ON TESTING EXPONENTIALITY AGAINST NBARFR LIFE DISTRIBUTIONS

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1. INTRODUCTION AND DEFINITIONS

Testing exponentiality against various classes of life distributions has got a good deal of attention. With respect to testing against IFR, see Proschan and Pyke (1967), Barlow (1968), and Ahmed (1975) among others. For testing against IFRA, see Deshpande (1983), Linmk (1989), Aly (1989), and Ahmed (1994). For testing against NBU, see Hollander and Proschan (1972), Koul (1977), Kumazawa (1983) and Ahmed (1994). For testing against NBUE, NBUFR and NBAFR classes, we refer to Klefsjo (1981, 1982), Deshpande *et al.* (1986), Aboammoh and Ahmed (1988), Loh (1984) and Hendi *et al.* (2000). Recently Mahmoud and Abdul Alim (2002) studied testing exponentiality against NBURFR based on a U-statistic for censored and noncensored data.

Now let T be a non negative random variable with life distribution $F(t)$, where $F(t)=0$ for t<0 and $F(0)$ may not be zero. The corresponding survival function of new system $\overline{F}(t)$, for t>0 and the density function is given by $f(t)$. The failure rate at time t is defined by $r_F(t)=f(t)/\overline{F}(t),\ t\geq 0.$ In the long run, if a device is replaced by sequence of mutually and identically distributed, the remaining life distribution of the system under operation at time t is given by stationary renewal distribution as follows:

$$W_F(t)=\mu_F^{-1}\int_0^t \overline{F}(u)du\ ,\ 0\leq t<\infty\ ,$$

where μ_F is the mean life of the random variable T, $\mu_F=\int_0^\infty \overline{F}(u)du<\infty$.

The corresponding renewal survival function is given by

$$\overline{W}_F(t)=\mu_F^{-1}\int_t^\infty \overline{F}(u)du\ .$$

The density function of the renewal distribution $W_F(t)$ is given by

$$w_F(t)=\frac{d}{dt}\left(\mu_F^{-1}\int_0^t \overline{F}(u)du\right)=\mu_F^{-1}\overline{F}(t)=-\frac{d}{dt}\overline{W}_F(t),\ 0\le t<\infty .$$

The failure rate of the renewal distribution $W_F(t)$ is given by

$$r_F(t)=\frac{w_F(t)}{\overline{W}_F(t)}=\frac{\overline{F}(t)}{\int_t^\infty \overline{F}(u)du}=\left(\mu_F(t)\right)^{-1}\ \text{for}\ 0\le t<\infty ,$$

where $\mu_F(t)$ is the mean remaining life distribution of a used unit at time *t*.

Definition 1.1. *F* is new better than average renewal failure rate (NBARFR) if

$$r_F(0)\le t^{-1}\int_0^t r_{W_F}(u)du,\ t>0 .$$

Equivalently $r_F(0)\le -t^{-1}\ln \overline{W}_F(t)$, where $\overline{W}_F(0)=\mu_F^{-1}\int_0^\infty \overline{F}(u)du=1$, *i.e* the failure rate of a new system is less than the average renewal failure rate of a used system.

Definition 1.2. *F* is new worth than used average renewal failure rate (NWARFR) if

$$r_F(0)\ge t^{-1}\int_0^t r_{W_F}(u)du,\ \mathrm{t}\ge 0 .$$

Equivalently $r_F(0)\ge -t^{-1}\ln \overline{W}_F(t)$, *i.e* the failure rate of a new system is greater than the average renewal failure rate of a used system (see Abouammoh and Ahmed, 1992).

Theorem 1.1. The life distribution *F* or its survival $\overline{F}$ having NBARFR iff

$$\int_t^\infty \overline{F}(u)du\le \mu_F e^{-tr_F(0)},\ \mathrm{t}\ge 0.$$

Proof. Let *F* be a life distribution with failure rate *r*(.), *F* is NBARFR means

$$r_F(0)\le -t^{-1}\ln \overline{W}_F(t)$$

then

$$-tr_F(0)\ge \ln \overline{W}_F(t),$$

$$\overline{W}_F(t)\le e^{-tr_F(0)} .$$

This is equivalent to the form

$$\int_t^\infty \overline{F}(u)du \le \mu_F e^{-tr_F(0)}, \text{ t} \ge 0 . \tag{1}$$

If (1) is satisfied, then it is easy to proof the NBARFR property.

