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ANALYSIS OF QUEUEING SYSTEM WITH DISCRETE AUTOREGRESSIVE ARRIVALS HAVING DML(α) AS MARGINAL DISTRIBUTION

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1. INTRODUCTION

In B-ISDN/ATM networks, IP packets or cells of voice, video, data etc are sent over a common transmission channel on statistical multiplexing basis. The performance analysis of statistical multiplexer whose input consists of a superposition of several packetized sources is not a straightforward one. The difficulty in modeling this type of traffic is due to the correlated structure of arrivals. A common approach is to approximate this complex non renewal input process by analytically tractable arrival process, namely discrete autoregressive process (DAR). The impact of autocorrelation in traffic processes on queueing performance measures such as mean queue length, mean waiting times and loss probabilities in finite buffers, can be very dramatic

The discrete auto regressive process of order 1 [DAR(1)] is known to be a good model for VBR coded teleconference traffic as in (Elwalid et al., 1995). Hwang et al., 2002 obtained the waiting time distribution of the discrete time single server queue with DAR(1) input. Again (Hwang and Sohraby, 2003) obtained the closed form expression for the stationary probability generating function of the system size of the discrete time single server queue with DAR(1) input. (Hwang et al., 2004) analyzed the queueing behavior of multiple first-order autoregressive sources. (Choi and Kim, 2004) analyzed a multiserver queue fed by DAR(1) input. (Kamoun, 2006) analyzed the discrete-time queue with autoregressive inputs revisited.

A queueing system with discrete autoregressive arrivals is analyzed by (Kim et al., 2007). Mean queue size in a queue with discrete autoregressive arrivals of order p is obtained by (Kim et al., 2008). Analytic approximations of queues with lightly- and heavily-correlated autoregressive service times is discussed in (Dieter et al., 2011). (Jose and Bindu, 2011) analyzed the DAR(1)/D/s queue with Quasi negative Binomial -II distribution as marginal.

In this paper we focus on the analysis of a multiserver ATM multiplexer queue with s servers $(s \geq 0)$ of constant service rate and VBR coded teleconference traffic as input. We can model the VBR coded teleconference traffic as discrete autoregressive process of order 1 [DAR(1)]. Here we analyze the multiserver queue in which the input process is DAR(1) with discrete Mittag-Leffler distribution DML(α) with parameter (α) as marginal distribution. The one step transition probability matrix of this Markov process is of M/G/1 type as presented in (Neuts, 1989). We construct a Markov renewal process at embedded epochs from the original Markov process. We then compute the stationary distribution of the constructed Markov renewal process by matrix analytic methods. From this stationary distribution we calculate the stationary distribution of the system size of the original Markov process by using the theory of Markov regenerative processes. Also the stationary distribution of the waiting time of the arbitrary packet is obtained..

The rest of the paper is arranged as follows. Discrete Mittag-Leffler [DML(α)] distribution is given in section 2. Input traffic as DAR(1) with DML(α) as marginal distribution is described in section 3. Analysis of DAR(1)/D/s Queue with DML(α) is given in section 4. Stationary distribution os system size and waiting time of an arbitrary packet is derived in section 5. The empirical analysis of the quantitative effect of the stationary distribution of system size and waiting time on the autocorrelation function as well as the parameters of the input traffic is illustrated numerically and graphically in section 6. The model is applied to a real data in section 7.

2. DISCRETE MITTAG-LEFFLER DISTRIBUTION

Recently Mittag-Leffler functions and distributions have received the attention of mathematicians, statisticians and scientists in physical and chemical sciences. (Pillai, 1990a) introduced the Mittag-Leffler distribution in terms of Mittag-Leffler functions. (Pillai and Jayakumar , 1995) introduced a new class of discrete distributions, namely discrete Mittag-Leffler distributions (DML(α)) which is a generalization of geometric distribution and developed a first order autoregressive process with discrete Mittag-Leffler marginal distribution. (Jayakumar, 2003) and (Jose and Pillai, 1996) have done extensive studies on Mittag-Leffler distribution and its applications. (Jose et al., 2010) extended this to develop a more generalized Mittag-Leffler model. Autoregressive process with marginals follow bivariate discrete Mittag-Leffler distribution is developed by (Jayakumar et al., 2010).

Consider a sequence of independent Bernoulli trials in which the kth trial has probability of success $\alpha / k, k = 1,2,3....$ Let N be the trial number in which the first success occurs. Then the probability mass function P[N=x] is given by

$$\begin{aligned} f(x) &= P[N = x] = (1-\alpha)(1-\frac{\alpha}{2})....(1-\frac{\alpha}{x-1})\frac{\alpha}{x} \\ &= \frac{(-1)^{x-1}\alpha(\alpha-1)...(\alpha-x+1)}{x!}, x = 1,2,... \end{aligned} \tag{1}$$

The probability generating function of N is given by $G(z) = 1-(1-z)^{\alpha}, 0 < \alpha \le 1$. Let $X_1, X_2 ... X_n$ be independent and identically distributed as N. Let G be geometric with parameter p so that $P(G = k) = q^k p, k = 0,1,2...0 < p < 1, q = 1-p$. Then $X_1 + X_2 + ... + X_G$ has generating function

$$P(z) = \frac{p}{1-q(1-(1-z)^{\alpha})} = \frac{1}{1+c(1-z)^{\alpha}} with\ p = 1/(1+c) \tag{2}$$

The distribution with probability generating function (2) is known as discrete Mittag-Leffler distribution with parameter (DML(α))

1. DML(α) is geometrically infinitely divisible and hence infinitely divisible.

2. DML(α) is normally attracted to stable α.
3. Mittag-Leffler distribution ML(α) is obtained as the limit of a sequence of DML(α)
4. The probability generating function of DML(α) is expanded as

$$P(z) = \sum_{n=0}^{\infty} p_n z^n = \frac{1}{1+c(1-z)^{\alpha}}$$

5. DML(α) distribution function [F_α] satisfies the renewal equation

$F * F_\alpha$ where $F_\alpha = \frac{1}{1+c} + \frac{c}{1+c}$

and F is the distribution function of N as (1).

6. By Steutel and van Harn [1979] DML(α) is in discrete class L.

$$\frac{P(z)}{P(1-\rho+\rho z)} = \rho^{\alpha} + \frac{(1-\rho^{\alpha}}{(1+c(1-z)^{\alpha})} \quad (3)$$

The right hand side of (3) is a probability generating function being the weighted average of two probability generating functions.

