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TESTING FOR SEASONAL FRACTIONAL INTEGRATION IN QUARTERLY TIME SERIES

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1. INTRODUCTION

In recent years, much attention has been given to the study of fractionally integrated or long-memory processes[1], which is a characteristic of time series that exhibits strong dependency between distant observations.

The discovery of the so-called Hurst effect (Hurst (1951)) initiated the development of stochastic models with this property.

Consequently, Granger (1980, 1981), Granger-Joyeux (1980), Diebold-Rudebusch (1989) and Hosking (1981) studied separately the autoregressive fractionally integrated moving average (ARFIMA) process, given as $\phi(B)(1-B)^d Y_t = \theta(B)e_t$, with d as the integration parameter. When d lies in the range 0.1 to 0.5, this process is known to possess the long-memory property, which is found to be useful in modelling and forecasting time series (see Baillie (1996) for a recent survey of long-memory processes).

This survey also reviews the growing number of applications in a variety of fields. It has been found in the literature that many climatological time series have the tendency for large values to be followed by large values of the same sign (Mandelbrot and Wallis, 1968, 1969).

This property, persistent statistical dependence, does not appear to result from ordinary serial dependence, but rather from a special kind of dependence with an infinite memory called `non-cyclic long-run statistical dependence'.

The series, which are long-term-dependent, exhibit `trends' and `cycles' of varying lengths (Mandelbrot and Wallis, 1969, Mandelbrot, 1972).

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[1] There are several definitions of the property of long memory in the literature.

According to McLeod-Hipel (1978), a time series (say) y_t, is said to possess the long memory property if $\sum_{k=0}^{\infty}|\rho_k|$ is infinite, where k is the autocorrelation function at lag k or, alternatively, the spectral density of y_t (say) f(ω) is unbounded at low frequencies. For other definitions of long memory, see Baillie (1996).

Many economic time series have been found to possess long memory, which is not surprising, since many economic variables (e.g., agricultural production and commodity prices) are related directly to climatological variables (see Mandelbrot, 1970). Most applied work using the above-mentioned techniques depends on the asymptotic results to make small-sample inferences.

Therefore, it is very important that the tests used to detect fractional unit roots at different frequencies in a time series model have correct sizes and good powers, particularly in finite samples, because failure to reject the null hypothesis when it is false may lead to unsubstantiated claims regarding the stochastic behaviour of the time series.

Cheung (1993) examined the finite sample behaviour of the fractional integration tests such as the GPH test, the modified rescaled range (MRR) test developed by Lo (1991) and the two LM tests proposed by Robinson (1991), however they were limited to the zero frequency. His results show that both the GPH and MRR tests perform better than Robinson's LM tests.

Since the long-run characteristics of a process can be captured by the fractional differencing parameter *d*, its testing is necessary for modelling the time series. A number of procedures have been suggested in the literature for testing the parameter d at zero and seasonal frequencies (see, for example, Porter-Hudak, 1990, Hassler, 1994, Robinson, 1994, Silvapulle, 1999, 2001 and others).

Porter-Hudak extended the non-seasonal estimation and testing procedure of d, developed by Geweke and Porter-Hudak (1983) (GPH), to the seasonal case, but only at the zero frequency. Hassler used the GPH technique to estimate and test the fractional parameters at zero and seasonal frequencies in a rigid and/or flexible time series model. On the other hand, Robinson proposed a frequency domain score statistic for testing fractional integration at seasonal frequencies. Furthermore, based on ideas presented by Robinson (1994), Silvapulle more recently derived time domain score statistics for testing fractional integration at zero and seasonal frequencies.

The score test is computationally attractive, particularly for the current testing problem and is likely to be preferred by applied researchers unfamiliar with the frequency domain approach.

The score-type tests statistics are very popular for two reasons. Firstly, only the model under the null hypothesis needs to be estimated, which is considerably simpler than the model under the alternative hypothesis. Other tests, such as likelihood ratio and Wald-type, are generally difficult to apply because the full model under the alternative hypothesis needs to be estimated.