By using " ≥ " instead of " ≤ " , the proof of the following theorem can be conducted.

Theorem 1.2. The life distribution F or its survival $\overline{F}$ having NWARFR iff

$$\int_t^\infty \overline{F}(u)du \ge \mu_F e^{-tr_F(0)}, \text{ t} \ge 0.$$

The main purpose of this article is testing H_0: *F* is exponential against H_1: *F*∈NBARFR and not exponential, based on a random sample $X_1, X_2, \ldots X_n$ from a continous life distribution *F* (noncensored data), and also (censored data) based on (Z_i, δ_i), *i*=1,2,3,…*n*, where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1 & \text{if } \mathrm{Z_i} = X_i \\ 0 & \text{if } \mathrm{Z_i} = Y_i \end{cases},$$

where, $Y_1, Y_2, \ldots Y_n$ be i.i.d according to a distribution *G*.

2. TESTING AGAINST NBARFR CLASS FOR NONCENSORED DATA

Nonparametric testing for classes of life distributions have been considered by many authors (see Ahmed, 1975, 1994, 1995; Ebrahimi *et al.*, 1992; Hendi, 1999; Hendi *et al.*, 2000; Mahmoud and Abdul Alim, 2002). In this section we derive a nonparametric U-statistic test for testing :

H_0 : *F* is exponential against H_1 : *F* ∈ NBARFR and not exponential.

For more details about U-statistics see Lee (1989).

Here, the problem is based on sample $X_1, X_2, \ldots X_n$ from *F*. Since *F* is NBARFR, this means

$$\int_t^\infty \overline{F}(u)du \le \mu_F e^{-t\,r(0)} = \mu_F e^{-t\,f(0)} \text{ for all } t,$$

we use the following measure of departure from H_0

$$\delta_F = E(\mu_F e^{-t\,f(0)} - \int_t^\infty \overline{F}(u)du) = \int_0^\infty \omega(0) e^{-t\,f(0)} dF(t) - \int_0^\infty \omega(t) dF(t) \ge 0 .$$

Note that $\delta_F = 0$ under H_0 and under H_1 , $\delta_F > 0$. To estimate δ_F let $X_1, X_2, \ldots X_n$ be a random sample from *F*; then *F*(*t*), $\omega(t)$ and *f*(0) will be empirically estimated. So the empirical form of δ_F is as follows:

$$\hat{\delta}_{F_n} = \frac{1}{n^2} \sum_{i}^{n} \sum_{j}^{n} X_i e^{-X_j \hat{f}_n(0)} - (X_i - X_j) I(X_i \geq X_j). \tag{2}$$

Since (Hardle, 1991)

$$\hat{f}_n(0) \xrightarrow{p} f(0), \quad \text{as } n \to \infty ,$$

therefore we can write

$$\phi(X_1, X_2) = X_1 e^{-X_2 f(0)} - (X_1 - X_2) I(X_1 \geq X_2)$$

and define the symmetric kernel

$$\psi(X_1, X_2) = \frac{1}{2!} \sum_{R} \phi(X_{i1}, X_{i2}) ,$$

where, the summation over all arrangements of X_{i1}, X_{i2} , then $\hat{\delta}_{F_n}$ is equivalent to U-statistic

$$U_n = \frac{1}{\binom{n}{2}} \sum_{R} \psi(X_i, X_j). \tag{3}$$

Since the order of the kernel in (3) is two, this procedure is simple to calculate. It also has asymptotic properties. The following theorem summarizes the asyptotic normality of $\hat{\delta}_{F_n}$.

Theorem 2.1.

i) As $n \to \infty$, $\sqrt{n}\left(\delta_{F_n} - \delta_F\right)$, is asymptotically normal with mean 0 and variance σ^2 that is as in (4). Under H_0 , $\sigma_0^2 = 1/12$.

ii) If *F* is continuous NBARFR, then the test is consistent.