7. DML(α) is directly attracted to discrete stable α.

2.1. *Simulated sample path of DML(α)*

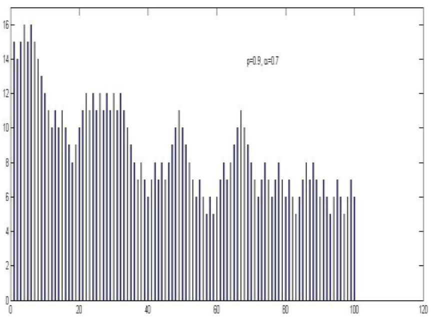
Let X be DML(α) with probability generating function $P(z)$. By expanding $P(z) = \sum_{n=0}^{\infty} P(X=n)z^n$, we can calculate the probabilities as

$$P(X=0) = \frac{1}{1+c}$$

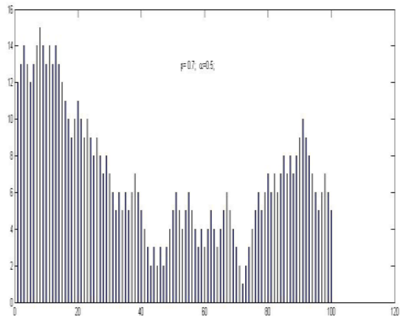
$$P(X=1) = \frac{c\alpha}{(1+c)^2}$$

$$P(X=n) = \frac{1}{1+c}\left[P[X=(n-1)]\alpha - \alpha C_2 P[X=(n-2)] + \ldots + (-1)^{n-1}\alpha C_n P[X=0]\right]$$

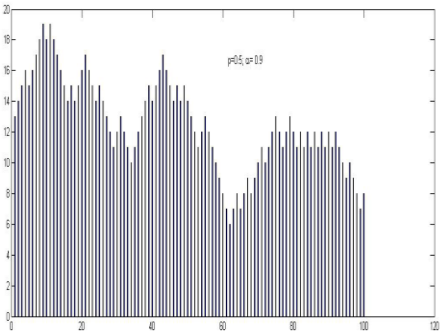
Simulated sample path of discrete Mittag-Leffler distribution is given in the figure 1.



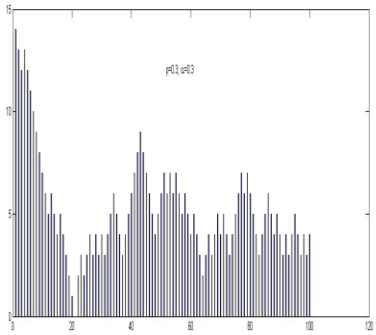
a) $p = 0.9$ and $\alpha = 0.7$



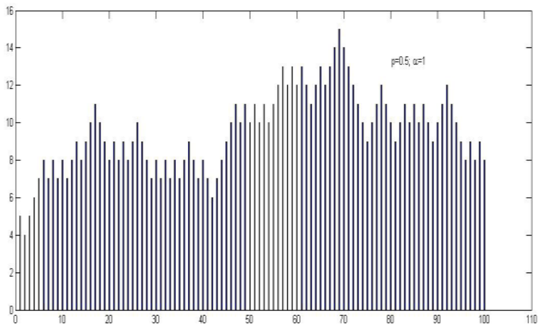
(b) $p = 0.7$ and $\alpha = 0.5$



(c) $p = 0.5$ and $\alpha = 0.9$



(d) $p = 0.3$ and $\alpha = 0.3$



(e) $p = 0.5$ and $\alpha = 1$

Figure 1 - Simulated sample path of discrete Mittag-Leffler distribution for n = 100 (a) $p = 0.9$ and $\alpha = 0.7$ (b) $p = 0.7$ and $\alpha = 0.5$ (c) $p = 0.5$ and $\alpha = 0.9$ (d) $p = 0.3$ and $\alpha = 0.3$ (e) $p = 0.5$ and $\alpha = 1$

3. INPUT TRAFFIC OF THE QUEUE AS DAR(1) WITH DML(α) AS MARGINAL

Various Markovian processes have been found useful models for traffic arising in telecommunication networks, especially when the traffic exhibits high autocorrelations. Many authors modeled correlated arrival processes as MAPs(Markovian arrival processes) which generalize MMPPs(Markov modulated Poisson processes) in a continuous time frame-work and MMBPs(Markov modulated Bernoulli process) in a discrete time frame work. However the draw back of these models is the need to estimate many parameters of MAPs which should be extracted from the marginal distribution and the correlation structure of the measured data and such estimations require time consuming work. Consequently in order to reduce the number of parameters , we use the two state MMPP or MMBP simply because they have only four parameters or fewer to estimate and their autocorrelations are exponentially or geometrically decaying, which is one of the salient features of the traffic in telecommunication networks such as ATM.

Among time series, the discrete autoregressive process of order 1[DAR(1)] is a Markov process with geometrically decaying autocorrelation function, which can exhibit any general distribution. The DAR process, constructed and analyzed by (Jacobs and Lewis, 1978) , has developed into one of several standard tools for modelling input traffic in telecommunication networks. Also DAR[1] is much simpler than the BMAP (Batch. Markovian arrival process) which can also exhibit general distribution The input ATM multiplexer with VBR coded teleconference traffic $\{X(t): t = t = 0,1...\}$ is assumed to be DAR(1) with Discrete Mittag - Leffler distribution as marginal. From McKenzie [2003], the first -order autoregressive equation can be in the following form.

$$X(t) = (1 - Z(t))X(t-1) + Z(t)Y(t), t = 1,2,.... \tag{4}$$

where $\{Z(t): t = 0,1,2,...\}$ are i.i.d. binary r.v.s with $P[Z(t) = 0] = \beta(0 \leq \beta < 1)$ and $P[Z(t) = 1] = 1 - \beta$. $\{Y(t): t = 0,1,2,...\}$ is a sequence of i.i.d random variables assume only positive values. $\{Z(t): t = 0,1,2,...\}$ is assumed to be independent of $\{Y(t): t = 0,1,2,...\}$, $b(x) = P[Y(t) = x]$

where $b(x)$ is taken as the discrete Mittag Leffler ($\alpha = 1$) distribution. DAR(1) is determined by the parameter β and the distribution $\{b(x): x = 0,1,2,...\}$ of $Y(t)$. So that

$$X(0) = Y(0)$$

$$X(t) = \begin{cases} X(t-1) & \text{with probability} \quad \beta \\ Y(t) & \text{with probability} \quad 1-\beta, 0 < \beta < 1 \end{cases}$$

The properties of DAR(1) are as follows

1. $\{X(t): t = 0,1,2..\}$ is stationary
2. The probability distribution of $X(t)$ is the same as the distribution of $Y(t)$

$$P[X(t) = x] = b_x, x = 0,1,2...$$

3. The autocorrelation function $Y(t)$ for $X(t)$ at lag t is $\beta^t, t = 0,1,2...$ the parameter β is the decay rate of the autocorrelation function.

