0.8	362.92	267.59	218.15	186.74	164.05	146.02	130.61
0.18	0.23	0.2	0.6	0.4	0.2	3184.61	1719.54	1050.00	685.71	466.66	326.13	231.81
0.18	0.23	0.2	0.6	0.4	0.4	1596.26	970.82	658.82	472.72	350.00	263.80	200.69
0.18	0.23	0.2	0.6	0.4	0.6	837.13	552.47	406.45	315.78	252.77	205.59	168.39
0.18	0.23	0.2	0.6	0.4	0.8	392.27	285.34	230.13	195.34	170.51	151.11	134.85
0.28	0.33	0.3	0.6	0.4	0.2	3511.57	1878.61	1140.86	743.09	505.65	354.09	252.70
0.28	0.33	0.3	0.6	0.4	0.4	1774.66	1067.92	718.59	512.42	377.90	284.39	216.54
0.28	0.33	0.3	0.6	0.4	0.6	929.75	606.32	441.51	340.14	270.51	219.06	179.08
0.28	0.33	0.3	0.6	0.4	0.8	430.11	308.30	245.71	206.58	178.96	157.72	140.25
0.38	0.43	0.4	0.6	0.4	0.2	3956.11	2097.55	1266.66	822.58	559.18	391.88	280.32
0.38	0.43	0.4	0.6	0.4	0.4	2014.28	1199.25	800.00	566.66	415.94	312.15	237.50
0.38	0.43	0.4	0.6	0.4	0.6	1053.73	678.71	488.88	373.17	294.55	237.20	193.22
0.38	0.43	0.4	0.6	0.4	0.8	480.68	339.08	266.66	221.73	190.38	166.60	147.41

TABLE 3

Percent relative efficiency of the proposed estimator $\hat{\pi}_c$ *with respect to the suggested* estimator $\hat{\pi}_c^*$

π_{S1}	π_{S2}	π_S	w_1	w_2	λ	P_1						
						$P_1 \rightarrow$ 0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.08	0.13	0.1	0.6	0.4	0.2	106.44	113.65	123.37	135.89	151.73	171.69	196.96
0.08	0.13	0.1	0.6	0.4	0.4	102.98	106.80	112.38	120.00	130.07	143.23	160.37
0.08	0.13	0.1	0.6	0.4	0.6	101.44	103.40	106.38	110.60	116.37	124.10	134.40
0.08	0.13	0.1	0.6	0.4	0.8	100.56	101.35	102.60	104.40	106.91	110.35	115.02
0.18	0.23	0.2	0.6	0.4	0.2	106.20	112.97	121.95	133.33	147.51	165.12	187.07
0.18	0.23	0.2	0.6	0.4	0.4	102.91	106.58	111.84	118.91	128.14	139.99	155.17
0.18	0.23	0.2	0.6	0.4	0.6	101.41	103.31	106.16	110.14	115.50	122.57	131.84
0.18	0.23	0.2	0.6	0.4	0.8	100.55	101.33	102.52	104.24	106.60	109.78	114.03
0.28	0.33	0.3	0.6	0.4	0.2	105.98	112.35	120.68	131.11	143.93	159.65	179.01
0.28	0.33	0.3	0.6	0.4	0.4	102.85	106.36	111.34	117.94	126.44	137.20	150.79
0.28	0.33	0.3	0.6	0.4	0.6	101.39	103.23	105.96	109.72	114.71	121.22	129.62
0.28	0.33	0.3	0.6	0.4	0.8	100.54	101.30	102.45	104.09	106.31	109.27	113.16
0.38	0.43	0.4	0.6	0.4	0.2	105.78	111.80	119.56	129.16	140.85	155.03	172.31
0.38	0.43	0.4	0.6	0.4	0.4	102.79	106.16	110.88	117.07	124.93	134.77	147.05
0.38	0.43	0.4	0.6	0.4	0.6	101.37	103.15	105.76	109.33	114.01	120.02	127.70
0.38	0.43	0.4	0.6	0.4	0.8	100.54	101.27	102.39	103.95	106.05	108.81	112.40

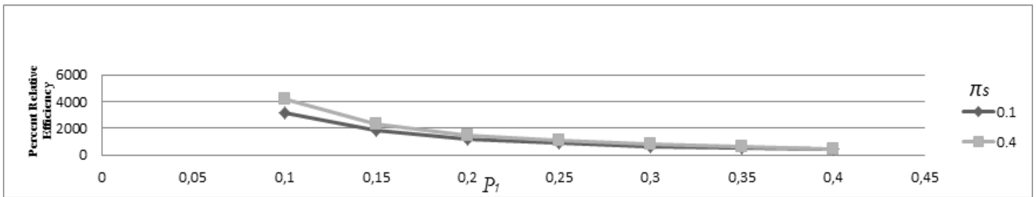

Figure 1 - Percent relative efficiency of the proposed estimator $\hat{\pi}_c$ with respect to Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$.