Proof.

i) Using standard theory of U-statistic (Lee, 1989), we need only evaluate the asymptotic variance, which is equal to

$$\sigma^2 = \text{Var}\left\{ \mathrm{E}\left[\phi(X_1, X_2) \mid X_1\right] + \mathrm{E}\left[\phi(X_1, X_2) \mid X_2\right]\right\}$$

Recall the definition of $\phi(X_1, X_2)$, thus it is not difficult to show that

$$\mathrm{E}\left[\phi(X_1,X_2)\,|\,X_1\right] = X_1 \int_0^\infty e^{-xf(0)} dF(x) - X_1 \int_0^{X_1} dF(x) + \int_0^{X_1} x dF(x).$$

Similarly, we have

$$\mathrm{E}\left[\phi(X_1,X_2)\,|\,X_2\right] = e^{-X_2 f(0)} \int_0^\infty x dF(x) - \int_{X_2}^\infty x dF(x) + X_2 \int_{X_2}^\infty dF(x).$$

Hence,

$$\sigma^2 = \mathrm{Var}\left\{X \int_0^\infty e^{-uf(0)} dF(u) + \int_0^X u dF(u) + \mu_F e^{-Xf(0)} + \right.$$

$$\left. X\big(1-2F(X)\big) - \int_{\mathrm{X}}^\infty u dF(u)\right\} \tag{4}$$

Under H_0 , $\sigma_0^2 = \mathrm{Var}\left\{-\frac{1}{2}X - e^{-X} + 1\right\} = 1/12$.

ii) δ_F can be written in the form

$$\delta_F = \int_0^\infty (\omega(0)e^{-t\,f\,0)} - \omega(t))dF(t).$$

Let $D(t) = \omega(0)e^{-t\,f\,0)} - \omega(t)$. Since F is NBARFR and continuous, then $D(t)$>0 and since F is not exponential then $D(t)$>0 for at least one t, call it t_0 .

Set $t_1 = \inf\left\{t \,|\, t \le t_0 \text{ and } \overline{F}(t) = \overline{F}(t_0)\right\}$. Thus

$$D(t_1) = \omega(0)e^{-t_1\,f\,0)} - \omega(t_1) \ge \omega(0)e^{-t_0\,f(0)} - \omega(t_0) \;=\; D(t_0) > 0$$

and $F(t_1+\delta) - F(t_1) > 0$, and since t_1 is the point of increase of F, thus δ_F >0.

To conduct the test, calculate $\sqrt{12\,n}\,\hat{\delta}_{F_n}$ and reject H_0 if this value exceeds Z_α , the standard normal variate at level α.

Lower and upper percentile points of the statistic $\hat{\delta}_{F_n}$ is based on 5000 simulated samples from the standard exponential distributions of order 5(1)50 are computed as in table 3.

3. TESTING AGAINST NBARFR CLASS FOR CENSORED DATA

In this section, a test statistic proposed to test H_0 versus H_1 with randomly right censored samples. In the censoring model, instead of dealing with $X_1, X_2 \ldots X_n$, we observe the pair (Z_i, δ_i), i=1,2,3,…n, where $Z_i = \min(X_i, Y_i)$ and $\delta_i = 1$ if $Z_i = X_i$, $\delta_i = 0$ if $Z_i = Y_i$, where $X_1, X_2 \ldots X_n$ denote their true life time from a distribution F and $Y_1, Y_2, \ldots Y_n$ be i.i.d according to distribution G. Also X's and Y's are independent. Let $Z_{(0)} = 0 \le Z_{(1)} \le Z_{(2)} \le \ldots Z_{(n)}$ denote the ordered Z's and $\delta_{(i)}$ is the δ_i corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier (1958) estimator in the case of censored data (Z_i, δ_i), i=1,2,… n:

$$\hat{\overline{F}}_n(X) = 1 - \hat{F}_n(X) = \prod_{(i < Z_{(i)} \le X)} \left[\frac{n-i}{n-i+1} \right]^{\delta_{(i)}}, \mathrm{X} \in \left[0, \mathrm{Z}_{(\mathrm{n})}\right],$$

and Tanner (1983), hazard rate estimate with censored data

$$\hat{r}(t) = \frac{1}{2R_k} \sum_{i=1}^{n} \left[\frac{\delta_{(i)}}{n-i+1} K\left(\frac{t - Z_{(i)}}{2R_k} \right) \right],$$

where:

R_k is the distance between point t and its k-th nearest failure point

K(.) is a function of bounded variation with compact support on the interval [-1,1].