If $X(0)$ is also sampled from DML $(\alpha = 1)$ then (4) generates a stationary process $X(t)$ whose marginal distribution is DML $(\alpha = 1)$. The model defines the current observation to be a mixture of two independent r.v.s: it is either the last observation, with probability β, or another, independent, sample from the same distribution. The sample path of DAR(1) process with discrete Mittag Leffler $(\alpha = 1)$ as marginal distribution is shown in figure (2). The conditional mean of $X(t)$ given $X(t-1)$ is linear in $X(t-1)$. The conditional variance is quadratic in $X(t-1)$. In addition, $X(t)$ is a Markov chain with transition probability matrix given by $\beta I + (1-\beta)Q$ where I is the identity matrix and Q is a matrix each of whose rows have the distribution DML $(\alpha = 1)$.

3.1. *DAR (p) model*

The model (4) can be extended to higher orders. The p-th order model, DAR(p), is given by

$$X(t) = (1 - Z(t))X(t - \varphi(t)) + Z(t)Y(t), t = 0,1,...$$

where $\{Z(t) : t = 0,1,2,...\}$ and $\{Y(t) : t = 0,1,2,...\}$ are same as before and $\{\varphi(t) : t = t = 0,1...\}$ is a sequence of i.i.d random variables taking values in the set $\{1,2..p\}$ with

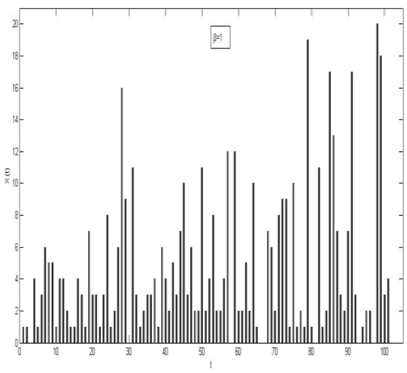
$$P[\varphi(t) = i) = \varphi_i, i = 1,2..p$$

The processes $\{Z(t) : t = 0,1,2,...\}$, $\{Y(t) : t = 0,1,2,...\}$ and $\{\varphi(t) : t = t = 0,1...\}$ are assumed to be independent The regression equation can be written as

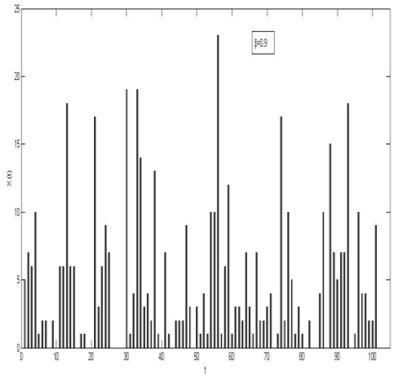
$$X(t) = \begin{cases} Y(t) & \textit{with probability} \quad 1-\beta \\ X(t-1) & \textit{with probability} \quad \beta\varphi_1 \\ X(t-2) & \textit{with probability} \quad \beta\varphi_2 \\ \vdots & \\ X(t-p) & \textit{with probability} \quad \beta\varphi_p \end{cases} \tag{5}$$

It is seen that $\{X(t) : t = t = 0,1...\}$ is stationary and stationary distribution of $\{X(t) : t = t = 0,1...\}$ is same as the distribution of $\{Y(t); t = 0,1...\}$. The autocorrelation function $r_X(k)$ satisfies the Yule Walker equations

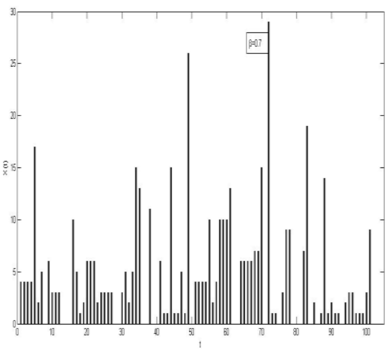
$$r_X(0) = 1, r_X(k) = \beta \sum_{i=1}^{p} \varphi_i r_X(k-i), k \geq 1$$



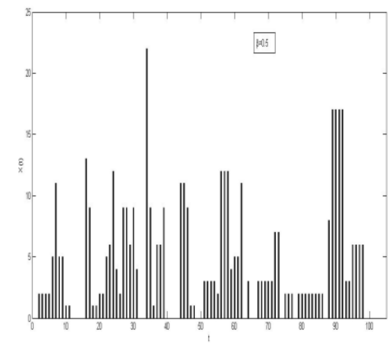
(a) $\beta = 1$



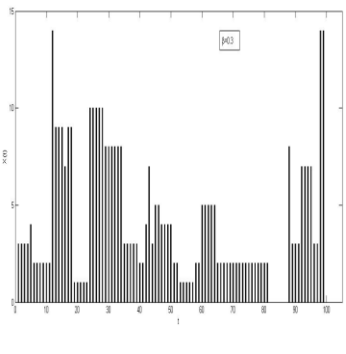
(b) $\beta = 0.9$



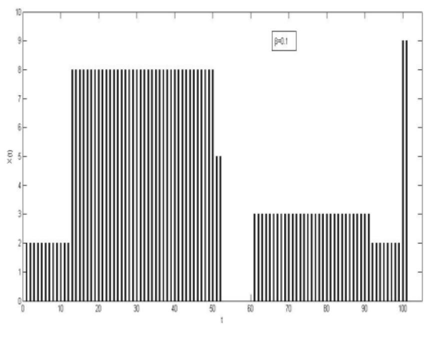
(c) $\beta = 0.7$



(d) $\beta = 0.5$



(e) $\beta = 0.3$



(f) $\beta = 0.1$

Figure 2: - Simulated sample path of the DAR(1) process with discrete Mittag-Leffler ($\alpha = 1$) as marginal distribution for n=100 (a) $\beta = 1$ (b) $\beta = 0.9$ (c) $\beta = 0.7$ (d) $\beta = 0.5$ (e) $\beta = 0.3$ (f) $\beta = 0.1$

