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BVAR MODELS AND FORECASTING: A QUARTERLY MODEL FOR THE EMU-11

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1. INTRODUCTION

In the last 20 years, vector autoregressive models (VAR) have encountered enormous success and they have been extensively used for forecasting purposes. They have become valid substitutes/complements of the structural macroeconometric models (SMMs). The main advantage of VAR models with respect to SMMs is their higher manageability in the specification, estimation and simulation stages.

On the other hand, the most serious limitation of VAR models is their inefficient parameterisation. The quest for more efficient estimation methods, capable of delivering more reliable forecasts, is the main motivation of the Bayesian approach to VAIZ modelling (Bayesian VAK models, or BVAR: see Litterman, 1979; Doan *et* al., 1984; Litterman, 1986). This approach is based on the combination of prior and sample information which may be obtained by resorting to the Kalman filter (Hamilton, 1994, chapter 13), given the state-space representation of the VAR.

This paper is devoted to the costruction and evaluation of a quarterly forecasting BVAR model for the EMU-11 member states treated as a single country. In the current completion stage of monetary union, all the key macroeconomic variables are affected by episodes of turbulence, and even the most basic macroeconomic relationships are characterised by structural instability. Within a reduced form model framework, like a VAR one, it is not possible to perceive these instability phenomena as modifications of the structural parameters. Nevertheless, in order to model these episodes of higher turbulence in a proper way, we used time varying BVAR models (see Doan *et* al., 1984; Amisano *et* al., 1997) in this paper.

There are still some signs that the models we have estimated have some limitations, in spite of their good forecasting properties. For this reason, we believe that it is necessary to refine the KVAR methodology with the purpose of making it more suitable to contexts undergoing gradual transitions. In fact, the original time varying parameter BVAR methodology has a crucial limitation: its limited ability to deal with periods in which the transition phenomena are concentrated in sub-samples or they are different across subperiods. To tackle this problem, in the second part of this paper, we present an innovative approach in which we extend the traditional BVAR time varying parameter models: the intensity of parameter variation is governed by a time varying variance covariance matrix of the state equation error terms. We achieve this by increasing the dimensionality of the hyperparameter space. We provide some preliminary evidence on how this proposal works, based on a set of simulated data and on a restricted version of the EMU-11 model.

The paper is organised as follows. In section (2) we describe the structure of the models for EMU-11 area. In section (3) we discuss the most important choices that we have made when aggregating the single-country series in order to obtain area-wide series. In section (4) we comment on the results obtained and the forecasting properties of the estimated models. In section (5) we describe the methodological features of our proposal on how to deal with gradual transition phenomena and the results of some preliminary applications. Section (6) contains some conclusions and the directions of our future research.

2. THE STRUCTURE OF THE MODELS

Our choice of working with the EMU-11 aggregates is strategically motivated: while we are aware of the usefulness of single country models, it is clear that the attention of policymakers, financial operators and academic researchers is increasingly focussing on the EMU area taken as a single entity.

Moreover, the alternative choice of running separate country specific models and then pooling the forecasts has some obvious disadvantages. First of all, managing 11 forecasting models could be extremely time consuming, especially when one takes into consideration that data updating is not synchronous across different countries. Secondly, it would be necessary to formulate (and defend!) 11 different scenarios on the exogenous variables, with the unavoidable consequence of multiplying the possible sources of forecasting errors. Thirdly, running separate models, one loses the covariance structure across countries, so that the uncertainty around point forecasts is not properly measured: this is a big problem in the EMU area, where the single countries are deeply interdependent. When EMU-11 aggregates are used this cross-country correlation is synthetically accounted for.

In the light of these considerations, we have chosen to work with the EMU-11 aggregates, as if we were dealing with a single country. Our forecasting model is articulated in two blocks. The first one is devoted to forecasting a set of real variables (real variable section, henceforth RVS), whereas the second one (deflators section, henceforth DS) deals with the corresponding deflators and some other price indicators. Taking jointly into consideration the forecasts produced by the two blocks, it is possible to produce forecasts on nominal variables.