Then the proposed test statistic is given by

$$\hat{\delta}^c_{F_n} = \int_0^\infty \left[e^{-tf(0)} \int_0^\infty \overline{F}_n(u) du \right] dF(t) - \int_0^\infty \left[\int_{\mathrm{t}}^\infty \overline{F}_n(u) du \right] dF(t) \tag{5}$$

For computation use, $\hat{\delta}^c_{F_n}$ in (5) can be written as

$$\hat{\delta}^c_{F_n} = \sum_{i=1}^{n} \left\{ \sum_{j=1}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right) - \sum_{j=i}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right) \right\}$$
$$\cdot \left\{ \prod_{m=1}^{i-2} \left(C_{(m)}\right)^{\delta_{(m)}} - \prod_{mm=1}^{i-1} \left(C_{(mm)}\right)^{\delta_{(mm)}} \right\} e^{-Z_{(i)} \hat{f}(0)},$$

where

$$C_k = (n-k)/(n-k+1) \text{ and } dF_n(Z_j) = \overline{F}_n(Z_{j-1}) - \overline{F}_n(Z_j).$$

Table 4 gives the percentiles of $\hat{\delta}^c_{F_n}$ for sample size 5(1)50, 60, 70, 80, 81.

4. ASYMPTOTIC RELATIVE EFFICIENCY AND POWERS

Since the above test is new and no other tests known for NBARFR we compare our test to the smaller classes and choose NBU, NBUFR and NBURFR classes proposed in Ahmed (1994), Hendi *et al.* (2000) and Mahmoud and Abdul Alim (2002). We choose the following alternatives:

i) Linear failure rate family : $\overline{F}(t) = \exp\left(-t - \theta\, t^2 / 2\right)$, t > 0, $\theta \geq 0$

ii) Makeham family $\overline{F}(t) = e^{-t-\theta(t+e^{-t}-1)}$

iii) Pareto family $\overline{F}(t) = (1-\theta\, t)^{1/\theta}$

iv) Weibull family $\overline{F}(t) = e^{-t^{\theta}}$, t > 0, $\theta \geq 0$

v) Gamma family $\overline{F}(t) = \int_t^{\infty} e^{-u} u^{\theta-1} du / \Gamma(\theta)$, t > 0, $\theta \geq 0$

Note that H_0 is attained at $\theta = 1$, in *iv*) and *v*), is attained at $\theta = 0$ in *i*) and *ii*), and is attained when $\theta \to 0$ in *iii*).

Direct calculations of the asymptotic efficiencies of the NBARFR class compared with NBU (Ahmed, 1994), NBUFR (Hendi *et al.*, 2000) and NBURFR (Mahmoud and Abdul Alim, 2002) in table 2.

For the previous alternatives, the powers for the proposed test are tabulated as in table 1 using simulated number of sample 5000 for sample sizes 10,20 and 30 and θ values 2,3 and 4.

TABLE 1

Powers for NBARFR class test

	N		*Powers*		*Powers*		*Powers*
	10	θ =2	.914		.882	θ =4	.858
LFR	20		.911	θ =3	.860		.792
	30		.880		.795		.680
	10		.862	θ =3	.686		.404
Pareto	20	θ =2	.805		.446	θ =4	.070
	30		.698		.197		.002
Weibull	10		1.000		1.000		1.000
	20	θ =2	1.000	θ =3	1.000	θ =4	1.000
	30		1.000		1.000		1.000
	10		1.000		1.000		1.000
Gamma	20	θ =2	1.000	θ =3	1.000	θ =4	1.000
	30		1.000		1.000		1.000

It is clear from table 1 that our test has a good powers specially in the case of Weibull and Gamma families. Table 2 shows that our test has much higher asymptotic efficacy for the linear failure rate and Makeham families compared with other two tests (Ahmed, 1994 and Hendi *et al.*, 2000). Also it shows acceptable AE for Weibull and Gamma families.

In section 1 and 2 the test statistics (2) and (5) for uncensored and right censored data respectively are derived. Using (2), (5) and tables 3 and 4 applications in medical science are presented to illustrate the theoretical results in section 5.