4. ANALYSIS OF DAR(1) /D/S QUEUE WITH DML(α) AS MARGINAL

Assume that the input process is DAR(1) with discrete Mittag-Leffler distribution as the marginal distribution and there are s servers $(s > 0)$ whose service occurs at constant rate. In this integer valued time queue, the time is divided into slots of equal size and one slot is needed to serve a packet by a server. Assume that packet arrivals occur at the beginning of slots and departures occur at the end of the slots. Here $\{X(t) : t = t = 0,1...\}$ represents packet arrivals so that $X(t)$ is the number of packets arriving at the beginning of the t th slot. Let $N(t)$ be the number of packets in the system say system size , immediately before arrivals at the beginning of the t th slot. Then $\{(N(t), X(t) : t = 0,1,2...\}$ is a two dimensional Markov process of M/G/1 queue type. The state space is $\bigcup_{n\geq 0} l(n) = \bigcup_{n,i\geq 0}\{(n,i)\} = E\{0,1,2,...\}\times\{0,1,2...\}$. The number of phases is infinity. So the computation of stationary distribution of $\{(N(t), X(t)) : t = 0,1,2...\}$ is not easy to work out. The stationary distribution of system size and waiting time of an arbitrary packet is computed by matrix analytical method and using the Markov regenerative processes

In practice by matrix analytical method and using the theory Markov regenerative processes we compute the stationary distribution of a new process at the embedded epochs $\{t_\tau, \tau = 0,1,2...\}$ $0 < t_0 < t_1 < t_2 < t_3....$ as follows

$$t_\tau = \begin{cases} 0, & \tau = 0 \\ inf\{t > t_{\tau-1} : Z(t) = 1 or 0 \leq X(t) \leq s-1\}, & \tau = 1,2... \end{cases}$$

Let

$$N_\tau = N(t_\tau), \tau = 0,1,2...$$

$$J_0 = s$$

$$J_\tau = \begin{cases} X(t_{\tau},) & if Z(t_\tau) = 0 \quad \tau = 1,2,3... \\ s & if Z(t_\tau) = 1, \quad \tau = 1,2,3... \end{cases}$$

The packet arrivals at and after t_τ are independent of the information prior to t_τ given J_τ. From this, it is observed that $\{(N_\tau, J_\tau) : \tau = 0,1,2,...\}$ is the new Markov renewal process with state space E $= \{0,1,2...\}\times\{0,1...s\}$. The probability transition matrix of the Markov renewal process is computed as follows.

1. For n=0,1,2... and i=0,1,...s-1

$$(n,i) \rightarrow \begin{cases} (\; max\;\{n-s+i,0\}, i) & with\ probability \quad \beta \\ \\ (\; max\;\{n-s+i,0\}, s) & with\ probability \quad 1-\beta \end{cases}$$

2. For n= 0,1,2.....

$$
(n,s) \rightarrow \begin{cases} (\max\{n-s+i,0\}, i) & \text{with probability } b_i\beta \quad 0 \le i \le s-1, \\ (n-s+i, s) & \text{with probability } b_i(1-\beta) \quad s-n+1 \le i \le s-1, \\ (0,s) & \text{with probability } \sum_{i=0}^{\min\{s-n,s-1\}} b_i(1-\beta) + g_0\delta_{n0} \\ (n+l, s) & \text{with probability } g_l \quad l \ge 0, n+l > 0 \end{cases}
$$

where

$$
\delta_{n0} = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \ge 1 \end{cases}
$$

$$
g_0 = b_s
$$

$$
g_l = \sum_{i|l} b_{i+s}(1-\beta)\beta^{\frac{l}{i}-1}, l = 1,2,\ldots
$$

The transition probability matrix P is as follows

$$
P = \begin{pmatrix}
B_s & A_{s+1} & A_{s+2} & \cdots & A_{2s-1} & A_{2s} & A_{2s+1} & \cdots \\
B_{s-1} & A_s & A_{s+1} & \cdots & A_{2s-2} & A_{2s-1} & A_{2s} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
B_1 & A_2 & A_3 & \cdots & A_s & A_{s+1} & A_{s+2} & \cdots \\
A_0 & A_1 & A_2 & \cdots & A_{s-1} & A_s & A_{s+1} & \cdots \\
0 & A_0 & A_1 & \cdots & A_{s-2} & A_{s-1} & A_s & \cdots \\
0 & 0 & A_0 & \cdots & A_{s-3} & A_{s-2} & A_{s-1} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots
\end{pmatrix}
$$

where

$$A_i = \begin{array}{c} \\ 0 \\ \vdots \\ i \\ \vdots \\ s \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & \dots & i & \dots & s \end{array} \\ \begin{pmatrix} 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \beta & \vdots & 1-\beta \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & b_i\beta & \vdots & b_i(1-\beta) \end{pmatrix} \end{array} 0 \le i \le s-1$$

$$A_i = \begin{array}{c} \\ 0 \\ \vdots \\ s \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \dots & s \end{array} \\ \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \dots & g_{i-s} \end{pmatrix} \end{array} i \ge s, B_i = \sum_{j=0}^{i} A_j \, 1 \le i \le s$$

4.1. *Stability condition*

Here the input process is taken as DAR(1)/D/s with discrete Mittag-Leffler ($\alpha = 1$) as marginal distribution, then

$$E(X) = \frac{1-p}{p}, V(X) = \frac{1-p}{p^2}$$

The parameter p can be estimated by the method of moments, which in this case happens to yield maximum likelihood estimates of p. For the first variant let $k = k_1, \dots, k_n$ be a sample where $k_i \ge 1$ for $i = 1, \dots, n.$ Then p can be estimated as $\hat{p} = \left(\frac{1}{n} \sum_{n}^{i=1} k_i \right)^{-1}$ In order to derive the stationary distribution, the stability condition $\rho = \frac{\lambda}{s\mu} < 1.$ should be satisfied. Here the service rate is constant then the offered load should be $\frac{\lambda}{s} < 1$ which means

$$\lambda = E[X(t)] = \sum_{x=1}^{\infty} x b_x = \frac{1-p}{p} < s$$

is the stability condition to be satisfied in the DAR(1)/D/s queue with discrete Mittag-Leffler distribution ($\alpha = 1$) as the marginal distribution.

4.2. *The stationary distribution of the Markov renewal process*

Consider $\{(N_\tau, J_\tau), \tau = 0,1,2\dots\}$, and $\pi_{ni} = lim_{\tau \to \infty} P\{N_\tau = n, J_\tau = i\}, n \ge 0, 0 \le i \le s$

Here we assume non exponential interarrival and service time distribution. In order to find the steady state probabilities of the queueing systems we apply matrix analytic method as described below: The transition probability matrix P has infinite order, so that it would have to be truncated before we implement matrix analytic method. We assume that there exists some index N such that $A_N = 0$ for all $n > N$. That is we assume that the Markov chain does not jump more than N steps at a time so that the matrix is of finite order, see (Latouche & Ramaswamy, 1991).