Secondly, the score-type tests are asymptotically equivalent to the likelihood ratio test. In this paper, we assess and compare the finite sample behaviour of Hassler's extension of the GPH semi-parametric test, Robinson's frequency domain score test and Silvapulle's time domain test for testing hypotheses in the following cases: (i) H_0:: I(0) process against H_1: fractional integration at different frequencies and (ii) the same hypotheses in (i), but with AR(1) present under both hypotheses.

This paper is organised as follows: (i) Section briefly discusses the models and the tests; (ii) Section 3 presents the experimental design; (iii) Section 4 reports the results and (iv) the final section provides concluding remarks.

2. MODELS AND TEST PROCEDURES

In this section, we present a brief description of the data-generating processes and procedures developed for testing fractional integration at zero and seasonal frequencies in quarterly time

2.1 *Models: definitions and properties*

Consider the following Auto-Regressive Seasonally Fractionally Integrated Moving average (ARSFIMA) model: $\phi(B)(1-B)^{d_0}(1+B)^{d_1}(1+B^2)^{d_2}y_t = \theta(B)e_t$, $t = 1,\ldots\ldots n$, where $\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ is a *p-th* order AR polynomial, $\theta(B) = 1 + \theta_1 B + \ldots + \theta_q B^q$ is a q th order MA polynomial, B is the lag operator defined as $By_t = y_{t-1}$, d_0, d_1 and d_2 are unknown fractionally integrated parameters at zero and seasonal frequencies $\pi/2$ and π respectively and e_t is a white noise process. For $d_0 = d_1 = d_2$, (Hassler 1994) argued that the type of filter $(1-B^4)^d$ is fairly rigid in that the contributions of half-yearly and yearly oscillations and long-run behaviour are all governed by one common d.

He argued further that the importance of fractional integration at zero and seasonal frequencies can be separated by means of a flexible filter $(1-B)^{d_0}(1+B)^{d_1}(1+B^2)^{d_2}$, where d_0, d_1 and d_2 may not be integers.

This class of model is useful, as it is a natural generalisation of seasonally integrated models introduced by Hylleberg et al. (1990).

The memory property of (1) depends on the value of d_i, $i = 0, 1, 2$.

The process y_t is both stationary and invertible if all roots of $\phi(B)$ and $\theta(B)$ areoutside the unit circle and $d_i \in (-1/2,1/2)$ for all i. Model (1) includes the seasonal ARIMA model as a special case where $d_i = 1$ for all i ; this model reduces to a standard ARMA model for $d_i = 0$ for all i. When $d_i = 0$ for all i, the autocorrelations of y_t decay geometrically at a rate proportional to k.

Thus, a stationary ARMA model has a short memory, because the dependence between observations k periods apart decays rapidly as k increases.

For $d_i \in (0, 1/2)$ for all i , y_t is still stationary, however the autocorrelations of y_t show a hyperbolic decay at a rate proportional to k^{2d-1}, in contrast to a faster geometric decay of a stationary ARMA process (see Hosking, 1981).

Due to the presence of such significant dependence between distant observations, the ARSFIMA process is often called a long-memory process, providing superior long-run forecasts to the ARIMA models.

The long-memory behaviour of y_t in (1) can be seen also from its spectral density, say, $f_y(\lambda_i)$. For $d_i > 0$ for all i, $f_y(\lambda_i)$ is unbounded at frequency $\lambda_i = 0$, rather than bounded as for a stationary ARMA process. When $d_i \in (1/2)$,1) for all i, the process y_t is covariance-non-stationary, because its variance is not finite (see Hosking, 1981).

Nonetheless, the process is mean-reverting, since an innovation has no permanent effect on the value of y_t. This is in contrast to an $I(1)$ process, which is both covariance-non-stationary and mean-averting.

The effect of an innovation on an $I(1)$ process is persistent forever (Cheung and Lai, 1993).

2.2 *Tests*

In this study, we consider the following procedures for testing fractional integration at zero and seasonal frequencies in quarterly time series models: (a) the generalised GPH tests proposed by Hassler (1994); (b) the frequency domain score test developed by Robinson (1994) and (c) the time domain score test derived by Silvapulle (1995).