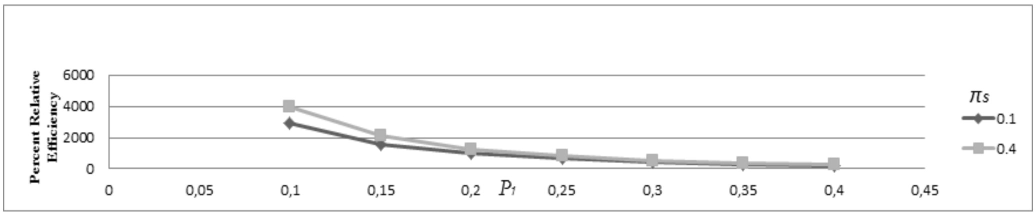

Figure 2 - Percent relative efficiency of the proposed estimator $\hat{\pi}_c^*$ with respect to Kim and Warde's (2005) estimator $\hat{\pi}_{kw}$.

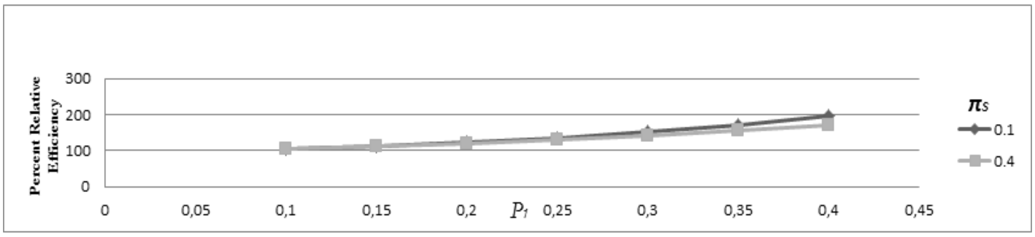

Figure. 3 - Percent relative efficiency of the proposed estimator $\hat{\pi}_c$ with respect to the suggested estimator $\hat{\pi}_c^*$.

4. A MIXED RANDOMIZED RESPONSE MODEL USING STRATIFICATION

4.1. A mixed stratified RR model

Stratified random sampling is generally obtained by divided the population into non-overlapping groups called strata and selecting a simple random sample from each stratum. An RR technique using a stratified random sampling gives the group characteristics related to each stratum estimator. Also, stratified sampling protects a researcher from the possibility of obtaining a poor sample. Hong *et al.* (1994) suggested a stratified RR technique using a proportional allocation. Kim and Warde (2004) presented a stratified RR model based on Warner (1965) model that has an optimal allocation and large gain in precision. Kim and Elam (2005) suggested a two – stage stratified Warner's randomized response model using optimal allocation. Further Kim and Warde (2005) suggested a mixed stratified randomized response model.

In the proposed model, the assumptions for a stratified mixed RR model are similar to Kim and Warde (2004) and Kim and Elam (2005) model. We have proposed two stratified randomized response techniques named as SMRRT1 and SMRRT2 which are described below:

SMRRT1: An individual respondent in a sample from each stratum is instructed to answer a direct question "I am a member of the innocuous trait group". Respondents answer the direct question by "Yes" or "No". If a respondent answers "Yes", then she or he is instructed to go to the randomization device R_{k1} consisting of statements: (i) "I am the member of the sensitive trait group" and (ii) "I am a member of the innocuous trait group" with pre-assigned probabilities Q_k and $(1-Q_k)$, respectively. If a respondent answers

"No", then the respondent is instructed to use the randomization device R_{k2} consisting of the statement (i) "I belong to the sensitive group" with known probability P_k, exactly the same probability as used by Warner (1965) and the statement "Yes" with probability (1- P_k). The interviewee is instructed to use the device and report "Yes" or "No" for the random outcome of the sensitive statement according to his actual status. Otherwise, he is simply to report the "Yes" statement observed on the randomization device. To protect the respondent's privacy, the respondents should not disclose to the interviewer the question they answered from either R_{k1} or R_{k2}. The randomization device R_{k2} is due to Singh (1993) and it is designated as RRT1. Finally, we designate the proposed mixed randomized response technique as SMRRT1.Suppose we denote m_k as the number of units in the sample from stratum k and n as the total number of units in samples from all strata. Let m_{k1} be the number of people responding "Yes" when respondents in a sample m_k were asked the direct question and m_{k2} be the number of people responding "No' when respondents in a sample m_k were asked the direct question so that $n=\sum_{k=1}^{r} m_k=\sum_{k=1}^{r}(m_{k1}+m_{k2})$. Under the assumption that these "Yes" or "No" reports are made truthfully, and Q_k and $(P_k \neq 0.5)$ are set by the researcher, then the proportion of "Yes" answer from the respondents using the randomization device R_{k1} will be