TABLE 2

Asymptotic relative efficiency of δ_{F_n} to $\delta_{F_n^{(1)}}$, $\delta_{F_n^{(2)}}$ and $\delta_{F_n^{(3)}}$ of Ahmed (1994), Hendi et al. (2000) and Mahmoud and Abdul Alim (2002)

Efficienciy	F_1 Linear failure rate	F_2 Makeham	F_3 Pareto	F_4 Weibull	F_5 Gamma
$\hat{\delta}F_n^{(1)}$ (Ahmed, 1994)	0.8056	0.2854	………	………	………
$\hat{\delta}F_n^{(2)}$ (Hendi et al., 2000)	0.433	0.289	0.1880	………	……….
$\hat{\delta}_{F_n^{(3)}}$	1.2990	0.5774	1.2990	………	………
δF_n	1.2990	0.5774	0.4330	0.9699	0.5196
$e(\delta F_n, \delta F_n^{(1)})$	1.6125	2.2277	………	………	………
$e(\delta F_n, \delta F_n^{(2)})$	3.000	1.9975	2.3094	……..	………
$e(\delta F_n, \delta F_n^{(3)})$	1.000	1.000	0.33333	……..	……..

5. APPLICATIONS

Consider the data in Abouammoh *et al.* (1994). These data represent a set of 40 patients suffering from blood cancer (Leukemia) from one of Ministry of Health Hospitals in Saudi Arabia and the ordered values are:

115	181	255	418	441	461	516	739	743	789
807	865	924	983	1024	1062	1063	1165	1191	1222
1222	1251	1277	1290	1375	1369	1408	1455	1478	1549
1578	1599	1603	1605	1696	1735	1799	1815	1852	1599

It was found that the test statistic for the data set, by formula (2) is $\hat{\delta}_{F_n} = 869.0544$ and exceeds the critical value of table 3. Then we reject the null hypothesis of exponentiality.

Consider the data in Susarla and Vanryzin (1978). These data represent 81 survival times of patients of melanoma. Of them 46 represent whole life times (non-censored data) and the ordered values are:

13	14	19	19	20	21	23	23	25	26	26	27
27	31	32	34	34	37	38	38	40	46	50	53
54	57	58	59	60	65	65	66	70	85	90	98
102	103	110	118	124	130	136	138	141	234		

The ordered censored observations are:

16	21	44	50	55	67	73	76	80	81	86	93
100	108	114	120	124	125	129	130	132	134	140	147
148	151	152	152	158	181	190	193	194	213	215	

Now ignoring censored data, one can apply the methodology of section 2 to test the hypothesis Ho: the survival times are exponential against H_1: the survival times follow nbarfr and not exponential.

Computing $\hat{\delta}_{F_n}$ from (2) , we get $\hat{\delta}_{F_n}$ =38.949430 exceeds the critical point in table 3 at 95% upper percentiles. Then we accept H_1 which states that the set data have nbarfr property.

A simple computer program is written to calculate $\hat{\delta}^c_{F_n}$ for these data and the value we get leads to a $\hat{\delta}^c_{F_n}$ =0.67681x10-20 less than the critical value in table 4 at 95% upper percentile. Then we reject H_1: which states that the set of data have nbarfr property.

6. CONCLUSIONS

Testing exponentiality against NBARFR distributions is considered. The percentiles and powers of our test are tabulated. Comparisons between our test and tests of Ahmed (1994), Hendi *et al.* (2000) and Mahmoud and Abdul Alim (2002) are given. Test for this problem when right censored data is available is handled. Our study explained that our test performs higher AE with respect to Ahmed (1994) and Hendi *et al.* (2000) tests. It gives a very good powers for the most common alternatives.