Following is a brief description of the above tests:

(a) The generalised GPH test Hassler (1994) generalised the GPH procedure to test the fractional parameter at zero and seasonal frequencies, as in Hylleberg *et al.* (1990), who extended the procedures for testing unit roots at the zero frequency to those at the zero and/or seasonal frequencies.

To describe the testing procedure, let us consider the following regression models:

$$I_{kj} = c_k + d_k R_j + u_{kj},\ k = 0, 1, 2 \text{ and } j = 1,\dots, T,$$

where $I_{kj} = \ln\{I_y(\lambda_{(kj)})\}$, $I_y(\lambda_{kj}) = n^{-1}\left|\sum_{t=1}^{n} y_t \exp\{k\lambda_{kj}t\}\right|^2$ is the estimated spectral periodogram of y^t at frequency λ_{kj} , d_k are the fractionally integrated parameters at 0 and seasonal frequencies $\pi/2$ and π respectively (belonging to the sets of harmonic seasonal frequencies λ_{kj}), $\lambda_{0j} = 2\pi j/n$, $\lambda_{1j} = (\pi/2) + 2\pi j/n$, $\lambda_{2j} = \pi - 2\pi j/n$, $R_j = -\ln\{4\sin^2(n^{-1}\pi j)\}$, $T = n\mu$ is the number of low frequency periodogram ordinates with $\mu = 0.60$, 0.50 and 0.40 considered in the simulation study conducted in Section \ref{sec-3} and n is the sample size.

To test H_0: $d_k = 0$ against H_1: $d_k \# 0$, $k = 0, 1, 2$ in (2), the test statistics are defined as $\dfrac{\hat{d}_k}{\sqrt{V(\hat{d}_k)}}$, where $\hat{d}_k$ are the OLS estimators of d_k and $V(\hat{d}_k)$ are the estimated variance of $\hat{d}_k$, defined as $\pi^2/6S_R^2$ for 0 and π frequencies and $\dfrac{\pi^2}{12S_R^2}$ for $\pi/2$ frequency and $S_R^2 = \sum_{j=1}^{n}(R_j - \overline{R})^2$.

The asymptotic distribution of the above statistics is the standardised normal distribution.

b) *The frequency domain score test.* Robinson (1994) proposed the frequency domain score statistic for testing fractional integration at zero and seasonal frequencies.

In what follows, the testing procedure is described.

Consider model (1), where $y_t = 0$, $t \leq 0$; y_t may be an observable time series or the unobservable error term in the regression $x_t = \beta' z_t + y_t$, $t = 1, 2, \ldots, n$, z_t is a $k \times 1$ vector of stochastic or non-stochastic variables and β is a $k \times 1$ vector of unknown parameters. To test H_0: $d = 0$ against H_1: $d \neq 0$, where $d = (d_0, d_1, d_2)$ and $0 = (0, 0, 0)$in (1), the score statistics is defined as $\tilde{R} = \tilde{r}'\tilde{r}$, where $\tilde{r} = (n^{1/2}/\tilde{\sigma}^2)\tilde{A}^{-1/2}\tilde{a}, \tilde{a} = -(2\pi/n) \sum_j' \psi(\lambda_j) I_{\tilde{e}}(\lambda_j)$, $I_{\tilde{e}}(\lambda_j) = (2\pi n)^{-1} |\sum_{t=1}^{n} \tilde{e}_t e^{it\lambda_j}|^2$, $\tilde{A} = \frac{2}{n}\sum_j' \psi(\lambda_j)\psi(\lambda_j)'$, the primed sums are over $\lambda_j \in M, M = \{\lambda : -\pi \lhd \lambda \lhd \pi, \lambda \notin (\rho_\ell - 1, \rho_\ell + 1), \ell = 1, \ldots, s\}$, such that $\rho_\ell, \ell = 1, \ldots, s \lhd \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$, $\sigma^2 = \frac{1}{n}\sum_{t=1}^{n} \tilde{e}_t^2, \psi(\lambda_j) = \left[\log\left|2\sin\frac{1}{2}\lambda_j\right|, \log(2\cos\frac{1}{2}\lambda_j), \log\left|2\cos\lambda_j\right| \right]', \lambda_j = 2\pi j/n$ and $\tilde{e}_t$ are the OLS errors in (1).