$$Y_k = Q_k \pi_{S_k} + (1-Q_k)\pi_{1_k} \quad \text{for } k = 1,2,\ldots r, \tag{19}$$

where Y_k is the proportion of "Yes" answers in stratum k, π_{S_k} is the proportion of respondents with the sensitive trait in stratum k, π_{1_k} is the proportion of respondents with the innocuous trait in stratum k, and Q_k is the probability that a respondent in the sample stratum k is asked a sensitive question.

Since the respondent performing a randomization device R_{k1} respond "Yes" to the direct question of the innocuous trait, if he or she chooses the same innocuous question from R_{k1}, then π_{1_k} is equal to one $(i.e.\ \pi_{1_k}=1)$. Therefore, (4.1) becomes $Y_k = Q_k \pi_{S_k} + (1-Q_k)$.

The estimator of π_{S_k} is

$$\hat{\pi}_{c1_k} = \frac{\hat{Y}_k - (1-Q_k)}{Q_k} \quad \text{for } k = 1,2,\ldots,r, \tag{20}$$

where $\hat{Y}_k$ is the proportion of "Yes" answers in a sample in stratum k and $\hat{\pi}_{c1_k}$ is the proportion of respondents with the sensitive trait in a sample from stratum k. Since each

$\hat{Y}_k$ is a binomial distribution $B(m_{k1}, Y_k)$, the estimator $\hat{\pi}_{c1_k}$ is an unbiased estimator for π_{S_k} with the variance

$$V(\hat{\pi}_{c1_k}) = \frac{(1-\pi_{S_k})\,[\,Q_k\pi_{S_k} + (1-Q_k)]}{m_{k1}Q_k} \tag{21}$$

The proportion of "Yes" answers from the respondents using Singh (1993) RR technique RRT1 will be

$$X_k = P_k\pi_{S_k} + (1-P_k) \tag{22}$$

where X_k is the proportion of "Yes" responses in stratum k, π_{S_k} is the proportion of respondents with the sensitive trait in stratum k, and P_k is the probability that a respondent in the sample stratum k has a sensitive question card. The unbiased estimator in this case is

$$\hat{\pi}_{c2_k} = \frac{\hat{X}_k - (1-P_k)}{P_k} \tag{23}$$

where $\hat{X}_k$ is the proportion of "Yes" responses in a sample from a stratum k and $\hat{\pi}_{c2_k}$ is the proportion of respondents with the sensitive trait in a sample from stratum k. By using $m_k = m_{k1} + m_{k2}$ and $P_k = (2-Q_k)^{-1}$, the variance of $\hat{\pi}_{c2_k}$ is given by

$$V(\hat{\pi}_{c2_k}) = \frac{1}{(m_k - m_{k1})}\left[\pi_{S_k}(1-\pi_{S_k}) + (1-Q_k)(1-\pi_{S_k})\right] \tag{24}$$

The unbiased estimator of π_{S_k}, in terms of sample proportion of "Yes" responses $\hat{Y}_k$ and $\hat{X}_k$, is

$$\hat{\pi}_{mS_k} = \frac{m_{k1}}{m_k}\hat{\pi}_{c1_k} + \frac{m_k - m_{k1}}{m_k}\hat{\pi}_{c2_k} \quad , \mathit{for}\ 0 < \frac{m_{k1}}{m_k} < 1 \tag{25}$$

Its variance is

$$V(\hat{\pi}_{mS_k}) = \frac{\pi_{S_k}(1-\pi_{S_k})}{m_k} + \frac{1}{m_k}\left[\frac{(1-\pi_{S_k})(1-Q_k)\{\lambda_k + (1-\lambda_k)Q_k\}}{Q_k}\right] \tag{26}$$

where $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

Thus the unbiased estimator of π_{S_k} is given by

$$\hat{\pi}_{mS} = \sum_{k=1}^{r} w_k \hat{\pi}_{mS_k} = \sum_{k=1}^{r} w_k \left[\frac{m_{k1}}{m_k} \hat{\pi}_{c1_k} + \frac{m_k - m_{k1}}{m_k} \hat{\pi}_{c2_k} \right] \tag{27}$$

