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A CHAIN RATIO EXPONENTIAL TYPE ESTIMATOR IN TWO-PHASE SAMPLING USING AUXILIARY INFORMATION

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1. INTRODUCTION

It is a common practice to use auxiliary information at the estimation stage for increasing the efficiency of the estimators of the population mean $\bar{Y}$ of the study variable y. Out of many ratio, regression and product methods of estimation are good examples in this context. When the population mean $\bar{X}$ of the auxiliary variable x is known, a number of modified versions of ratio and product estimators have been proposed by various authors including Srivastava (1967), Chakraborty (1968), Gupta (1978), Vos (1980), Walsh (1970), Upadhyaya et al. (2004), Singh and Tailor (2005 a, b) and others.

In case when the population mean $\bar{X}$ of the auxiliary variable x is not known then it is often estimated from a preliminary large sample on which only the auxiliary character x is observed. After that a small sample is drawn from preliminary large sample for estimating the study variable y. This technique is known as the two-phase or double sampling. Prasad et al. (1996), Singh and Gangele (1999), Upadhyaya et al. (2006) and many others have reported the modified ratio estimators in two phase sampling.

The two-phase sampling is effective and economical procedure to find out the reliable estimate of the unknown population mean $\bar{X}$ of the auxiliary variable x.

Consider a finite population $U=(U_1,\ U_2,\ \ldots,\ U_N)$ of N units. Let $(y,\ x)$ be the study variable and the auxiliary variable respectively, taking values $(y_i,\ x_i)$ for the i^{th} unit U_i Let $\bar{Y}=(1/N)\sum_{i=1}^{N} y_i$ and $\bar{X}=(1/N)\sum_{i=1}^{N} x_i$ be the population means of study variable y and auxiliary variable x respectively.

When the population mean $\bar{X}$ of the auxiliary variable x is known, the classical ratio estimator for population mean $\bar{Y}$ is defined by

$$\bar{y}_r = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right);\ \bar{x} \neq 0 \tag{1}$$

where $\bar{y} = (1/n)\sum_{i=1}^{n} y_i$ and $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$.

When the population mean $\bar{X}$ of the auxiliary variable x is unknown, Sukhatme (1962) has defined the two-phase (or double) sampling ratio estimator of population mean $\bar{Y}$ as

$$\bar{y}_{rd} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right);\ \bar{x} \neq 0 \tag{2}$$

where $\bar{x}' = (1/n')\sum_{i=1}^{n'} x_i$ is the sample mean of x based on first phase sample of size n'.

Suppose that the population mean $\bar{Z}$ of the another auxiliary variable z is available which is closely related to x but compared to x remotely related to y (i.e. $\rho_{xz} > \rho_{yz}$; ρ_{xz} being the correlation coefficient between x and z).

In this situation, Chand (1975) has suggested a chain ratio-type estimator for the population mean $\bar{Y}$ defined as

$$\bar{y}_{rd}^{(C)} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right);\ \bar{x} \neq 0,\quad \bar{z}' \neq 0 \tag{3}$$

where $\bar{z}' = (1/n')\sum_{i=1}^{n'} z_i$ is the sample mean of the second auxiliary variable z based on n' observations.

This type of situation has been briefly discussed by Kiregyera (1980, 1984) Mukherjee et al. (1987), Srivastava et al. (1989), Srivenkataramana and Tracy (1989), Upadhyaya et al. (1990), Singh and Singh (1991), Singh et al. (1994), Upadhyaya and Singh (2001), Singh et al. (2006), Singh et al. (2009) and others.

Motivated by Bahl and Tuteja (1991), Singh and Vishwakarma (2007) have suggested the ratio exponential type estimator for finite population mean $\bar{Y}$ in two phase sampling as

$$\bar{y}_{red} = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right). \tag{4}$$