The asymptotic distribution of the statistic tilde{R} has a chi-square distribution with three degrees of freedom.

c) *The time domain score test.* Recently, Silvapulle (1995) derived a time domain score statistic for testing fractional integration at zero and seasonal frequencies in a quarterly time series model.

To describe the testing procedure, consider the ARSFIMA (*p,d,q*) model (1), where y_t is an observable time series or the unobservable error term in the regression $x_t = \beta' z_t + y_t, t = 1, \ldots, n, z_t$ is a k $\times$ 1 vector of stochastic or non-stochastic variables and β is a k $\times$ 1 vector of unknown parameters.

Let us suppose $\gamma = (d, \eta)$, where $d = (d_0, d_1, d_2)$ and $\eta = (\pi, \theta, \beta, \sigma_2)$.

Then, to test H_0:0 against *H1:d* > 0 at 0 and seasonal frequencies $\pi/2$ and π, the score statistic is defined as $T_0 = T^{-1} s_{d}^{'} \ell_{dd}^{-1} s_d$, where $T = (n - 2m)$, n is the number of observations, m is the number of terms chosen from the expansion of log(1+B), selected as 5, 10 and 15 in the simulation study, the score vector s_d is the slope of the log likelihood function L(γ) for n observations, defined as

$$s_d = \left(\frac{\partial L(\gamma)}{\partial d_0}, \frac{\partial L(\gamma)}{\partial d_1}, \frac{\partial L(\gamma)}{\partial d_2} \right),$$

$$\frac{\partial L(\gamma)}{\partial d_0} = (1/\hat{\sigma}^2)\sum_{t=1}^{n} \hat{e}_t \sum_{j=1}^{m} j^{-1}\hat{e}_{t-j}$$

$$\frac{\partial L(\gamma)}{\partial d_1} = -(1/\hat{\sigma}^2)\sum_{t=1}^{n} \hat{e}_t \sum_{j=1}^{m} j^{-1}(-1)^{j-1}\hat{e}_{t-j}$$

$$\frac{\partial(\gamma)}{\partial d_2} = -(1/\hat{\sigma}^2)\sum_{t=1}^{n} \hat{e}_t \sum_{j=1}^{m} j^{-1}(-1)^{j-1}\hat{e}_{t-2j}, \hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t^2 ,$$

$hate_t$ are the OLS errors in (1) under H_0, the information matrix of s_d under H_0 is defined as $-E_{H_0}\left(\frac{{}^2 L(\gamma)}{\partial \gamma^2} \right) = \begin{bmatrix} \iota_{dd}\iota_{d\eta} \\ \iota_{\eta d}\iota_{\eta\eta} \end{bmatrix}$,

and the lower triangular part of ι_{dd} is given as

$$\begin{bmatrix} \sum_{j=1}^{m} j^{-2} & & \\ \sum_{j=1}^{m} (-1)^{j-1} j^{-2} & \sum_{j=1}^{m} j^{-2} & \\ \sum_{j=1}^{\left[\frac{m}{2}\right]} (-1)^{j-1}(2j^2)^{-1} & \sum_{j=1}^{\left[\frac{m}{2}\right]} (-1)^{j}(2j^2)^{-1} & \sum_{j=1}^{m} j^{-2} \end{bmatrix}$$

where the elements of d are all zero when the error term is white noise.

The limiting distribution of the statistic T_0 has a chi-square distribution with 3 degrees of freedom; see Silvapulle[2] (2001) for the expressions of d and the corresponding test statistic when the error term follows a stationary ARMA process.

3. EXPERIMENTAL DESIGN

A Monte Carlo simulation experiment was conducted to assess the finite sample behaviour of the fractional integration tests considered in this paper for testing the null of stationary short-memory process against seasonal long-memory alternatives (i.e. H_0: $d_i = 0$ against H_1: $d_i > 0$, i = 0, 1 and 2) in (1) under various conditions.

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[2] The OPG-LM tests proposed by Silvapulle (1999) performed as well as the score test of Silvapulle (2001) when the error term is white noise, and under performed when the errors are correlated. The results are shown here for space reasons.

In this study, we considered sample sizes *n=52*, *n=100* and *n=252*. The simulation experiments were based on 2000 replications for calculations of rejection rates of statistics under the null and alternative hypotheses.