where N is the number of units in the whole population, N_k is the total number of units in stratum k, and $w_k = N_k / N$ for $k = 1, 2, ..., r$, so that $w = \sum_{k=1}^{r} w_k = 1$. It can be shown that the proposed estimator $\hat{\pi}_{mS}$ is unbiased for the population proportion π_S. The variance of an estimator $\hat{\pi}_{mS}$ is given by

$$V(\hat{\pi}_{mS}) = \sum_{k=1}^{r} \frac{w_k^2}{m_k} \times \left[\pi_{S_k}(1-\pi_{S_k}) + \frac{(1-\pi_{S_k})(1-Q_k)\{\lambda_k + (1-\lambda_k)Q_k\}}{Q_k} \right] \tag{28}$$

In order to do the optimal allocation of a sample size n, we need to know $\lambda_k = m_{k1} / m_k$ and π_{S_k}. Information on $\lambda_k = m_{k1} / m_k$ and π_{S_k} is usually unavailable. But if prior information about them is available from past experience it will help to derive the following optimal allocation formula.

Theorem 4.1 The optimal allocation of m *to* $m_1, m_2, \dots m_{r-1}$ *and* m_r to derive the minimum variance of the $\hat{\pi}_{mS}$ subject to $n = \sum_{k=1}^{r} m_k$ is approximately given by

$$\frac{m_k}{n} = \frac{w_k \left[\pi_{S_k}(1-\pi_{S_k}) + \frac{(1-\pi_{S_k})(1-Q_k)\{\lambda_k + (1-\lambda_k)Q_k\}}{Q_k} \right]^{1/2}}{\sum_{k=1}^{r} w_k \left[\pi_{S_k}(1-\pi_{S_k}) + \frac{(1-\pi_{S_k})(1-Q_k)\{\lambda_k + (1-\lambda_k)Q_k\}}{Q_k} \right]^{1/2}} \tag{29}$$

where $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

The minimal variance of the estimator $\hat{\pi}_{mS}$ is given by

$$V(\hat{\pi}_{mS}) = \frac{1}{n} \left[\sum_{k=1}^{r} w_k \left\{ \pi_{S_k}(1-\pi_{S_k}) + \frac{(1-\pi_{S_k})(1-Q_k)\{\lambda_k + (1-\lambda_k)Q_k\}}{Q_k} \right\}^{1/2} \right]^2 \tag{30}$$

where $n = \sum_{k=1}^{r} m_k$, $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

SMRRT2: An individual respondent in a sample from each stratum is instructed to answer a direct question "I am a member of the innocuous trait group". Respondents answer the direct question by "Yes" or "No". If a respondent answers "Yes", then she or he is instructed to go to the randomization device R_{k1} consisting of statements: (i) "I am the member of the sensitive trait group" and (ii) "I am a member of the innocuous trait group" with pre-assigned probabilities Q_k and $(1- Q_k)$, respectively. If a respondent answers "No", then the respondent is instructed to use the randomization device R_{k2} consisting of the statement (i) "I belong to the sensitive group" with known probability P_k, exactly the same probability as used by Warner (1965) and the statement "Yes" with probability $(1- P_k)$. The interviewee is instructed to use the device and report "Yes" or "No" for the random outcome of the sensitive statement according to his actual status. Otherwise, he is simply to report the "Yes" statement observed on the randomization device. To protect the respondent's privacy, the respondents should not disclose to the interviewer the question they answered from either R_{k1} or R_{k2}. The randomization device R_{k2} is due to Singh (1993) and it is designated as RRT2. Finally, we designate the proposed mixed randomized response technique as SMRRT2.Suppose we denote m_k as the number of units in the sample from stratum k and n as the total number of units in samples from all strata.

Let m_{k1} be the number of people responding "Yes" when respondents in a sample m_k were asked the direct question and m_{k2} be the number of people responding "No' when respondents in a sample m_k were asked the direct question so that $n = \sum_{k=1}^{r} m_k = \sum_{k=1}^{r} (m_{k1} + m_{k2})$. Under the assumption that these "Yes" or "No" reports are made truthfully, and Q_k and $(P_k \neq 0.5)$ are set by the researcher, then the proportion of "Yes" answer from the respondents using the randomization device R_{k1} will be

$$Y_k = Q_k \pi_{S_k} + (1 - Q_k)\pi_{1_k} \quad for\ k = 1,2,...r, \tag{31}$$

where Y_k is the proportion of "Yes" answers in stratum k, π_{S_k} is the proportion of respondents with the sensitive traits in stratum k, π_{1_k} is the proportion of respondents with the innocuous trait in stratum k, and Q_k is the probability that a respondent in the sample stratum k is asked a sensitive question.