To the first degree of approximation, the mean squared errors (MSEs) of the estimators $\bar{y}_{rd}$, $\bar{y}_{rd}^{(C)}$ and $\bar{y}_{red}$ are respectively given by

$$MSE(\bar{y}_{rd}) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + \left(\frac{1-f''}{n}\right)C_x^2(1-2K_{01})\right] \tag{5}$$

$$MSE\left(\bar{y}_{rd}^{(C)}\right) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + \left(\frac{1-f'}{n'}\right)C_z^2(1-2K_{02}) + \left(\frac{1-f''}{n}\right)C_x^2(1-2K_{01})\right] \tag{6}$$

$$MSE(\bar{y}_{red}) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + \frac{1}{4}\left(\frac{1-f''}{n}\right)C_x^2(1-4K_{01})\right] \tag{7}$$

where $f = (n/N)$, $f' = (n'/N)$, $f'' = (n/n')$ be sample fractions based on the sample of size n and n', $C_y = (S_y/\bar{Y})$, $C_x = (S_x/\bar{X})$, $C_z = (S_z/\bar{Z})$ be the coefficient of variations of x, y and z respectively, $\rho_{yx} = (S_{yx}/S_yS_x)$ and $\rho_{yz} = (S_{yz}/S_yS_z)$ be the correlation coefficients between (y, x) and (y, z) respectively, $K_{01} = \rho_{yx}(C_y/C_x)$ and $K_{02} = \rho_{yz}(C_y/C_z)$.

This paper deals with problem of estimating the finite population mean $\bar{Y}$ using auxiliary variables x and z in two-phase or double sampling. A family of chain ratio exponential type estimators has been proposed in two-phase (or double sampling) sampling and its properties have been discussed under large sample of approximation. Regions of preferences of the suggested class of estimators have been defined over others. An empirical study is carried out to judge the merits of the proposed class of estimators over two-phase ratio estimator $\bar{y}_{rd}$, Chand's (1975) estimator $\bar{y}_{rd}^{(C)}$ and Singh and Vishwakarma's (2007) estimator $\bar{y}_{red}$.

2. THE SUGGESTED CLASS OF ESTIMATORS

Motivated by Singh and Vishwakarma (2007), we have suggested a class of chain ratio exponential type estimators for population mean $\bar{Y}$ using information on two auxiliary variables x and z in two-phase or double sampling as

$$\hat{t}_{red} = \bar{y}\exp\left[\frac{\hat{\bar{X}}_{rd} - \bar{x}}{\hat{\bar{X}}_{rd} + \bar{x}}\right] \tag{8}$$

where $\hat{\bar{X}}_{rd} = \frac{\bar{x}'}{\left(a\bar{z}' + b\right)}\left(a\bar{Z} + b\right)$, 'a' and 'b' are the suitably chosen positive scalars.

Thus the proposed class of estimators can be expressed as

$$\hat{t}_{red} = \bar{y}\exp\left[\frac{\frac{\bar{x}'}{\left(a\bar{z}' + b\right)}\left(a\bar{Z} + b\right) - \bar{x}}{\frac{\bar{x}'}{\left(a\bar{z}' + b\right)}\left(a\bar{Z} + b\right) + \bar{x}}\right]. \tag{9}$$

To obtain the bias and mean squared error (MSE) of the suggested family of estimators $\hat{t}_{red}$, we write

$$\bar{y} = \bar{Y}\left(1 + e_0\right),\ \bar{x} = \bar{X}\left(1 + e_1\right),\ \bar{x}' = \bar{X}\left(1 + e_2\right),\ \bar{z}' = \bar{Z}\left(1 + e_3\right)$$

such that

$$\left.\begin{array}{lll}
E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = 0 & & \\
E\left(e_0^2\right) = \left(\frac{1-f}{n}\right)C_y^{\ 2}, & E\left(e_1^2\right) = \left(\frac{1-f}{n}\right)C_x^2, & E\left(e_2^2\right) = \left(\frac{1-f'}{n'}\right)C_x^2, \\
E\left(e_3^2\right) = \left(\frac{1-f'}{n'}\right)C_z^2, & E\left(e_0e_1\right) = \left(\frac{1-f}{n}\right)\rho_{yx}C_yC_x, & E\left(e_0e_2\right) = \left(\frac{1-f'}{n'}\right)\rho_{yx}C_yC_x, \\
E\left(e_0e_3\right) = \left(\frac{1-f'}{n'}\right)\rho_{yz}C_yC_z, & E\left(e_1e_2\right) = \left(\frac{1-f'}{n'}\right)C_x^2, & E\left(e_1e_3\right) = \left(\frac{1-f'}{n'}\right)\rho_{xz}C_xC_z, \\
E\left(e_2e_3\right) = \left(\frac{1-f'}{n'}\right)\rho_{xz}C_xC_z & &
\end{array}\right\}$$