These rejection rates were estimated at the 5 per cent nominal level. For all computations GAUSS programming software was used.

The experiment was conducted in the following stages:

3.1 *Experiment 1*

The stationary process $y_t = e_t, e_t$ sim N(0,1) is generated under H_0 for testing H_0: $d_1 = 0$ against H_1: $d_i > 0$, i = 0, 1 and 2.

Further, the seasonally fractionally integrated (SFI) process $(1-B)^{d_0}(1+B)^{d_1}(1+B^2)^{d_2} y_t$ is generated for d_i = 0.1, 0.2, 0.3 and 0.4 for i = 0, 1 and 2, under H_1.

When $d_0 = d_1 = d_2$, the expression of autocovariance function of the SFI of process y_t can be defined as:

$$\text{cov } (y_t, y_{t-k}) = \gamma_{4k} = \Gamma(1-2d)\Gamma(d+k)/\Gamma(d)\Gamma(1-d)\Gamma(1-d+k)\sigma_e^2 .$$

First, the desired T $\times$T covariance matrix Σ is constructed, then the rigid SFI process y_t generated as $y_t = Pe_t$, where P is the lower triangular Choleski decomposition of Σ. When not all d's are equal, the flexible seasonal filter is constructed using the following formulae:

i. $$(1-B)^{d_0}(1+B)^{d_1}(1+B^2)^{d_2} = \sum_{j=0}^{\infty}\delta_j B^j \approx \sum_{j=0}^{p}\delta_j B^j$$

where the coefficients $\delta_j = \left\{ \begin{array}{l} \sum_{k=0}^{m} c_{m-k}\beta_{2k}, j = 2m \\ \sum_{k=0}^{m} c_{m-k}\beta_{2k+1}, j = 2m+1, m = 0,1,2,.. \end{array} \right\}$

are defined by the convolution of

ii. $(1-B)^{d_0} = \sum_{j=0}^{\infty} a_j B^j$ with $a_0 = 1$ and $a_j = \dfrac{j-1-d_0}{j} a_{j-1}$

iii. $(1+B)^{d_1} = \sum_{j=0}^{\infty} b_j B^j$ with $b_0 = 1$ and $b_j = \dfrac{d_1 - j + 1}{j} b_{j-1}$

iv. $(1+B^2)^{d_2} = \sum_{j=0}^{\infty} c_j B^j$ with $c_0 = 1$ and $c_j = \dfrac{d_2 - j + 1}{j} c_{j-1}$

v. $(1-B)^{d_0}(1+B)^{d_1} = \sum_{j=0}^{\infty} \beta_j B_j$ with $\beta_j = \sum_{k=0}^{j} a_{j-k} b_k$.

The flexible SFI process y_t is generated as $y_t = e_t^2 / \sum_{j=0}^{p} \delta_j e_{t-j}, t = 1,\ldots,n$ where e_t and δ_t are defined as above.

The rejection rates of fractional integration tests under the null and alternative hypotheses (i.e, estimated sizes and power of the tests respectively) for testing H_0: d_i = 0 against H_1: d_i > 0 are reported in Tables 3 and 4 respectively.

3.2 *Experiment 2*

The finite sample behaviour of the tests is explored when the data are generated as an ARFISMA(1,*d*,0) process under the null H_0: d_i = 0 and alternative H_1: d_i > 0 hypotheses.

Under H_0, the stationary AR(1) process $(1-\phi B)y_t = e_t$, where e_t sim $N(0,1)$, is generated for values of ϕ = 0.2, 0.5 and 0.7, with initial observations adjusted to ensure stationarity. Under H_1, when $d_0 = d_1 = d_2$, the covariance matrix Σ of the SFI process is constructed as in Experiment 1.

Then the rigid ARSFIMA(1,d,0) process y_t is generated as $y_t = Pe_t$, where P is the lower triangular Choleski decomposition of Σ_1, which is the covariance matrix of the AR(1) process.