Since the respondent performing a randomization device R_{k1} respond "Yes" to the direct question of the innocuous trait, if he or she chooses the same innocuous question from R_{k1}, then π_{1_k} is equal to one. Therefore, (31) becomes $Y_k = Q_k \pi_{S_k} + (1 - Q_k)$. The estimator of π_{S_k} is

$$\hat{\pi}_{c1_k} = \frac{\hat{Y}_k - (1 - Q_k)}{Q_k} \quad for\ k = 1,2,...,r, \tag{32}$$

where $\hat{Y}_k$ is the proportion of "Yes" answers in a sample in stratum k and $\hat{\pi}_{c1_k}$ is the proportion of respondents with the sensitive trait in a sample from stratum k. Since each $\hat{Y}_k$ is a binomial distribution $B(m_{k1}, Y_k)$, the estimator $\hat{\pi}_{c1_k}$ is an unbiased for π_{S_k} with

$$V(\hat{\pi}_{c1_k}) = \frac{(1-\pi_{S_k})[\,Q_k\pi_{S_k} + (1-Q_k)]}{m_{k1}Q_k} \tag{33}$$

The proportion of "Yes" answers from the respondents using Singh (1993) RR technique RRT2 will be

$$Z_k = (1-P_k)\pi_{S_k} + P_k \tag{34}$$

where Z_k is the proportion of "Yes" responses in stratum k, π_{S_k} is the proportion of respondents with the sensitive trait in stratum k, and P_k is the probability that a respondent in the sample stratum k has a sensitive question card. The unbiased estimator in this case is

$$\hat{\pi}_{c3_k} = \frac{\hat{Z}_k - (1-P_k)}{(1-P_k)}, \tag{35}$$

where $\hat{Z}_k$ is the proportion of "Yes" responses in a sample from a stratum k and $\hat{\pi}_{c3_k}$ is the proportion of respondents with the sensitive trait in a sample from stratum k. By using $m_k = m_{k1} + m_{k2}$ and $P_k = (2-Q_k)^{-1}$, the variance of $\hat{\pi}_{c3_k}$ is given by

$$V(\hat{\pi}_{c3_k}) = \frac{1}{(m_k - m_{k1})}\left[\pi_{S_k}(1-\pi_{S_k}) + \frac{(1-\pi_{S_k})}{(1-Q_k)}\right] \tag{36}$$

The unbiased estimator of π_{S_k}, in terms of sample proportion of "Yes" responses $\hat{Y}_k$ and $\hat{Z}_k$, is

$$\hat{\pi}^*_{mS_k} = \frac{m_{k1}}{m_k}\hat{\pi}_{c1_k} + \frac{m_k - m_{k1}}{m_k}\hat{\pi}_{c3_k} \quad \textit{for } 0 < \frac{m_{k1}}{m_k} < 1 \tag{37}$$

Its variance is

$$V(\hat{\pi}^*_{mS_k}) = \frac{\pi_{S_k}(1-\pi_{S_k})}{m_k} + \frac{(1-\pi_{S_k})}{m_k}\left[\frac{\lambda_k(1-Q_k)}{Q_k} + \frac{(1-\lambda_k)}{(1-Q_k)}\right] \tag{38}$$

where $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

The unbiased estimator of π_{S_k} is shown to be

$$\hat{\pi}^*_{mS} = \sum_{k=1}^{r} w_k \hat{\pi}^*_{mS_k} = \sum_{k=1}^{r} w_k \left[\frac{m_{k1}}{m_k} \hat{\pi}_{c1_k} + \frac{m_k - m_{k1}}{m_k} \hat{\pi}_{c3_k} \right] \tag{39}$$

where N is the number of units in the whole population, N_k is the total number of units in stratum k, and $w_k = N_k / N$ for $k = 1,2,...,r$, so that $w = \sum_{k=1}^{r} w_k = 1$. It can be shown that the proposed estimator $\hat{\pi}^*_{mS}$ is unbiased for the population proportion π_S. The variance of an estimator $\hat{\pi}^*_{mS}$ is given by

$$V(\hat{\pi}^*_{mS}) = \sum_{k=1}^{r} \frac{w_k^2}{m_k} \times \left[\pi_{S_k}(1-\pi_{S_k}) + (1-\pi_{S_k}) \left[\frac{\lambda_k (1-Q_k)}{Q_k} + \frac{(1-\lambda_k)}{(1-Q_k)} \right] \right] \tag{40}$$

In order to do the optimal allocation of a sample size n, we need to know $\lambda_k = m_{k1} / m_k$ and π_{S_k} Information on $\lambda_k = m_{k1} / m_k$ and π_{S_k} is usually unavailable. But if prior information about them is available from past experience it will help to derive the following optimal allocation formula.