For a numerical illustration, consider the case when s=5 and N=14. Then the transition probability matrix P can be a matrix of M/G/1 type, which underlines the similarity to the embedded Markov chain of the M/G/1 queue. With respect to the levels , the Markov chain is called skip free to the left, since in one transition the level can be reduced only by one

$$\left(\begin{array}{ccccc|ccccc|ccccc}
B_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} & A_{19} \\
B_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\
B_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} \\
B_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\
B_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\
\hline
A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} \\
0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} & A_{13} \\
0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} & A_{12} \\
0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} \\
0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} \\
\hline
0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 \\
0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 & A_5 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 & A_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 & A_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 & A_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0 & A_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_0
\end{array}\right)$$

By arranging the transition probability matrix into $(s \times s)$ matrices we get

$$P = \begin{pmatrix} \hat{B}_0 & \hat{B}_1 & \hat{B}_2 \\ \hat{A}_0 & \hat{A}_1 & \hat{A}_2 \\ 0 & \hat{A}_0 & \hat{A}_1 \\ 0 & 0 & \hat{A}_0 \end{pmatrix} \textit{ or equivalently } P = \begin{pmatrix} \hat{B}_0 & \hat{A}_2 & \hat{A}_3 \\ \hat{A}_0 & \hat{A}_1 & \hat{A}_2 \\ 0 & \hat{A}_0 & \hat{A}_1 \\ 0 & 0 & \hat{A}_0 \end{pmatrix}$$

In general we can symbolize the transition matrix P as

$$P=\begin{pmatrix} \hat{B}_0 & \hat{B}_1 & \hat{B}_2 & \cdots & \hat{B}_{n^*-1} \\ \hat{A}_0 & \hat{A}_1 & \hat{A}_2 & \cdots & \hat{A}_{n^*-1} \\ 0 & \hat{A}_0 & \hat{A}_1 & \cdots & \hat{A}_{n^*-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \hat{A}_0 \end{pmatrix} n^* = \frac{N+s+1}{s}-1$$

or equivalently

$$P=\begin{pmatrix} \hat{B}_0 & \hat{A}_2 & \hat{A}_3 & \cdots & \hat{A}_{n^*} \\ \hat{A}_0 & \hat{A}_1 & \hat{A}_2 & \cdots & \hat{A}_{n^*-1} \\ 0 & \hat{A}_0 & \hat{A}_1 & \cdots & \hat{A}_{n^*-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \hat{A}_0 \end{pmatrix} n^* = \frac{N+s+1}{s}-1$$

The elements of P can be written as

$$\hat{B}_0=\begin{pmatrix} B_s & A_{s+1} & \cdots & A_{2s-1} \\ B_{s-1} & A_s & \cdots & A_{2s-2} \\ \vdots & \vdots & \vdots & \vdots \\ B_1 & A_2 & \cdots & A_s \end{pmatrix}$$

$$\hat{A}_n=\begin{pmatrix} A_{sn} & A_{sn+1} & \cdots & A_{s(n+1)-1} \\ A_{sn-1} & A_{sn} & \cdots & A_{s(n+1)-2} \\ \vdots & \vdots & \vdots & \vdots \\ A_{s(n-1)+1} & A_{s(n-1)+2} & \cdots & A_{sn} \end{pmatrix} n=0,1,2\ldots n^*,$$

$$\hat{B}_n=\hat{A}_{n+1}, n=1,2,\ldots n^*$$

By the matrix analytic method we proceed as follows
Step1: Find the minimal nonnegative solution G of the matrix equation

$$G=\sum_{n=0}^{\infty}\hat{A}_n G^n$$

G can be given by the following iteration See (Breuer, 2005)

$$G_0 = 0$$
$$G_1 = \hat{A}_0$$
$$G_k = \sum_{n=1}^{k-1} \hat{A}_n G_{k-1}^n, k = 2,3,...$$
$$G = \sum_{k=1}^{\infty} G_k$$

G is a stochastic matrix ,so we can stop the iteration procedure when $|1 - G.1| < \varepsilon$ reaches where $\varepsilon = 0.0001$. From this iteration we obtained the upper limit of k & *let* $n^* = k - 1$. From this n^* we come to know the truncated index N at which G become stochastic

Step 2: Find

$$H = \sum_{n=0}^{n^*} \hat{B}_n G^n$$

and a positive row vector h satisfying

$$hH = h$$

Step 3:

$$x_0 = h$$
$$x_n = \left(x_0 \sum_{i=0}^{n^*} \hat{B}_{n+i} G^i + \sum_{l=1}^{n-1} x_l \sum_{i=0}^{n^*} \hat{A}_{n-l+i+1} G^i \right) \left(I - \sum_{i=0}^{n^*} \hat{A}_{i+1} G^i \right)^{-1}, n = 1,2....n^*$$

Step 4: Finally

$$((\pi_{ns,0},...\pi_{ns,s})...(\pi_{(n+1)s-1,0}...\pi_{(n+1)s-1,s})) = Cx_n, n = 0,1,2...n^*$$

where $C = \left[\sum_{n=0}^{n^*} x_n e \right]^{-1}$ and e is the s x (s+1) dimensional column vector whose components are all ones

5. STATIONARY DISTRIBUTION OF $\{N(t), X(t), t = 0,1,2...\}$

Observe that $\{((N_\tau, J_\tau), t_\tau), \tau = 0,1,2...\}$ is a Markov renewal process and $\{((N(t+t_\tau), X(t+t_\tau)) : t = 0,1,2...\}$ given $\{(N(\upsilon), X(\upsilon)), 0 \le \upsilon < t_\tau, N_\tau, J_\tau) = (n,i)\}$ is stochastically equivalent to $\{N(t), X(t) : t = 0,1,2...\}$ given $\{N_0, J_0 = (n,i)\}$ Hence $\{N(t), X(t) : t = 0,1,2...\}$ is a discrete time Markov regenerative process with the Markov renewal sequence $\{((N_\tau, J_\tau), t_\tau) : \tau = 0,1,2...\}$. Now we consider the following theorem

THEOREM 5.1 *(Choi) Let* $\{Z(t), t \geq 0\}$ *be a discrete time Markov regenerative process on the countable state space* S *with Markov renewal sequence* $\{(Y_k, S_k), k = 0,1,2,...\}$. *Let* E *be the countable state space of the Markov process* $\{Y_k : k = 0,1,2...\}$ *and for* $i \in E$ *and* $j \in \mathsf{S}$,

$$\mu_i = E[S_1 / Y_0 = i], \tag{7}$$

$$\alpha_{ij} = E\left[\sum_{t=0}^{S_1 - 1} 1_{\{Z(t)=j\}} \mid Y_0 = i\right] \tag{8}$$