When $d_0 \neq d_1 \neq d_2$, the flexible seasonal filter is generated first using formulae (i) to (v) as in Experiment 1, then the flexible ARFISMA(1,*d*,0) process y_t is generated as $y_t = \sum_1 \left[e_t^2 / \sum_{j=0}^{p} \delta_j e_{t-j} \right], t = 1,2,3,\ldots,n$, where Σ_1, e_t and δ_t are defined as above. The estimated rejection rates of the tests under the null and alternative hypotheses are reported in Tables 3 and 4 respectively.

Since the rejection rates of the tests under the null are very high in many situations, we estimate the 5 per cent critical values, which in turn are used in the power calculations.

4. RESULTS

In this section, we discuss the results of the estimated sizes and powers of the seasonal fractional integration tests, namely, the generalized GPH tests at 0, $\pi/2$ and π frequencies, denoted by $t(\hat{d}_0)$, $t(\hat{d}_1)$ and $t(\hat{d}_2)$ respectively, the frequency domain score test denoted by $\tilde{R}$ and the time domain score test denoted by T_0 for testing H_0: d_i = 0 against H_1: d_i > 0, i = 0, 1 and 2 in (1) at the 5 per cent nominal level under various conditions discussed in the previous section.

Table 1 contains the empirical rejection rates of the tests under the null hypothesis, computed using the 5% asymptotic x_3^2 critical value when the data were generated from the ARSFIMA(0,0,0) process under the null hypothesis.

TABLE 1

Estimated rejection rates of the seasonal fractional integration tests under the null for testing $H_o: d_i = 0$ against $H_1: d_i > 0$, $i = 0, 1, 2$ in ARSFIMA (0,d,0) at the 5 per cent level of significance

n	$t(\hat{d}_0)$ μ 0.60 0.50 0.40	$t(\hat{d}_1)$ μ 0.60 0.50 0.40	$t(\hat{d}_2)$ μ 0.60 0.50 0.40	$\tilde{R}$	T_0 m 5 10 15
52	.04 .05 .04	.11 .11 .10	.04 .05 .04	.10	.06 .06 .09
100	.05 .04 .04	(.05) (.05) (.05)	.04 .04 .04	.15	.05 .06 .06
252	.04 .04 .04	.12 .11 .11	.04 .04 .05	.18	.04 .05 .04
		(.04) (.05) (.06)			
		.11 .10 .10			
		(.04) (.04) (.04)			

Notes:

(a) $t(dhat_0), t(dhat_1)$ and $t(dhat_2)$ are the generalised GPH statistics at the frequencies 0, $\pi/2$ and π respectively, proposed by Hassler (1994); $\tilde{R}$ is the frequency domain score statistic developed by Robinson (1994); T_0 is the time domain score statistic proposed by Silvapulle (2001).

(b) n is the sample size, μ is a constant used to compute the number of low frequency periodogram ordinates and m is the number of terms chosen from the expression of log(1+B). Values in parenthesis are the nominal size of the Hassler statistic at frequency $\pi/2$ when the error variance is assumed to be $\pi^2/6$ instead of $\pi^2/12$.

It is clear from Table 1 that, for all sample sizes, the tests had rejection rates close or equal to 5 per cent, except for the $\tilde{R}$ test and the $t(\hat{d}_1)$ when the variance was assumed to be $\pi^2/12$.

The rejection rates of the $\tilde{R}$ test had the tendency to increase as the sample size increased.

The finite sample behaviour of the seasonal fractional integration tests, used for testing H_0: $d_i = 0$ against H_1: $d_i > 0$, i = 0, 1 and 2 in (1), were also explored when the data were generated from the ARSFIMA(1,d,0) process; the estimated rejection rates are presented in Table 2.

It is clear from these results that the rejection rates of the generalised GPH tests at all frequencies were approximately similar to those obtained in Experiment 1.

The $\tilde{R}$ test suffers from changes in the size of the samples, but above all its rejection rates increase with increasing ϕ.

On the other hand, the T_0 test had smaller rejection rates than the nominal level when compared to those observed in the Experiment 1.