Expressing (9) in terms of e's we have

$$\hat{t}_{red} = \bar{Y}(1+e_0)\exp\left[\left\{(1+e_2)(1+\theta_0 e_3)^{-1} - (1+e_1)\right\}\left\{(1+e_2)(1+\theta_0 e_3)^{-1} + (1+e_1)\right\}^{-1}\right]$$

where $\theta_0 = \left(\dfrac{a\bar{Z}}{a\bar{Z}+b}\right)$.

$$= \bar{Y}\left(1+e_0\right)\left[1+\left\{\left(1+e_2\right)\left(1+\theta_0 e_3\right)^{-1} - \left(1+e_1\right)\right\}\left\{\left(1+e_2\right)\left(1+\theta_0 e_3\right)^{-1} + \left(1+e_1\right)\right\}^{-1}\right.$$

$$\left. +\frac{1}{2}\left\{\left(1+e_2\right)\left(1+\theta_0 e_3\right)^{-1} - \left(1+e_1\right)\right\}^2\left\{\left(1+e_2\right)\left(1+\theta_0 e_3\right)^{-1} + \left(1+e_1\right)\right\}^{-2} + \cdots\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1+\left\{\left(1+e_2\right)\left(1-\theta_0 e_3+\theta_0^2 e_3^2\right) - \left(1+e_1\right)\right\}\left\{\left(1+e_2\right)\left(1-\theta_0 e_3+\theta_0^2 e_3^2\right) + \left(1+e_1\right)\right\}^{-1}\right.$$

$$\left. +\frac{1}{2}\left\{\left(1+e_2\right)\left(1-\theta_0 e_3+\theta_0^2 e_3^2\right) - \left(1+e_1\right)\right\}^2\left\{\left(1+e_2\right)\left(1-\theta_0 e_3+\theta_0^2 e_3^2\right) + \left(1+e_1\right)\right\}^{-2} + \ldots\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1+\frac{1}{2}\left(e_2 - e_1 - \theta_0 e_3 + \frac{e_1^2}{2} - \frac{e_2^2}{2} + \frac{\theta_0^2 e_3^2}{2}\right)+\right.$$

$$\left. +\frac{1}{8}\left(e_1^2 + e_2^2 + \theta_0^2 e_3^2 - 2e_1 e_2 - 2\theta_0 e_2 e_3 + 2\theta_0 e_1 e_3\right) + \ldots\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1 - \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}\theta_0 e_3 + \frac{3}{8}e_1^2 - \frac{1}{8}e_2^2 +\right.$$

$$\left. +\frac{5}{8}\theta_0^2 e_3^2 - \frac{1}{4}e_1 e_2 - \frac{1}{4}\theta_0 e_2 e_3 + \frac{1}{4}\theta_0 e_1 e_3 + \ldots\right]$$

Neglecting terms of e's having power greater than two, we have

$$\hat{t}_{red} \cong \bar{Y}\left[1 + e_0 - \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}\theta_0 e_3 + \frac{3}{8}e_1^2 - \frac{1}{8}e_2^2 + \frac{5}{8}\theta_0^2 e_3^2 - \frac{1}{2}e_0 e_1 + \frac{1}{2}e_0 e_2\right.$$
$$\left. -\frac{1}{4}e_1 e_2 - \frac{1}{2}\theta_0 e_0 e_3 + \frac{1}{4}\theta_0 e_1 e_3 + \frac{1}{4}\theta_0 e_2 e_3\right]$$

or

$$\left(\hat{t}_{red} - \bar{Y}\right) = \bar{Y}\left[e_0 - \frac{1}{2}e_1 + \frac{1}{2}e_2 - \frac{1}{2}\theta_0 e_3 + \frac{3}{8}e_1^2 - \frac{1}{8}e_2^2 + \frac{5}{8}\theta_0^2 e_3^2 - \frac{1}{2}e_0 e_1 + \frac{1}{2}e_0 e_2\right.$$
$$\left. -\frac{1}{4}e_1 e_2 - \frac{1}{2}\theta_0 e_0 e_3 + \frac{1}{4}\theta_0 e_1 e_3 + \frac{1}{4}\theta_0 e_2 e_3\right] \quad . \quad (10)$$