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APPENDIX

TABLE 3

Critical value for $\hat{\delta}_{F_n}$

N	.01	.05	.10	.90	.95	.98	.99
5	.0907	.1328	.1643	.6425	.7830	.9742	1.1308
6	.0991	.1435	.1741	.5989	.7119	.8619	.9560
7	.1061	.1505	.1774	.5482	.6427	.7813	.8794
8	.1091	.1482	.1764	.5356	.6252	.7266	.8288
9	.1103	.1550	.1814	.5150	.5995	.7068	.8083
10	.1149	.1593	.1827	.4990	.5765	.6772	.7521
11	.1206	.1633	.1904	.4886	.5527	.6426	.7170
12	.1217	.1641	.1890	.4682	.5251	.6150	.6856
13	.1264	.1653	.1870	.4541	.5167	.5989	.6416
14	.1341	.1698	.1922	.4472	.5080	.5736	.6378
15	.1310	.1704	.1941	.4403	.4989	.5675	.6259
16	.1393	.1730	.1953	.4363	.4847	.5520	.5948
17	.1406	.1776	.1976	.4270	.4760	.5451	.5837
18	.1390	.1736	.1963	.4223	.4625	.5230	.5776
19	.1384	.1751	.1973	.4210	.4669	.5213	.5738
20	.1438	.1785	.1997	.4062	.4510	.5024	.5482
21	.1485	.1809	.2014	.4055	.4475	.4965	.5311
22	.1482	.1825	.2029	.4013	.4357	.4855	.5254
23	.1454	.1810	.2034	.4033	.4427	.4876	.5204
24	.1531	.1868	.2060	.3959	.4365	.4796	.5200
25	.1531	.1874	.2053	.3957	.4338	.4812	.5143
26	.1583	.1872	.2065	.3873	.4304	.4767	.5100
27	.1578	.1918	.2087	.3894	.4263	.4759	.5080
28	.1570	.1905	.2078	.3816	.4174	.4600	.4872
29	.1640	.1927	.2091	.3821	.4163	.4599	.4893
30	.1596	.1916	.2107	.3823	.4160	.4560	.4854
31	.1619	.1971	.2139	.3746	.4067	.4461	.4782
32	.1670	.1928	.2097	.3729	.4078	.4447	.4711
33	.1648	.1958	.2118	.3769	.4047	.4395	.4683
34	.1670	.1965	.2133	.3760	.4071	.4450	.4672
35	.1689	.1972	.2144	.3730	.4045	.4438	.4729
36	.1669	.1963	.2140	.3702	.3991	.4351	.4632
37	.1679	.1968	.2146	.3668	.3972	.4296	.4493
38	.1728	.2004	.2164	.3664	.3955	.4319	.4583
39	.1726	.2007	.2163	.3634	.3914	.4214	.4470
40	.1727	.2008	.2172	.3670	.3949	.4288	.4573
41	.1753	.2020	.2170	.3594	.3857	.4196	.4408
42	.1741	.2005	.2172	.3625	.3909	.4251	.4485
43	.1760	.2015	.2185	.3581	.3821	.4122	.4332
44	.1763	.2039	.2194	.3573	.3831	.4138	.4346
45	.1774	.2024	.2188	.3605	.3855	.4242	.4462
46	.1781	.2040	.2201	.3555	.3816	.4125	.4317
47	.1790	.2046	.2188	.3539	.3761	.4080	.4276
48	.1812	.2061	.2206	.3534	.3769	.4072	.4330
49	.1808	.2055	.2207	.3518	.3746	.4053	.4228
50	.1830	.2067	.2222	.3510	.3736	.4016	.4213