TABLE 2

Estimated rejection rates of the seasonal fractional integration tests under the null for testing $H_o: d_i = 0$ against $H_1: d_i > 0$, $i = 0, 1$ and 2 in ARSFIMA(1,d,0) at the 5 per cent level of significance

N	ϕ	$t(\hat{d}_0)$ μ =0.50	$t(\hat{d}_1)$ μ =0.50	$t(\hat{d}_2)$ μ =0.50	$\tilde{R}$	T_0 m = 10
52	0.2	.04	.11(.05)	.04	.07	.05
	0.5	.03	.10(.05)	.04	.12	.03
	0.7	.03	.11(.06)	.03	.15	.03
100	0.2	.03	.12(.06)	.04	.05	.03
	0.5	.04	.10(.05)	.04	.09	.02
	0.7	.03		.04	.13	.02
252	0.2	.04	.11(.06)	.04	.08	.02
	0.5	.03	.11(.05)	.04	.12	.03
	0.7	.03	.11(.05)	.04	.15	.02

See footnotes (a) and (b) for Table 1. ϕ is the autoregressive parameter

Since the tests had varying rejection rates under the null hypotheses, power comparison using the asymptotic critical values was not informative.

We therefore estimated the 5 per cent critical values under the null for all cases considered in this study, and these values were used in subsequent power calculations.

The estimated powers of the fractional integration tests at different frequencies when the data were generated from the rigid and flexible ARSFIMA (0,*d*,0) processes are reported in Table 3.

TABLE 3

Estimated powers of the seasonal fractional integration tests for testing $H_0: d_i = 0$ against $H_1: d_i > 0$, $i = 0, 1$ and 2 in ARSFIMA (0,d,0) at the 5 per cent level of significance

(d_0,d_1,d_2)	N	$t(\hat{d}_0)$ μ =0.50	$t(\hat{d}_1)$ μ =0.50	$t(\hat{d}_2)$ μ =0.50	$\tilde{R}$	T_0 m=10
(.1,.1,.1)	52	.07	.12(.06)	.08	.12	.10
	100	.06	.13(.07)	.07	.18	.16
(.1,.2,.3)	52	.08	.14(.07)	.09	.17	.15
	100	.09	.13(.08)	.09	.22	.21
(.2,.2,.2)	52	.06	.13(.08)	.09	.29	.32
	100	.07	:18(.08)	.08	.38	.41
(.3,.3,.3)	52	.07	.18(.09)	.08	.53	.54
	100	.11	.25(.16)	.12	.62	.65
(.4,.3,.2)	52	.08	.12(.09)	.06	.62	.61
	100	.06	.11(.07)	.08	.75	.81
(.4,.4,.4)	52	.12	.21(.09)	.09	.78	.82
	100	.17	.36(.20)	.21	.89	.95

See footnotes (a) and (b) for Table 1

There is no notable changes in the powers of the $t(\hat{d}_i)$ test across all d_i, i = 0, 1 and 2 and/or the sample size except when all $d_i s$ are 0.4 and the sample size is 100.

Of three tests $t(\hat{d}_1)$ dominates the other two counterparts.

The powers of the $\tilde{R}$ and T_0 are much higher than the *t(.)* tests, particularly when the values of *d's* and the sample size are large.

Comparing the performances of the $\tilde{R}$ and T_0 tests, we can say that $\tilde{R}$ dominates T_0 in terms of power for small values of $d_i s$, and, clearly, the reverse is true for large values of $d_i s$.

Table 4 reports the estimated powers of the tests when the data were generated from the flexible ARSFIMA (1,*d*,0) process for varying values of *d*.

TABLE 4

Estimated powers of the seasonal fractional integration tests for testing $H_0: d_i = 0$ against $H_1: d_i > 0$, i = 0, 1 and 2 in ARSFIMA (1,d,0) at the 5 per cent level of significance