Taking expectation of both sides of (10), we get the bias of the estimator $\hat{t}_{red}$ to the first degree of approximation is given by

$$B\left(\hat{t}_{red}\right)=\bar{Y}\left[\frac{1}{2}\left(\frac{1-f}{n}\right)C_x^2\left(\frac{3}{4}-K_{01}\right)\right.$$
$$\left.-\frac{1}{2}\left(\frac{1-f'}{n'}\right)\left(\frac{3}{4}C_x^2-\frac{5}{4}\theta_0^2C_z^2-\rho_{yx}C_yC_x+\theta_0\rho_{yz}C_yC_z-\theta_0\rho_{xz}C_xC_z\right)\right] \tag{11}$$

Squaring both sides of (10) and neglecting terms of e's having power greater than two, we have

$$\left(\hat{t}_{red}-\bar{Y}\right)^2\cong\bar{Y}^2\left(e_0-\frac{e_1}{2}+\frac{e_2}{2}-\frac{\theta_0e_3}{2}\right)^2$$
$$=\bar{Y}^2\left[e_0^2-\left(e_0e_1-e_0e_2+\theta_0e_0e_3\right)+\right. \tag{12}$$
$$\left.+\frac{1}{4}\left(e_1^2+e_2^2+\theta_0^2e_3^2\right)-\frac{1}{2}\left(e_1e_2-\theta_0e_1e_3+\theta_0e_2e_3\right)\right]$$

Taking expectation of both sides of (12), we get the MSE of to the first degree of approximation as

$$MSE\left(\hat{t}_{red}\right)=\bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2+\frac{\theta_0}{4}\left(\frac{1-f'}{n'}\right)C_z^2\left(\theta_0-4K_{02}\right)+\right.$$
$$\left.+\frac{1}{4}\left(\frac{1-f''}{n}\right)C_x^2\left(1-4K_{01}\right)\right] \tag{13}$$

The MSE of $\hat{t}_{red}$ at (13) is minimized for

$$\theta_0=\left(\frac{2\rho_{yz}C_yC_z}{C_z^2}\right)=2K_{02}=\theta_0^* \quad \text{(say)} \tag{14}$$

Thus the resulting minimum MSE of $\hat{t}_{red}$ is given by

$$\min MSE(t)=\bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2+\frac{1}{4}\left(\frac{1-f''}{n}\right)C_x^2\left(1-4K_{01}\right)-\left(\frac{1-f'}{n'}\right)\rho_{yz}^2C_y^2\right] \tag{15}$$

3. Efficiency comparison

3.1. When the scalar θ_0 does not coincide with its exact optimum value θ_0^*

From (5), (6), (7) and (13), we have

(i) $MSE\left(\hat{t}_{red}\right) - MSE\left(\bar{y}_{rd}\right) < 0$ if

$$\left.\begin{array}{ll}\text{either} & L_1 < \theta < L_2 \\ \text{or} & L_2 < \theta < L_1\end{array}\right\} \tag{16}$$

or equivalently, $\min\left(L_1, L_2\right) < \theta < \max\left(L_1, L_2\right)$, (17)

where $L_1 = \left[2K_{02} - \left(\frac{n'}{1-f'}\right)\sqrt{4\left(\frac{1-f'}{n'}\right)^2 K_{02}^2 + \left(\frac{1-f'}{n'}\right)\left(\frac{1-f''}{n}\right)\frac{C_x^2}{C_z^2}\left(3-4K_{01}\right)}\right]$

and $L_2 = \left[2K_{02} + \left(\frac{n'}{1-f'}\right)\sqrt{4\left(\frac{1-f'}{n'}\right)^2 K_{02}^2 + \left(\frac{1-f'}{n'}\right)\left(\frac{1-f''}{n}\right)\frac{C_x^2}{C_z^2}\left(3-4K_{01}\right)}\right]$.

(ii) $MSE\left(\hat{t}_{red}\right) - MSE\left(\bar{y}_{rd}^{(C)}\right) < 0$ if

$$\left.\begin{array}{ll}\text{either} & L_1^* < \theta < L_2^* \\ \text{or} & L_2^* < \theta < L_1^*\end{array}\right\} \tag{18}$$

or equivalently, $\min\left(L_1^*, L_2^*\right) < \theta < \max\left(L_1^*, L_2^*\right)$, (19)

where

$$L_1^* = \left[2K_{02} - 2\left(\frac{n'}{1-f'}\right)\sqrt{\left(\frac{1-f'}{n'}\right)^2\left(1-K_{02}\right)^2 + \frac{1}{4}\left(\frac{1-f'}{n'}\right)\left(\frac{1-f''}{n}\right)\frac{C_x^2}{C_z^2}\left(3-4K_{01}\right)}\right]$$

and

$$L_2^* = \left[2K_{02} + 2\left(\frac{n'}{1-f'}\right)\sqrt{\left(\frac{1-f'}{n'}\right)^2\left(1-K_{02}\right)^2 + \frac{1}{4}\left(\frac{1-f'}{n'}\right)\left(\frac{1-f''}{n}\right)\frac{C_x^2}{C_z^2}\left(3-4K_{01}\right)}\right]$$