Theorem 4.2 The optimal allocation of m *to* $m_1, m_2, \ldots m_{r-1}$ *and* m_r to derive the minimum variance of the $\hat{\pi}_{mS1}$ subject to $n = \sum_{k=1}^{r} m_k$ is approximately given by

$$\frac{m_k}{n} = \frac{w_k \left[\pi_{S_k}(1-\pi_{S_k}) + (1-\pi_{S_k}) \left[\frac{\lambda_k (1-Q_k)}{Q_k} + \frac{(1-\lambda_k)}{(1-Q_k)} \right] \right]^{1/2}}{\sum_{k=1}^{r} w_k \left[\pi_{S_k}(1-\pi_{S_k}) + (1-\pi_{S_k}) \left[\frac{\lambda_k (1-Q_k)}{Q_k} + \frac{(1-\lambda_k)}{(1-Q_k)} \right] \right]^{1/2}} \tag{41}$$

where $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

The minimal variance of the estimator $\hat{\pi}^*_{mS}$ is given by

$$V(\hat{\pi}^*_{mS}) = \frac{1}{n} \left[\sum_{k=1}^{r} w_k \left\{ \pi_{S_k}(1-\pi_{S_k}) + (1-\pi_{S_k}) \left[\frac{\lambda_k (1-Q_k)}{Q_k} + \frac{(1-\lambda_k)}{(1-Q_k)} \right] \right\}^{1/2} \right]^2 \tag{42}$$

where $n = \sum_{k=1}^{r} m_k$, $m_k = m_{k1} + m_{k2}$ and $\lambda_k = m_{k1} / m_k$.

To have tangible idea about the performance of the proposed stratified estimators $(\hat{\pi}_{ms}, \hat{\pi}^*_{mS})$ over Kim and Warde (2005) stratified estimator $\hat{\pi}_{kw}$, we have computed percent relative efficiency by using the formulae:

$$(i)\ PRE\,(\hat{\pi}_{mS}, \hat{\pi}_{kw}) = \frac{V(\hat{\pi}_{kw})}{V(\hat{\pi}_{mS})} \times 100;\ (ii)\ PRE\,(\hat{\pi}^*_{mS}, \hat{\pi}_{kw}) = \frac{V(\hat{\pi}_{kw})}{V(\hat{\pi}^*_{mS})} \times 100$$

$$(iii)\ PRE\,(\hat{\pi}_{mS}, \hat{\pi}^*_{mS}) = \frac{V(\hat{\pi}^*_{mS})}{V(\hat{\pi}_{mS})} \times 100$$

Where

$$V(\hat{\pi}_{kw}) = \frac{1}{n}\left[\sum_{k=1}^{2} w_k \left\{\pi_{S_k}(1-\pi_{S_k}) + \frac{(1-Q_k)\{\lambda_k Q_k(1-\pi_{S_k}) + (1-\lambda_k)\}}{Q_k^2}\right\}^{1/2}\right]^2$$

and $V(\hat{\pi}_{mS})$,$V(\hat{\pi}^*_{mS})$ are respectively defined in (30) and (42) with r = 2.

We have obtained the values of the percent relative efficiencies $PRE\,(\hat{\pi}_{mS}, \hat{\pi}_{kw})$, $PRE\,(\hat{\pi}^*_{mS}, \hat{\pi}_{kw})$ and $PRE\,(\hat{\pi}_{mS}, \hat{\pi}^*_{mS})$ for different cases of π_S, λ , n, n_1 *and* P_1. Findings are shown in Table 4 - 6.

Table 4 - exhibits that the percent relative efficiency of the proposed stratified estimator $\hat{\pi}_{mS}$ with respect to the Kim and Wardes (2005) stratified estimator $\hat{\pi}_{kw}$ decrease as sample size and value of P_1 increase. Larger gain in efficiency is observed for small as well as moderately large sample sizes. However, the percent relative efficiency is more than 100 for all parametric values considered here; therefore the proposed estimator $\hat{\pi}_{mS}$ is better than the Kim and Wardes (2005) stratified estimator $\hat{\pi}_{kw}$. Also the values of the percent relative efficiencies $PRE\,(\hat{\pi}_{mS}, \hat{\pi}_{kw})$ decrease as the value of λ increase for fixed values of P_1 .