ϕ	(d_0, d_1, d_2)	N	$t(\hat{d}_0)$ μ =0.50	$t(\hat{d}_1)$ μ =0.50	$t(\hat{d}_2)$ μ =0.50	$\tilde{R}$	T_0 m=0
0.2	(.1,.1,.1)	52	.08	.09(.07)	.06	.21	.15
		100	.07	.11(.06)	.07	.32	.28
	(.1,.2,.3)	52	.06	.15(.08)	.07	.25	.23
		100	.07	.15(.08)	.06	.40	.39
	(.2,.3,.4)	52	.06	.14(.09)	.08	.36	.38
		100	.09	.20(.15)	.12	.52	.61
	(.4,.4,.4)	52	.09	.20(.17)	.13	.70	.82
		100	.16	.31(.25)	.17	.89	.95
0.7	(.1,.1,.1)	52	.10	.07(.06)	.06	.27	.29
		100	.09	.12(.10)	.06	.43	.52
	(.1,.2,.3)	52	.08	.17(.09)	.06	.45	.48
		100	.07	.16(.06)	.08	.89	.99
	(.2,.3,.4)	52	.14	.14(.11)	.10	.61	.70
		100	.13	.17(.13)	.12	.88	.97
	(.4,.4,.4)	52	.13	.19(.12)	.11	.83	.89
		100	.18	.26(.19)	.20	.93	1.0

See footnotes (a) and (b) for Table 1, and footnote (a) for Table 2

The results show that, in the case of the rigid ARSFIMA (1,*d*,0) process, all tests had good power properties, except for the generalised GPH tests $t(\hat{d}_i)$ test across all *i = 0, 1* and *2*.

Being similar to the results reported in Table 3, $t(\hat{d}_0)$ and $t(\hat{d}_2)$ were found to have low powers in comparison with the $t(\hat{d}_1)$ test.

The estimated powers of the $\tilde{R}$ and T_0 tests are much higher than those of the t(.) tests for all values of d_i, and these powers increase as the values of ϕ and the sample size increase.

Further, we note that the power of $\tilde{R}$ test is higher than those of the T_0 test for small values of ϕ and d_i regardless of sample sizes, and the reverse is true for large values of and d_i.

5. CONCLUSION

In this paper, we examine the finite sample behaviour of the seasonal fractional integration tests for testing the following hypotheses: (i) H_0 : *I(0)* process against H_1 : fractional integration at different frequencies and (ii) the same hypotheses in (i) but with AR(1) present under both hypotheses in quarterly time series models.

We consider the generalised version of the GPH test proposed by Hassler (1994), the frequency domain score test developed by Robinson (1994) and the time domain score test derived by Silvapulle (2001).

Evidence from a Monte Carlo simulation study indicates that all tests have desirable finite sample value properties, except for the frequency domain score test.

The values of the latter can be much higher than the nominal level in large samples.

On the other hand, both the frequency and time domain score tests have better powers against the fractionally integrated processes than the generalised GPH tests.

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RIASSUNTO

Test per valutare le proprietà di serie storiche trimestrali

Numerose serie storiche, come quelle riguardanti i prezzi dei prodotti agricoli, nonché le serie economiche e finanziarie, presentano una forte dipendenza a lunga memoria. Poiché molte di esse presentano anche regolarità stagionali, vengono presi in considerazione alcuni test statistici, e precisamente l'estensione di Hassler al test semi-parametrico GPH di Geweke e Porter-Hudak (1983), il test *frequency domain score* di Robinson e il test *domain time* di Silvapulle, per valutare le proprietà di lunga memoria delle serie storiche trimestrali al tempo iniziale e le frequenze stagionali. Le proprietà statistiche finite di questi test sono poco conosciute. In uno studio di simulazione, si trova che il test *domain time* è più potente degli altri. Valutare l'affidabilità di questi test nel finito è utile nella ricerca applicata.

SUMMARY

Testing for seasonal fractional integration in quarterly time series

Many series, such as agricultural commodity prices and economic and financial series, exhibit strong dependence--long memory property.

Since many time series also exhibit seasonal patterns, this paper considers a number of tests - namely Hassler's extension of Geweke and Porter-Hudak's (1983) (GPH) semi-parametric test, Robinson's frequency domain score test and Silvapulle's time domain test - to assess the long memory properties of quarterly time series at zero and seasonal frequencies. Very little is known about the finite sample statistical properties of these tests. In a simulation study, we find that time domain and semi-parametric tests generally have the rejection rates under the null hypothesis close to the nominal level, with the latter tests' rejection rates higher than the nominal level at the semi-annual frequency. In terms of power, the time domain score test was shown to be superior with respect to the others. Establishing the reliability of these tests in finite samples is very useful to applied researchers.