(iii) $MSE\left(\hat{t}_{red}\right) - MSE\left(\bar{y}_{red}\right) < 0$ if

$$\left.\begin{array}{ll}\text{either} & 0 < \theta < 4K_{02} \\ \text{or} & 4K_{02} < \theta < 0\end{array}\right\} \tag{20}$$

or equivalently, $\min(0,\ 4K_{02}) < \theta < \max(0,\ 4K_{02})$. (21)

3.2. *When the scalar θ_0 exactly coincides with its true optimum value θ_0^**

From (5), (6), (7) and (15), we get

(iv) $\min MSE(\hat{t}_{red})$ - $MSE(\bar{y}_{rd}) < 0$ if (22)

$$\left\{\left(\frac{1-f''}{n}\right)(4K_{01}-3)C_x^2 - 4\left(\frac{1-f'}{n'}\right)\rho_{yz}^2 C_y^2\right\} < 0$$

which is always satisfied as long as $K_{01} < (3/4)$ (23)

(v) $\min MSE(\hat{t}_{red})$ - $MSE(\bar{y}_{rd}^{(C)}) < 0$ if (24)

$$\left\{\left(\frac{1-f''}{n}\right)(4K_{01}-3)C_x^2 - 4\left(\frac{1-f'}{n'}\right)C_z^2(1-K_{02})^2\right\} < 0$$

which is always true if $K_{01} <\ (3/4)$ (25)

(vi) $\min MSE(\hat{t}_{red})$ - $MSE(\bar{y}_{red}) < 0$ if

$$=\left(\frac{1-f'}{n'}\right)\rho_{yz}^2 C_y^2 > 0 \tag{26}$$

$\rho_{yz} > 0.$

Thus if the correlation between y and z is positive, the proposed class of estimators $\hat{t}_{red}$ (at optimum) is always better than the estimator $\bar{y}_{red}$ due to Singh and Vishwakarma (2007).

4. EMPIRICAL STUDY

To judge the merits of the proposed class of estimators $\hat{t}_{red}$ over the two-phase ratio estimator $\bar{y}_{rd}$, Chand's (1975) chain ratio type estimator $\bar{y}_{rd}^{(C)}$ and Singh and Vishwakarma's (2007) estimator $\bar{y}_{red}$, we have taken the four natural population data sets. The parameters of the populations are defined below:

Population I [Source: Anderson (1958), pp.97]

The variables are

y: Head length of second son

x: Head length of first son

z: Head breadth of first son

$N = 25,\quad n' = 10,\quad n = 7,\quad \bar{Y} = 183.84,\quad \bar{X} = 185.72,$

$\bar{Z} = 151.12,\quad C_y = 0.0546,\quad C_x = 0.0526,\quad C_z = 0.0488,\quad \rho_{yx} = 0.7108,$

$\rho_{yz} = 0.6932,\quad \rho_{xz} = 0.7346.$

Population II [Source: Singh (1967)]

The variables are

y: Number of females employed

x: Number of females in service

z: Number of educated females

$N = 61,\quad n' = 20,\quad n = 10,\quad \bar{Y} = 7.46,\quad \bar{X} = 5.31,$

$\bar{Z} = 179.00,\quad C_y = 0.7103,\quad C_x = 0.7587,\quad C_z = 0.2515,\quad \rho_{yx} = 0.7737,$

$\rho_{yz} = -0.2070,\quad \rho_{xz} = -0.0033.$

Population III [Source: Murthy (1967) pp.226]

The variables are

y: Output

x: Number of workers

z: Fixed capital

$N = 80,\quad n' = 25,\quad n = 10,\quad \bar{Y} = 5182.638,\quad \bar{X} = 283.875,$

$\bar{Z} = 1126.00,\quad C_y = 0.3520,\quad C_x = 0.9430,\quad C_z = 0.7460,\quad \rho_{yx} = 0.9136,$

$\rho_{yz} = 0.9413,\quad \rho_{xz} = 0.9859.$

Population IV [Source: Cochran (1977)]

The variables are

y: Number of 'placebo' children.

x: Number of paralytic polio cases in the placebo group.

z: Number of paralytic polio cases in the 'not inoculated' group.