Table 5 - exhibits that the percent relative efficiency of the proposed stratified estimator $\hat{\pi}^*_{mS}$ with respect to the Kim and Wardes (2005) stratified estimator $\hat{\pi}_{kw}$ decrease as the value of λ increase for fixed values of P_1. Higher gain in efficiency is observed for small as well as moderately large sample sizes. However, the percent relative efficiency is more than 100 % for all parametric values considered here; therefore the proposed estimator $\hat{\pi}^*_{mS}$ is better than the Kim and Wardes (2005) stratified estimator $\hat{\pi}_{kw}$.

We note from Table 6 that the values of the percent relative efficiencies $PRE\,(\hat{\pi}_{mS}, \hat{\pi}^*_{mS})$ increase as the value of P_1 increases. Also the values of the percent relative efficiencies $PRE\,(\hat{\pi}_{mS}, \hat{\pi}^*_{mS})$ decrease as the value of λ increase for fixed values of P_1 .

Finally from the above discussion we conclude that the envisaged model SMRRT1 is to be preferred over the suggested model SMRRT2 and the Kim and Warde's (2005) stratified mixed randomized response model.

5. DISCUSSION

In this paper, we have envisaged a mixed randomized response models as well as its stratified randomized response models to estimate the proportion of qualitative sensitive character. It has been shown that the proposed mixed randomized response models and the stratified randomized response models are better than the Kim and Warde (2005) mixed randomize response models with larger gain in efficiency.

TABLE 4

Percent relative efficiency of the proposed stratified estimator $\hat{\pi}_{mS}$ with respect to Kim and Warde (2005) stratified estimator $\hat{\pi}_{kw}$.

						$P_1 = Q_1 = Q_2$						
			$_1$	$_2$		0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.08	0.13	0.1	0.6	0.4	0.10	2345.30	1348.75	907.50	669.17	524.34	429.19	363.16
0.08	0.13	0.1	0.6	0.4	0.15	1827.17	1105.34	771.07	584.32	467.82	389.70	334.60
0.08	0.13	0.1	0.6	0.4	0.20	1469.10	923.10	662.91	514.08	419.44	354.97	308.92
0.08	0.13	0.1	0.6	0.4	0.25	1206.77	781.50	575.02	454.96	377.54	324.17	285.70
0.18	0.23	0.2	0.6	0.4	0.10	2499.83	1418.77	944.88	690.77	537.14	436.56	366.88
0.18	0.23	0.2	0.6	0.4	0.15	1965.30	1172.87	809.21	607.44	482.23	398.56	339.66
0.18	0.23	0.2	0.6	0.4	0.20	1589.82	985.68	699.91	537.42	434.57	364.72	314.89
0.18	0.23	0.2	0.6	0.4	0.25	1311.51	838.38	609.94	477.74	392.79	334.36	292.25
0.28	0.33	0.3	0.6	0.4	0.10	2713.74	1521.85	1004.28	728.59	562.73	454.50	379.69
0.28	0.33	0.3	0.6	0.4	0.15	2151.00	1267.66	865.84	644.40	507.69	416.67	352.74
0.28	0.33	0.3	0.6	0.4	0.20	1749.73	1071.27	752.75	572.75	459.36	382.61	327.99
0.28	0.33	0.3	0.6	0.4	0.25	1449.08	914.93	658.59	511.00	416.55	351.78	305.16
0.38	0.43	0.4	0.6	0.4	0.10	3014.24	1671.93	1094.10	788.20	604.97	485.79	403.57
0.38	0.43	0.4	0.6	0.4	0.15	2406.94	1401.96	948.65	700.43	547.93	446.75	375.85
0.38	0.43	0.4	0.6	0.4	0.20	1967.91	1190.59	828.36	624.90	497.33	411.29	350.19
0.38	0.43	0.4	0.6	0.4	0.25	1635.65	1020.54	727.19	559.18	452.11	378.92	326.34

TABLE 5

Percent relative efficiency of the proposed stratified estimator $\hat{\pi}^*_{mS}$ with respect to Kim and Warde (2005) stratified estimator $\hat{\pi}_{kw}$.