The choice of splitting up the model into two blocks parallels the one made in Amisano et al. (1997) for the Italian economy and it is mainly justified by the aim of limiting the dimension of each single model for practical purposes.

The type of interdependence between the two sections of the model is such that some variables which are endogenous in the first section appear as exogenous variables in the second one. In this way, we keep the need to produce scenarios for the exogenous variables at a minimum.

The deflators section of the model is upstream and the real variable section is downstream. The real variable section (RVS) has 7 endogenous variables; six of them are the key quarterly accounts indicators, *i.e.* GDP (Y) and its main constituents: investments, disaggregated into business and machinery investments (MI) and construction investments (CI), private consumptions (C) and finally exports (X) and imports (Q). Of course, these two variables do not record intra-EMU trade, which is recorded as consumption or investment. The seventh variable is industrial production (IP). We include this variable in the model in order to verify whether our simplified forecasting structure can produce sensible indications on the tendencies of the productive system.

To avoid collinearity problems, the endogenous set does not contain all GDP components: the excluded components are changes in stocks and public consumptions.

The exogenous variables set contains 11 variables. A first block of variables is needed to measure the degree of competitiveness of the EMU-11 area products, and to forecast the international trade flows. In this group we have two exchange rates (German Mark/Japanese Yen, MY, and German Mark/US Dollar, MD), the terms of trade (ToT), measured as ratio between EMU-11 import and export deflators, and an indicator of world demand (WDEM), which is obtained by summing the imports of those areas ([1]) with high levels of imports from the EMU-11.

A second block of exogenous variables aims at forecasting aggregate demand and activity. This group of variables comprises a measure of the degree of market confidence (MCONF), public sector revenues and expenditures ([2]) (PSREV and PSEXP), an area-wide measure of inflation (INFL) which is relevant for its effects on internal demand. In this group we have also the EURO 3 month interbank interest rate (3ME), the 10 year German benchmark rate (10YD), and a measure of degree of capacity utilisation (CU), which is very important to forecast IP and usually leads investment.

All exogenous variables are included with their current values and the first two lags.

It should be noted that the true number of exogenous variables is less than 11: in fact, ToT and INFL are exogenous in this section of the model but they are endogenous in the deflators one. Moreover, other 5 exogenous variables (MY, MD, 3ME, 10YD and the WDEM indicator) appear as exogenous variables in the DS section.

The specification of RVS allows for a deterministic part formed by an intercept term and 3 impulse dummies ([3]).

The deflators section has 8 equations. A first block of 6 equations deals with the deflators of the corresponding variables appearing in the first section of the model:

([1]) USA, UK, Switzerland, Japan, Brasil, Argentina, Russia, Poland and all Asia.

([2]) Comprising interest paid on public sector debt.

([3]) These step dummies are introduced to deal with abnormal observations, *i.e.* 1983Q4, 1991Q1 and 1993Q1.

the GDP deflator (DGDP), the deflators of the two investiment aggregates (DMI and DCI), the consumption deflator (DC), the import and export deflators (DQ and DX).

The last two equations of the model are devoted to the harmonised CPI index (CPI) and a PPI index (PPI). The role of PPI is similar to that of IP in the real variable section of the model, i.e. to anticipate productive tensions.

There are 8 exogenous variables, five of which are in common with the other section (MD, MY, 3ME, 10YD and WDEM). In addition, we have two raw materials price indexes, NONOIL and OIL, with obvious meanings. The last exogenous variable is a nominal wage variable, measuring the state of the labour market.

As in the previous section of the model, exogenous variables are included with their current values and their first two lags, and the deterministic part includes an intercept term and some impulse dummies.

All variables included in either section of the model are seasonally adjusted (when needed) and appear in logs, bar the interest rates. The two sections of the model are VARs with 4 lags and the sample period is 1980Q1-1999Q1. All series concerning national or EMU-11 aggregates come from Eurostat or Datastream.