$N = 34$, $n' = 15$, $n = 10$, $\overline{Y} = 4.92$, $\overline{X} = 2.59$,

$\overline{Z} = 2.91$, $C_y = 1.0123$, $C_x = 1.2318$, $C_z = 1.0720$, $\rho_{yx} = 0.7326$,

$\rho_{yz} = 0.6430$, $\rho_{xz} = 0.6837$.

For the purpose of efficiency comparison of the estimators $\overline{y}_{rd}$, $\overline{y}_{rd}^{(C)}$, $\overline{y}_{red}$ and the proposed class of estimators $\hat{t}_{red}$ over the usual sample mean estimator $\overline{y}$, we have computed the percent relative efficiencies (PREs) of the estimators w.r.t $\overline{y}$ using the formula:

$$PRE(\,.\,,\overline{y}) = \frac{MSE(\overline{y})}{MSE(.)} \times 100\,; \qquad \text{where } (.) = \overline{y},\ \overline{y}_{rd},\ \overline{y}_{rd}^{(C)},\ \overline{y}_{red} \text{ and } \hat{t}_{red}.$$

Findings are given in the Table 1.

TABLE 1

PREs of the estimators $\overline{y}$, $\overline{y}_{rd}$, $\overline{y}_{rd}^{(C)}$, $\overline{y}_{red}$ and $\hat{t}_{red}$ with respect to $\overline{y}$

S. No.	Estimator	$PRE(.,\overline{y})$ Population I	Population II	Population III	Population IV
1.	$\overline{y}$	100.00	100.00	100.00	100.00
2.	$\overline{y}_{rd}$	122.5362	144.1780	38.8489	116.6397
3.	$\overline{y}_{rd}^{(C)}$	178.8180	124.5562	36.6101	137.0018
4.	$\overline{y}_{red}$	123.2272	147.7192	180.6020	132.7205
5.	$\hat{t}_{red}$	188.3079	151.4388	364.8648	187.0466

From Table 1, it is observed that the proposed class of estimators $\hat{t}_{red}$ is more efficient than the sample mean estimator $\overline{y}$, two-phase ratio estimator $\overline{y}_{rd}$, Chand's (1975) chain ratio estimator $\overline{y}_{rd}^{(C)}$ and Singh and Vishwakarma's (2007) estimator $\overline{y}_{red}$ for the given data sets.

5. CONCLUSION

In this paper, we have suggested a class of chain ratio exponential type estimators in two-phase sampling. Its bias and mean squared error (MSE) are obtained upto the first degree of approximation. Theoretically, it has shown that both the proposed class of estimators $\hat{t}_{red}$ and asymptotic optimum estimator (AOE) in class of estimators $\hat{t}_{red}$ are more efficient than the two-phase ratio estimator $\overline{y}_{rd}$, Chand's (1975) chain-type estimator $\overline{y}_{rd}^{(C)}$ and Singh and Vishwakarma's (2007) estimator $\overline{y}_{red}$ in double sampling scheme under some realistic conditions. In support of the proposed class of estimators, an empirical study is carried out. It is observed that the suggested class of estimators $\hat{t}_{red}$ is more precise than the usual unbiased estimator $\overline{y}$, two-phase ratio estimator $\overline{y}_{rd}$, Chand's (1975) chain-type estimator $\overline{y}_{rd}^{(C)}$ and Singh and Vishwakarma's (2007) estimator $\overline{y}_{red}$. Thus we recommended our study for its use in practice.

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SUMMARY

A chain ratio exponential type estimator in two-phase sampling using auxiliary information

This paper advocates the problem of estimating the population mean $\overline{Y}$ of the study variable y using the information on two auxiliary variables x and z. We have suggested the family of chain ratio exponential type estimators in two-phase (or double) sampling. The bias and mean squared error (MSE) are obtained upto the first order of approximation. The suggested class of estimators is more efficient than the two-phase ratio estimator $\overline{y}_{rd}$, Chand's (1975) chain-type ratio estimator $\overline{y}_{rd}^{(C)}$ and Singh and

Vishwakarma's (2007) estimator $\overline{y}_{red}$ in two-phase (or double) sampling. An empirical study is given to justify the superiority of the proposed estimator.