						P_1 = Q_1= Q_2						
			1	2		0.1	0.1 5	0. 2	0. 25	0. 3	0. 35	0. 4
0.08	0.13	0.1	0.6	0.4	0.10	2122.44	1119.01	681.24	449.52	311.91	223.66	163.87
0.08	0.13	0.1	0.6	0.4	0.15	1690.21	948.00	603.68	412.15	293.69	215.11	160.36
0.08	0.13	0.1	0.6	0.4	0.20	1380.12	812.21	537.27	378.24	276.39	206.70	156.79
0.08	0.13	0.1	0.6	0.4	0.25	1146.74	701.73	479.75	347.34	259.94	198.41	153.16
0.18	0.23	0.2	0.6	0.4	0.10	2273.60	1190.34	722.97	477.37	332.18	239.31	176.44
0.18	0.23	0.2	0.6	0.4	0.15	1823.90	1014.06	642.97	438.49	312.91	230.00	172.39
0.18	0.23	0.2	0.6	0.4	0.20	1496.86	872.47	573.89	403.03	294.55	220.83	168.27
0.18	0.23	0.2	0.6	0.4	0.25	1248.25	756.21	513.61	370.53	277.04	211.78	164.10
0.28	0.33	0.3	0.6	0.4	0.10	2479.31	1289.55	781.33	516.01	359.81	260.12	192.70
0.28	0.33	0.3	0.6	0.4	0.15	2002.19	1104.00	697.03	474.69	338.99	249.79	187.97
0.28	0.33	0.3	0.6	0.4	0.20	1650.84	953.41	623.67	436.80	319.10	239.60	183.17
0.28	0.33	0.3	0.6	0.4	0.25	1381.26	828.72	559.22	401.92	300.07	229.53	178.30
0.38	0.43	0.4	0.6	0.4	0.10	2765.17	1429.31	863.83	570.33	398.18	288.57	214.51
0.38	0.43	0.4	0.6	0.4	0.15	2246.60	1229.00	772.66	525.26	375.12	276.83	208.88
0.38	0.43	0.4	0.6	0.4	0.20	1860.29	1064.89	692.77	483.75	353.03	265.23	203.17
0.38	0.43	0.4	0.6	0.4	0.25	1561.29	927.93	622.16	445.36	331.84	253.76	197.37

TABLE 6

Percent relative efficiency of the proposed stratified estimator $\hat{\pi}_{mS}$ with respect to the suggested stratified estimator $\hat{\pi}^{*}_{mS}$.

						$P_1 = Q_1 = Q_2$						
			1	2		0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.88	0.93	0.9	0.6	0.4	0.1	459.32	581.99	762.43	1043.92	1520.34	2430.07	4554.23
0.88	0.93	0.9	0.6	0.4	0.2	403.98	511.85	671.00	919.92	1342.09	2149.59	4037.40
0.88	0.93	0.9	0.6	0.4	0.3	353.73	446.94	584.95	801.45	1169.52	1874.91	3526.28
0.88	0.93	0.9	0.6	0.4	0.4	307.91	386.69	503.81	688.14	1002.36	1605.83	3020.75
0.89	0.94	0.91	0.6	0.4	0.1	401.91	497.14	633.04	837.54	1168.70	1766.99	3067.71
0.89	0.94	0.91	0.6	0.4	0.2	357.40	441.68	562.21	743.87	1038.34	1570.71	2728.24
0.89	0.94	0.91	0.6	0.4	0.3	316.38	389.67	494.77	653.48	911.14	1377.33	2391.26
0.89	0.94	0.91	0.6	0.4	0.4	278.46	340.80	430.47	566.22	786.97	1186.80	2056.76
0.90	0.95	0.92	0.6	0.4	0.1	360.33	437.71	545.97	705.27	956.72	1397.94	2325.84
0.90	0.95	0.92	0.6	0.4	0.2	323.21	391.95	488.26	630.09	854.05	1247.01	2073.05
0.90	0.95	0.92	0.6	0.4	0.3	288.62	348.64	432.88	557.10	753.38	1097.82	1821.66
0.90	0.95	0.92	0.6	0.4	0.4	256.33	307.57	379.69	486.20	654.65	950.36	1571.65
0.91	0.96	0.93	0.6	0.4	0.1	328.83	393.76	483.37	613.25	814.96	1162.77	1880.93
0.91	0.96	0.93	0.6	0.4	0.2	297.04	354.87	434.73	550.51	730.31	1040.18	1679.54
0.91	0.96	0.93	0.6	0.4	0.3	267.18	317.79	387.78	489.33	647.03	918.75	1479.05
0.91	0.96	0.93	0.6	0.4	0.4	239.08	282.41	342.44	429.63	565.10	798.47	1279.44

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SUMMARY

An alternative to Kim and Warde's mixed randomized response technique

The paper proposes two mixed randomized response techniques as an alternative to the Kim and Warde's (2005) randomized response technique. The properties of the models have been studied and found that the proposed mixed randomized response models are better than the Kim and Warde's (2005) mixed randomized response models in some realistic situations. We extend the proposed model to stratified sampling. Numerical illustration is given in support of the present study.