Both blocks are modeled as time varying parameters BVARs. According to the standard BVAR approach (Litterman, 1979; Doan et al., 1984; Litterman, 1986, Sims, 1989), each equation of the VAR is estimated separately and the prior distribution is Gaussian with unit prior mean on the first lag coefficients of the dependent variable, whereas all other parameters are given zero prior mean (Minnesota *prior*). The prior variance-covariance matrix ($\mathbf{Q}_{i0}$, $i = 1, 2, \ldots, n$) of the parameters in each equation is diagonal with diagonal elements specified by means of a small vector of hyperparameters: iocussing on the i-th equation, and calling ψ_{ij} the coefficient on the j-th deterministic variable, $a_{ij,k}$ and $b_{ij,k}$ the coefficients on the k-th lag of the j-th endogenous variable and of the j-th exogenous variable respectively, the prior variances are set as follows:

$$[v(\psi_{ij})]^{1/2} = \pi_3^i, [v(a_{ij,k})]^{1/2} = \pi_1^i \cdot k^{\pi_4^i} \cdot \pi_2^{ij} \cdot \frac{\sigma_{ii}}{\sigma_{jj}}, [v(b_{ij,k})]^{1/2} = \pi_1^i \cdot k^{\pi_5^i} \tag{2.1}$$

$$\pi_1^i > 0, \pi_2^i > 0, \pi_3^i > 0, \pi_4^i < 0, \pi_5^i < 0,$$

where σ_{ii} is the i-th diagonal element of Σ, the variance-covariance matrix of the disturbance terms of the VAK.

As for parameter time variability, let us indicate β_{it} the coefficient vector of the i-th equation of the VAR; the state equation is:

$$\beta_{it} - \hat{\beta}_{i0} = \pi_8^i(\beta_{it-1} - \hat{\beta}_{i0}) + \eta_{it}, \eta_{it} \sim N(0, \Omega_i), \quad 0 \le \pi_8^i \le 1, \tag{2.2}$$

where $\hat{\beta}_{i0}$ is the prior expected value of β_{i0}; the hyperparameter π_8^i tunes the decay of the state vector towards its prior mean. The state equation variance-covariance matrix R, is specified by means of the hyperparameter π_7^i:

$$\Omega_i = \pi_7^i Q_{i0} \tag{2.3}$$

The hyperparameters (collected in the vector ξ) are chosen so as to optimise the forecasting performances of the model: in this case we maximize the Theil's-U index at a given forecasting horizon h, i.e. the ratio between the h-step ahead Root Mean Square Errors (KMSE) of the model and the one of a random walk model (naiveforecast).

The hyperparameters of both models have been calibrated using the 1991:1-1999:1 subsample as forecasting properties assessment period. Obviously, this subsample contains all the relevant turbulences that have affected the European economies in the last years.

3. DATA DESCRIPTION AND AGGREGATION PROBLEMS

In order to deal with the EMU-11 area as a single country, it is necessary to solve some preliminary problems, concerning the construction of appropriate economic time series.

Although in the EMU-11 case Eurostat already produces the relevant aggregates, we have chosen to re-construct them by aggregation of country indicators. We decided to do so for two reasons. Firstly, some Eurostat series are available only for the 1990s; secondly, we felt the need to explore the technical characteristics of the data set, and to make it possible to carry out a quick update of the aggregate series in the case of other economies joining the EMU.

This choice meant that the following 2 problems had to be solved: (1) how to convert country data into a common currency; (2) how to aggregate [4] the country indicators to obtain an area wide indicator.

In order to better understand the implications of how to solve (1), we will start by describing (2). The approach we have chosen is coherent with the one adopted by many international organisations (such as Eurostat) and by many applied researchers (see for instance Bikker, 1998). All magnitudes expressed as pure numbers and harmonised across countries (for instance national accounts indicators), the EMU-11 series were obtained as simple sums of the country values expressed in a single currency. Calling Y a given nominal variable, we have:

$$Y_{EMU}^{N} = \sum_{i \in EMU} Y_i^N \tag{3.1}$$

The same holds for real aggregates so that the corresponding deflator is obtained as:

$$DY_{EMU} = \frac{Y_{EMU}^{N}}{Y_{EMU}^{R}} = \frac{\sum_i Y_i^N}{\sum_i Y_i^R}, \qquad i \in EMU \tag{3.2}$$