Suppose that the discrete time semi-Markov process $\{Y(t) : t = 0,1,2,...\}$ defined by $Y(t) = Y_k$ for $S_k \leq t < S_{k+1}$ is irreducible, aperiodic and positive recurrent. Then for $k \in E$ and $j \in \mathsf{S}$,

$$lim_{t\to\infty} P\{Z(t) = j \mid Y_0 = k\} = \frac{\sum_{i\in E} \pi_i \alpha_{ij}}{\sum_{i \in E} \pi_i \mu_i} \tag{9}$$

where $\pi = (\pi_i)_{i \in E}$ is a stationary measure of the Markov process $\{Y_k : k = 0,1,2...\}$

THEOREM 5.2 *The stationary distribution or the limiting probabilities*
$p_{nj} = lim_{t\to\infty} P\{(N(t), X(t)) = (n, j)\}, n, j = 0,1,2...$ *of* $\{(N(t), X(t)) : t = 0,1,2...\}$

are given by

$$p_{nj} = \begin{cases} \mu^{-1}(\pi_{nj} + \pi_{ns} b_j), & 0 \leq j \leq s-1 \\ \mu^{-1} \dfrac{\pi_{ns} b_s}{1-\beta}, & j = s \\ \mu^{-1} \sum_{i=0}^{\left|\frac{n}{j-s}\right|} \pi_{n-i(j-s),s} b_j \beta^i, & j \geq s+1 \end{cases}$$

$$\text{where } \left(1 - \beta \sum_{r=s}^{\infty} b_r\right)^{-1} = \mu$$

THEOREM 5.3 *The distribution of the waiting time W of an arbitrary packet is given by P(W=w)*

$$= \frac{1}{\lambda}\left(\sum_{n=0}^{sw} \sum_{j=sw-n+1}^{\infty} p_{nj} \; min \; \{n = j - sw, s\} + \sum_{n=sw+1}^{s(w+1)-1} \sum_{j=1}^{\infty} p_{nj} \; min \; \{s(w+1) - n, j\} \right)$$

$for \; w = 0,1,2...$

6. EMPIRICAL ANALYSIS

Assume the number of servers to be s=3, and $\alpha = 1$. To satisfy the stability condition assume the mean $\lambda = 1.5, 1.8, 2, 2.5, \; and \; 2.7$ so that $\frac{\lambda}{s} < 1$. Figure (3)(a) displays the complementary distribution function of the stationary system size when $\lambda = 2.7(p = 0.27027) \; and \; \beta = 0.1, 0.3, 0.5, 0.7, \; and \; 0.9$ and figure (3)(b) displays the complementary distribution function of the stationary system size when $\beta = 0.5 \; and \; \lambda = 1.5, 1.8, 2, 2.5, \; and \; 2.7$ (p=0.4, 0.3571, 0.3333, 0.2857, and 0.27027) respectively. The parameter β gives the information on the strength of correlation of the input process. Stationary system size is larger for the large β. Also stationary system size is stochastically larger for large variance of the input process ($\lambda = 2.7(p = 0.27027)$ & variance= 9.99) than those for the case that the stationary distribution of the input process has a small variance($\lambda = 1.5(p = 0.4)$ & variance= 3.75). Figure (2) support this intuitive facts.

Table (1) and (2) display the stationary probabilities of the system size for different values of λ and β. Table (2) and (3) display the stationary probabilities of waiting time of an arbitrary packet for different values of λ and β.

Figure (4)(a) displays the complementary distribution function of the waiting time of an arbitrary packet,when $\lambda = 2.7 \; and \; \beta = 0.1, 0.3, 0.5, 0.7, \; and \; 0.9$ and figure (4)(b) displays the complementary distribution function of the stationary system size when $\beta = 0.5 \; and \; \lambda = 1.5, 1.8, 2, 2.5, \; and \; 2.7$ respectively. Stationary waiting time of an arbitrary packet,is larger for the large β . Also stationary waiting time of an arbitrary packet,is stochastically larger for large variance of the input process (($\lambda = 2.7(p = 0.27027)$ & variance= 9.99) than those for the case that the stationary distribution of the input process has a small variance($\lambda = 1.5(p = 0.4)$ & variance= 3.75). Figure (3)support this intuitive facts.

TABLE 1

p_{nj} in DAR(1)/D/s with discrete Mittag-Leffler as marginal for $\alpha = 1$, mean= $\lambda = 2.7$, p=0.27027, $\beta = 0.9$ and s=3

n	0	1	2	3	4	5	6	7
0	0.19350	0.14136	0.25616	0.07162	0.00523	0.00381	0.00278	0.00203
1	0.00074	0.00056	0.01755	0.00260	0.00489	0.00014	0.00010	0.00007
2	0.00017	0.00014	0.01898	0.00044	0.00444	0.00346	0.00002	0.00001
3	0.00012	0.00012	0.00014	0.00033	0.00402	0.00014	0.00252	0.00001
4	0.00008	0.00007	0.00010	0.00024	0.00363	0.00312	0.00010	0.00183
5	0.00002	0.00005	0.00004	0.00018	0.00328	0.00014	0.00002	0.00007
6	0.00003	0.00004	0.00004	0.00013	0.00296	0.00282	0.00227	0.00001
7	0.00002	0.00002	0.00002	0.00009	0.00267	0.00013	0.00009	0.00001
8	0.00002	0.00001	0.00001	0.00007	0.00241	0.00254	0.00002	0.00165
9	0.00001	0.00001	0.00002	0.00005	0.00217	0.00012	0.00205	0.00007
10	0.00001	0.00001	0.00001	0.00004	0.00196	0.00229	0.00009	0.00001
11	0.00001	0.00001	0.00000	0.00003	0.00177	0.00011	0.00002	0.00001
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