TABLE 4

Critical values of $\hat{\delta}^{c}_{F_n}$ *-(60,70,80,81)* 10^{-10}

n	.01	.05	.10	.90	.95	.98	.99
7	-.033224850	-.006854016	-.000363972	.034631130	.047285330	.066254680	.083703170
8	-.026360480	-.006859757	-.001182442	.026659040	.037861920	.059381370	.072423510
9	-.017087790	-.003589807	-.000429099	.018481340	.026649020	.038333800	.048232600
10	-.012231820	-.003243839	-.000839866	.013385580	.019586600	.031247070	.039666950
11	-.008208380	-.001749931	-.000258584	.009754967	.014587610	.023781910	.035158330
12	-.007353045	-.001528120	-.000294497	.007214207	.011197040	.017245920	.023316420
13	-.004310631	-.001016936	-.000196094	.004395628	.007264028	.012585690	.017582220
14	-.003235959	-.000694326	-.000133666	.003257281	.005511061	.009911915	.014207200
15	-.002310727	-.000445835	-.000096936	.002178582	.003670595	.006414143	.009494149
16	-.001696247	-.000330039	-.000055226	.001394978	.002454675	.004789334	.006922268
17	-.001159436	-.000180608	-.000040388	.000966653	.001679410	.003153126	.004936925
18	-.000777893	-.000162165	-.000031014	.000637148	.001165656	.002320627	.003420803
19	-.000564952	-.000081704	-.000015544	.000418034	.000795973	.001579985	.002696489
20	-.000541604	-.000065617	-.000013085	.000302319	.000575975	.001184818	.001829348
21	-.000327746	-.000039479	-.000006069	.000177988	.000318361	.000700496	.001164032
22	-.000173208	-.000029070	-.000006219	.000116634	.000242577	.000554178	.000886036
23	-.000147275	-.000017554	-.000002872	.000068272	.000146279	.000307182	.000480807
24	-.000092759	-.000010057	-.000002058	.000050388	.000107198	.000249742	.000414588
25	-.000078820	-.000008991	-.000001246	.000030834	.000073689	.000165397	.000280827
26	-.000041054	-.000005265	-.000000812	.000021187	.000046338	.000098940	.000166527
27	-.000035200	-.000003089	-.000000481	.000013199	.000028078	.000078044	.000147713
28	-.000018615	-.000002011	-.000000318	.000008496	.000020342	.000049468	.000095454
29	-.000012805	-.000001230	-.000000138	.000006097	.000013966	.000037505	.000069044
30	-.000012505	-.000000776	-.000000094	.000003919	.000009696	.000028402	.000049956
31	-.000010275	-.000000558	-.000000093	.000002273	.000006249	.000015806	.000033306
32	-.000005310	-.000000418	-.000000063	.000001575	.000003610	.000010167	.000018587
33	-.000004034	-.000000239	-.000000021	.000000989	.000002422	.000006158	.000013730
34	-.000002273	-.000000155	-.000000021	.000000577	.000001706	.000004898	.000008196
35	-.000001546	-.000000115	-.000000012	.000000393	.000000956	.000002644	.000004637
36	-.000001059	-.000000070	-.000000009	.000000274	.000000818	.000002256	.000004447
37	-.000001082	-.000000035	-.000000004	.000000178	.000000490	.000001478	.000003279
38	-.000000400	-.000000022	-.000000002	.000000121	.000000313	.000000962	.000001705
39	-.000000344	-.000000016	-.000000001	.000000079	.000000226	.000000732	.000001326
40	-.000000173	-.000000010	-.000000001	.000000045	.000000126	.000000401	.000000818
41	-.000000102	-.000000007	-.000000001	.000000032	.000000092	.000000314	.000000644
42	-.000000092	-.000000004	-.000000001	.000000017	.000000055	.000000173	.000000358
43	-.000000049	-.000000002	-.000000000	.000000012	.000000039	.000000118	.000000271
44	-.000000040	-.000000001	-.000000000	.000000007	.000000022	.000000069	.000000137
45	-.000000028	-.000000001	-.000000000	.000000005	.000000016	.000000049	.000000114
46	-.000000012	-.000000001	-.000000000	.000000003	.000000009	.000000032	.000000079
47	-.000000010	-.000000000	.000000000	.000000002	.000000006	.000000019	.000000041
48	-.000000009	-.000000000	.000000000	.000000001	.000000004	.000000015	.000000041
49	-.000000005	-.000000000	.000000000	.000000001	.000000003	.000000010	.000000023
50	-.000000004	-.000000000	.000000000	.000000001	.000000002	.000000006	.000000013
60	-.222418800	-.006300647	-.000299028	.048864510	.214923300	.827251800	1.81293300
70	-.006451606	-.000125200	-.000004093	.000526405	.001927479	.007590745	.022914550
80	-.000017509	-.000000325	-.000000014	.000003376	.000014502	.000102074	.000237834
81	-.000012132	-.000000153	-.000000011	.000001666	.000009875	.000076221	.000318292

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REFERENCES

A. M. ABOUAMMOH, A. N. AHMED (1992), *On renewal failure rate classes of life distributions.* "Statistcs & Probability Letter" 14, pp. 211-217.

A. M. ABOUAMMOH, A. N. AHMED (1988), *The new better than used failure rate class of life distribution*. "Adv. Prob.", 20, pp. 237-240.

A. M. ABOUAMMOH, S. A. ABDULGHAN, I. S. QAMBER (1994), *On partial orderings and testing of new better than renewal used classes.* "Reliability Eng. Syst. Safety", 43, pp. 37-41.

I. A. AHMED (1975), *A nonparametric test for the monotincity of a failure rate function,* "Communication in Statistics", 4, pp. 967-974.