As for the variables expressed as indexes (e.g. IP or wages), the EMU-11 measure is a weighted sum of the single country series, with weights given by the GDP (expressed in current prices, common currency) quota of each country:

(4) Having solved (1).

$$Y_{EMU} = \sum_{i \in EMU} \beta_i Y_i\,, \qquad \beta_i = \frac{GDP_i}{GDP_{EMU}}\,, \qquad i = 1,\ldots,11 \tag{3.3}$$

The problem of how to convert all the single country series into a common currency is more complicated. To this end, we can alternatively use the bilateral nominal exchange rates, or the bilateral PPPs (5) with respect to the currency chosen as the denominator. In either case, the most important choice is the one between a constant rate and a time series of conversion rates. The literature on this topic (see for instance Winder, 1997) suggests that to obtain aggregate variables in real terms it is necessary to use a constant conversion rate (6). In this way, we ensure that no price dynamics is introduced arbitrarily in the resulting real aggregates. In fact, if we used the current exchange rate, the behaviour of the resulting aggregate would be directly influenced by the evolution of the bilateral exchange rate.

Moreover, the choice of a constant conversion rate, together with the aggregation described by (3.1) and by (3.3), is supported by some general properties. which hold for both resulting nominal and real aggregates To illustrate these properties, let us consider a variable, for instance GDP, and by means of (3.1) let us compute its EMU-11 aggregate, by using the nominal exchange rate of a base year (e_{by}):

$$Y_{EMU} = \sum_{i \in EMU} Y_i = \sum_{i \in EMU} \frac{Y_i}{e^i_{by}} \tag{3.4}$$

Note that the percentage growth rate of the EMU-11 GDP ($\tilde{Y}/Y$) can be written as:

$$\frac{\tilde{Y}_{EMU}}{Y_{EMU}} = \frac{\sum_{i \in EMU} \frac{Y_i}{e^i_{by}}}{Y_{EMU}} = \sum_{i \in EMU} \left(\frac{\frac{Y_i}{e^i_{by}}}{Y_{EMU}} \right) \frac{\tilde{Y}_i}{Y_i} \tag{3.5}$$

In other words, the EMU-11 GDP growth rate (and the growth rate of all area wide aggregates) is a weighted average of the growth rates of the single country components, with weights given by the quota of each country variable on the area wide aggregate Note also that the resulting growth rate is invariant with respect to the choice of the common currency used for the aggregation These properties disappear if the conversion rate is the current exchange rate (or PPP) These properties are particularly appealing for interpreting the resulting forecasts in the light of those of its single country components.

The good properties of using a constant conversion rate hold also when dealing with aggregate deflators, obtained as the ratio between current and constant price series. To provide an example, let us consider the EMU-11 GDP deflator(DY_{EMU})

(5) Purchasing Power Parities.

(6) Nominal exchange rate or PPP of a base year.

$$DY_{EMU} = \frac{Y^N_{EMU}}{Y^R_{EMU}} = \frac{\sum \frac{Y_i^N}{e^i_{by}}}{\sum \frac{Y_i^R}{e^i_{by}}} = \sum_{i \in EMU} \left(\frac{\frac{Y_i^R}{e^i_{by}}}{Y^R_{EMU}} \right) DY_i \tag{3.6}$$

Expression (3.6) states that even the aggregate deflator (and, with a small approximation (7), also its variations) is a weighted average of the single country deflators, with weights given by the quota of each country variables on the area-wide aggregate.

In the light of these considerations, in this paper we adopt as conversion rates the bilateral exchange rates with respect to the German Mark, using 1995 as a base year.

4. THE RESULTS OF THE EMU-11 MODEL

4.1. *General comments*

We have already pointed out that the last part of the sample period is affected by instability phenomena which are mainly (but not completely) concentrated in the 1992/1993 period. This is particularly evident for the constant price variables, whereas the behaviour of the deflators seems to be less influenced by such phenomena.