TABLE 2

p_{nj} in DAR(1)/D/s with discrete Mittag-Leffler as marginal for $\alpha = 1$, mean= $\lambda = 1.5$, p=0.4, $\beta = 0.5$ and s=3

n	0	1	2	3	4	5	6	7
0	0.34617	0.20768	0.14007	0.07064	0.02119	0.01272	0.00763	0.00458
1	0.01305	0.00805	0.02210	0.00535	0.01224	0.00096	0.00058	0.00035
2	0.00395	0.00255	0.03445	0.00153	0.00656	0.00663	0.00017	0.00019
3	0.00258	0.00180	0.00127	0.00101	0.00358	0.00066	0.00392	0.00007
4	0.00159	0.00101	0.00081	0.00063	0.00198	0.00343	0.00036	0.00233
5	0.00019	0.00061	0.00040	0.00038	0.00118	0.00046	0.00012	0.00028
6	0.00057	0.00040	0.00028	0.00024	0.00062	0.00176	0.00199	0.00007
7	0.00034	0.00021	0.00016	0.00014	0.00035	0.00023	0.00019	0.00004
8	0.00020	0.00013	0.00008	0.00009	0.00026	0.00089	0.00007	0.00117
9	0.00014	0.00010	0.00007	0.00006	0.00012	0.00012	0.00145	0.00014
10	0.00009	0.00005	0.00004	0.00004	0.00007	0.00045	0.00016	0.00003
11	0.00005	0.00003	0.00002	0.00002	0.00004	0.00007	0.00004	0.00002
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

TABLE 3

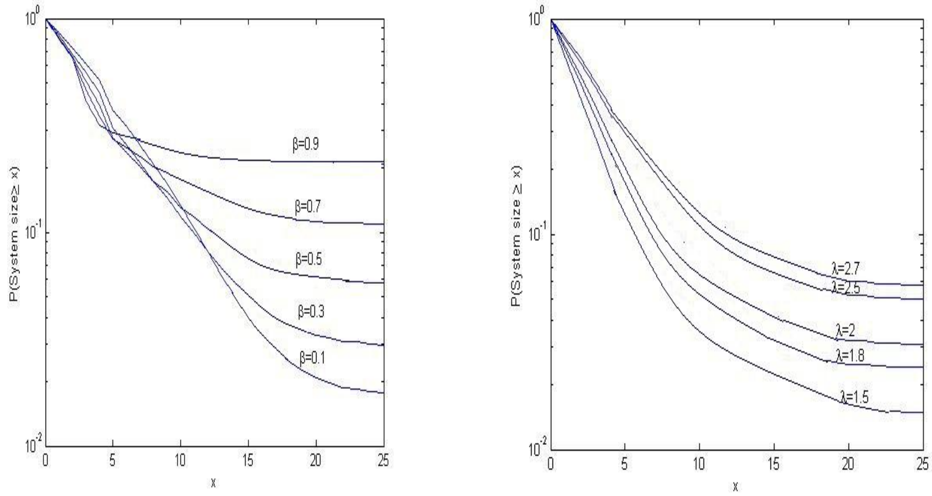
$P(W=\omega)$ *in DAR(1)/D/s with discrete Mittag-Leffler as marginal for different values of* β *and* $\lambda = 2.7$, $\alpha = 1$, *and s=3.*

	β				
ω	0.1	0.3	0.5	0.7	0.9
0	0.3892	0.4149	0.4181	0.403	0.372
1	0.2597	0.2353	0.1938	0.1353	0.0536
2	0.1437	0.1218	0.1028	0.0816	0.0416
3	0.0855	0.0704	0.0621	0.0543	0.0338
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

TABLE 4

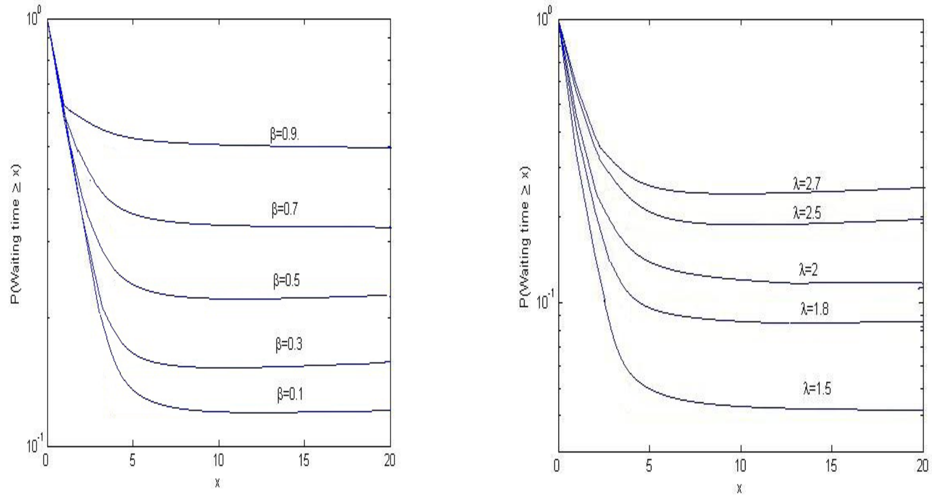
$P(W=\omega)$ *in DAR(1)/D/s with discrete Mittag-Leffler as marginal for different values of* λ , $\beta = 0.5$, $\alpha = 1$, *and s=3*

	λ				
ω	1.5	1.8	2	2.5	2.7
0	0.6684	0.5914	0.5455	0.4497	0.4181
1	0.1846	0.1978	0.2006	0.1978	0.1938
2	0.0721	0.0855	0.0921	0.1014	0.1028
3	0.0338	0.0433	0.0488	0.0593	0.0621
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$



(a) (b)

Figure 3 - Complementary distribution function of the stationary system size in DAR(1)/D/s with discrete Mittag-Leffler as marginal when (a) $\alpha = 1$, $\lambda = 2.7$ (b) $\alpha = 1$, $\beta = 0.5$



(a) (b)

Figure 4 - Complementary distribution function of the waiting time of an arbitrary packet in DAR(1)/D/s with discrete Mittag-Leffler as marginal,when (a) $\alpha = 1$ and $\lambda = 2.7$ (b) $\alpha = 1$ and $\beta = 0.5$

7. REAL DATA ANALYSIS

Apply the model to a data on the passenger arrivals at a subway bus terminal in Santiago de Chile for each 15-minute interval between 6:30 hours and 22 hours from 1-21 March 2005. This includes all the busy periods as well as idle periods. http://robjhyndman.com/TSDL /BICUP2006.DAT is the source of the data. The data can be treated as DAR(1), where $X(t)$ be the number of arrivals waiting for the service is given in figure (5).