I. A. AHMED (1994), *A class of statistics useful in testing increasing failure rate average and new beter than used life distribution*, "J. Statist.. Plant. Inf.", 41, pp. 141-149.

I. A. AHMED (1995), *Nonparametric testing of class of life distributions derived from a variability ordering*, Paisankhyan Samikkha, 2, pp. 13-18.

E.E. ALY (1989), *On testing exponentiality against IFRA alternative,* "Metrika", 36, pp. 255-267.

R. E. BARLOW (1968), *Likelihood ratio tests for restricted families of probability distributions,* "Annals of Mathematical Statistics" 39, pp. 547-560.

J. V. DESHPANDE (1983), *A class of tests for exponentiality against failure rate average*. Biometrika, 70, pp. 514-518.

J. V. DESHPANDE, S. C KOCHAR, H. SINGH (1986), *Aspects of positive aging,* "Journal of Applied Probability", 28, pp. 773-779.

N. EBRAHIMI, M. HABIBULLAH, E. SOFI (1992), *Testing exponentiality based on Kullback-Leibler information,* "Journal of the Royal Statistical Society" , 54 B, pp. 739-748.

W. HARDLE (1991), *Smothing Techniques With Implementation In S*, Spring-Verlag, New York.

M. I. HENDI (1999), *Testing general harmonic new better than used in expectation using kernal method,* "Egypt . computer. Sc. J.", 21, pp. 1-16.

M. I. HENDI, H. ALNACHAWATI, M. N. AL-GRAIAN (2000), *Testing NBUFR and NBAFR classes of life distributions using kernal methods,* "Arab J. Math. Sc.", 6, pp. 37-57.

M. HOLLANDER, F. PROSCHANE (1972), *Testing whether new better than used*, "Annals of Mathematical Statistics", 43, pp. 1136-1146.

E. L. KAPLAN, P. MEIER (1958), *Nonparametric* estimation *from incomplete observations*, "J. Amer. Assoc", 53, pp. 457-481.

B. KLEFSJO (1981), *HNBUE survival under some shoch models,* "Scandinavian Journal of Statistics", 8, pp. 34-47.

B. KLEFSJO (1982), *The HNBUE and HNWUE classes of life distributions*, Naval. Res. Logistics Quartely. 24, pp. 331-344.

H. L. KOUL (1977), *A new test for new better than used,* "Comm. Statist. Theor. Math", 6, pp. 563-573.

Y. KUMAZAWA (1983), *Testing for new is betterthan used,* "Comm. Statist. Theor. Math.", 12, pp. 311-321.

A. J. LEE (1989), *U-Statistics*, Marcel Dekker, New York, NY.

W. A.LINMK (1989), *Testing for exponentiality against monoton failure rateaverage alternative*, "Comm. Statist. Theor. Math.", 18, pp. 3009-3017.

W. Y. LOH (1984), *A new generalization of the class of NBU distribution,* "IEEE Trans. Reli.", R3, pp. 419-422.

M. A. W. MAHMOUD AND N. A. ABDUL ALIM, *On testing exponentiality against NBURFR class of life distributions*, Submitted for publication.

F. PROSCHAN and R. PYKE (1967), *Tests for monoton failure rate,* Proc. 5th BereklySymp., pp. 3293-3312.

V. SUSARLA, J. VANRYZIN (1978), *Empirical bayes estimation of survival function right censored oservations,* "Annals of Statistics", 6, pp. 710-755.

M. A. TANNER (1983), *A note on the variable kernel estimator of the hazard function from randomly censored data*, "Annals of Statistics", 11, pp. 994-998.

SUMMARY

On testing exponentiality against NBARFR life distributions

This paper considers testing exponentiality against new better than average renewal failure rate (NBARFR) alternatives. The percentiles of this test statistic are tabulated for sample sizes 5(1)50. Pitman's asymptotic efficiencies relative to the tests of the new better than used (NBU), new better than used failure rate (NBUFR) and new better than used renewal failure failure rate (NBURFR) (Ahmed, 1994; Hendi *et al.,* 2000 and Mahmoud and Abdul Alim, 2002). The powers of this test are also calculated for some used life distributions. The problem when right-censored data is available is handled. Practical applications of our tests in the medical sciences are present.