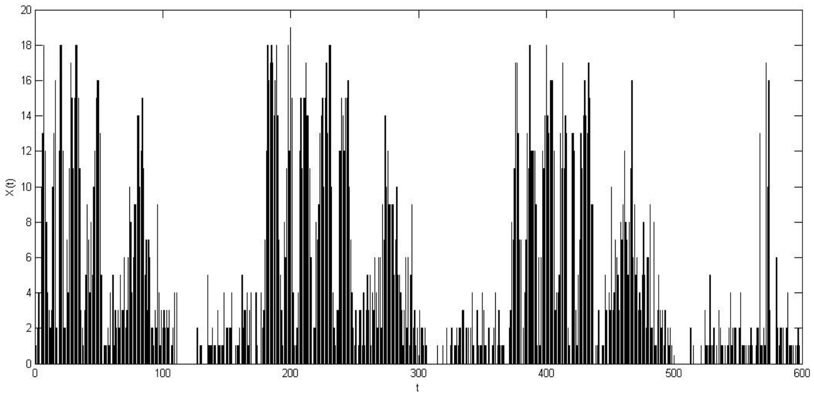


Figure 5 - Real data

The data set can be fitted as DAR(1) with marginal as discrete Mittag Leffler $\alpha = 1$ distribution as follows. To test whether there is a significant difference between an observed data and the DAR(1) with marginal as discrete Mittag Leffler $\alpha = 1$, use Kolmogorov-Smirnov [K.S.] test for H_0: DAR(1) with marginal discrete Mittag Leffler $\alpha = 1$ distribution with parameter $\hat{p} = 0.1756$ and $\hat{q} = 0.8244$ and $\beta = 0.8$ is a good fit for the given data. Here the calculated value of the K.S. test statistic is 0.0633 and the critical value corresponding to the significance level 0.01 is 0.0665, showing that the assumption for number of arrivals follow DAR(1) with marginal discrete Mittag Leffler $\alpha = 1$ distribution is valid which is given in figure (6)

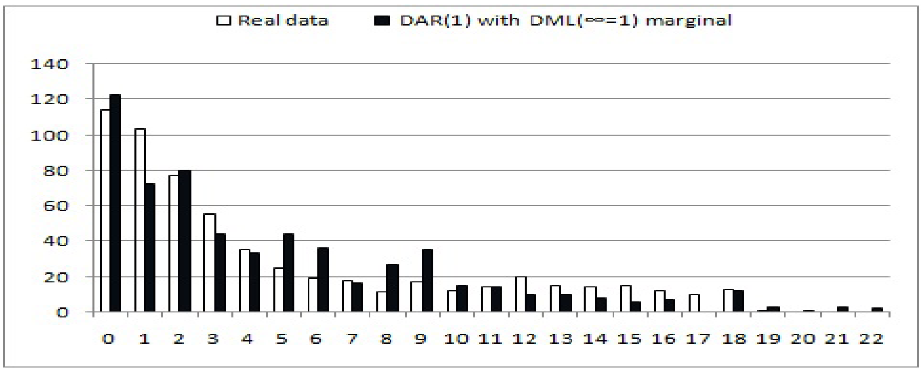


Figure 6 - The Probability histogram of Real data and DAR(1) with marginal Discrete Mittag Leffler $\alpha = 1$ distribution with $\hat{p} = 0.1756$ and $\hat{q} = 0.8244$, $\beta = 0.8$

By applying matrix analytic method obtain the stationary distribution of system size and waiting time of an arbitrary customer for the DAR(1)/D/s queue with marginal as discrete Mittag Leffler $\alpha = 1$ distribution. Here the mean = $\lambda = 4.855$. To satisfy the stability condition assume the number of servers as $s = 5$. i.e $\lambda / s = 0.971 < 1$ Also assume the value of autocorrelation function $\beta = 0.8$, $\hat{p} = 0.1756$ and $\hat{q} = 0.8244$. Table(5) and (6) display the stationary distribution of waiting time of an arbitrary customer and system size.

TABLE 5

p_{nj} *in real data when* $\beta = 0.8$, $\lambda = 4.855$, *s=5.*

	0	1	2	3	4	5	6	7
0	0.16506	0.1486	0.11094	0.07893	0.06589	0.02652	0.00403	0.00382
1	0.00065	0.00047	0.00045	0.00053	0.00749	0.00055	0.00008	0.00008
2	0.01017	0.00916	0.00912	0.00903	0.01476	0.01096	0.00167	0.00158
3	0.00019	0.0002	0.00014	0.00012	0.05716	0.00006	0.00001	0.00001
4	0.00019	0.0002	0.00014	0.00012	0.05716	0.00006	0.00001	0.00001
5	0.00004	0.00004	0.00012	0.00012	0.00015	0.00004	0.00001	0.00382
6	0.00018	0.00016	0.00016	0.00023	0.00025	0.0002	0.00003	0.00003
7	0.00021	0.00019	0.00019	0.00019	0.00023	0.00023	0.00003	0.00003
8	0.00016	0.00014	0.00014	0.00014	0.00014	0.00017	0.00003	0.00002
9	0.00016	0.00014	0.00014	0.00014	0.00014	0.00017	0.00003	0.00002
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

TABLE 6

$P(W = \omega)$ in real data when $\beta = 0.8$, $\lambda = 4.855$, s=5.

w	p(w)
0	0.3602
1	0.1685
2	0.0452
3	0.0214
⋮	⋮

7. CONCLUSIONS

A DAR(1)/D/s queue with discrete Mittag-Leffler distribution as the marginal distribution is analyzed in this chapter. ATM multiplexer with VBR coded teleconference traffic is taken as the input traffic. DAR(1) model is a good mathematical model for a multiserver ATM multiplexer with VBR coded teleconference traffic. The stationary distributions of the system size and the waiting time of an arbitrary packet are obtained with the help of the matrix analytic methods and the Markov regenerative theory. From the definition of autocorrelation function, the larger the parameter β , the slower the decay of the autocorrelation of the input process. So it is expected that stationary system size and waiting time for the case of large β are stochastically larger than those for the case of small β. Also the stationary system size and waiting time for the case that the stationary distribution of the input process has a large variance are stochastically larger than those for the case that the stationary distribution of the input process has a small variance.

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SUMMARY

Analysis of queueing system with discrete autoregressive arrivals having DML(α) as marginal distribution

In this paper we analyze DAR(1)/D/s Queue with Discrete Mittag-Leffler [DML(α)] as marginal distribution. Simulation study of the sample path of the arrival process is conducted. For this queueing system, the stationary distribution of the system size and the waiting time distribution of an arbitrary packet is obtained with the help of matrix analytic methods and Markov regenerative theory. The quantitative effect of the stationary distribution on system size, waiting time and the autocorrelation function as well as the parameters of the input traffic is illustrated empirically. The model is applied to a real data on the passenger arrivals at a subway bus terminal in Santiago de Chile and is established that the model well suits this data.