The turbulence period starts immediately after the signing of the Maastricht rrcaty and the completion of the Single Market, and it does not stop at the outset of the European recession of 1993 For these reasons, we suspect that it might signal a deeper transition process brought about by the EMU

The behaviour of the two models (as gauged by Theil's U indexes and by control forecasts) seems to be satisfactory. The frequent turning points are correctly anticipated without significant delays, and the signs of predicted quarter-over-corresponding quarter (qcy) growth rates (8) arc almost always in line with those of the actual series. The forecasting performance is uniformly superior to that of the corresponding non-Hayesian VARs.

However, there are some signs of properties of the model which are not completely satisfactory, which we summarise as follows:

1. In the RVS, the CI equation has Theil's U's marginally above one.

2. In some cases, hyperparameter configurations that are capable of reducing further Theil's U's have a negative influence on control forecasts in the last 4 quarters. In other words, the hyperparameter configuration is not optimal with respect to the last 4 observations. Moreover, the optimal hyperparameter configuration is not robust to the insertion of new observations.

(7) Winder (1997)

(8) With this expression, we mean the growth rate of a variable with respect to its value 4 quarters before.

3. Sometimes, some relevant trade-offs across different forecasting horizons arise.

4. The forecasting performance is highly sensitive to the calibration of the hyperparameters on the deterministic variables and of those governing the degree of time variability.

Taking all these points into consideration, we believe that in our context it is necessary to adopt a modified strategy to account for parameter time variability. We need to model the gradual transition processes connected to the EMU with a more appropriate framework, as documented in the proposal contained in section (5) of this paper.

4.2. *Forecasting properties of the real model*

In table 1, Theil's U indexes from 1 to 4 step ahead are reported. Table 2 contains the control forecasts for the endogenous variables in logs and table 3 for the qcq changes.

In general, the model has good forecasting properties for almost all equations. The first equation (Y) has good Theil's U's values at all forecasting horizons (from 0.712 to 0.534). The forecasts for the series in levels and the qcq changes show that the model can forecast GDP very well.

The second equation (MI) also shows good results. Theil's U's are satisfactory (from 0.642 to 0.521), but we note that, although the model is capable of producing forecasts very close to the actual values on the control period, the forecasts follow the slowdown in the growth rate of the actual series occurred in 1998:4 only with a delay.

The third equation (CI) deserves the title of the "worst equation of the model". Beside the values of Theil's U's, which points at the difficult forecastability of the CI series (from the 2 step ahead onwards, the indexes are marginally above 1), we note a clear tendency of the model to over-predict CI, even if there is a gradual narrowing of the gap between forecasts and actual values.

The forecasts produced by the fourth equation (C) have much better properties. Theil's U's are clearly satisfactory (from 0.704 to 0.615) and show that the estimated equation can closely mimic the behaviour of actual consumption data. This is particularly evident from the analysis of the qcq growth rates: the deceleration of the private consumption growth rate is correctly picked-up.

TABLE 1

Theil's U's of the real model

Steps	GDP	IM	IC	C	Q	X	PI
1	0.712	0.642	0.984	0.704	0.933	0.783	0.602
2	0.544	0.459	1.028	0.594	0.783	0.694	0.69
3	0.516	0.466	1.047	0.577	0.662	0.548	0.769
4	0.534	0.521	1.041	0.615	0.629	0.479	0.845

TABI F 2

Control forecasts for the real model, logs

Date	FOR(Y)	Y	Date	FOR(Q)	Q
98.02	14.71571388	14.716455	98.02	12.77992	12.738779
98.03	14.72244482	14.722427	98.03	12.78424	12.710312
98.04	14.7255072	14.724498	98.04	12.78226	12.676373
99.01	14.73132344	14.727506	99.01	12.78356	12.65531
Date	FOR(MI)	MI	Date	FOR(X)	X
98.02	12.28198733	12.280695	98.02	12.84335	12.858467
98.03	12.29887116	12.303887	98.03	12.76789	12.814487
98.04	12.31881087	12.302671	98.04	12.75001	12.778376
99.01	12.3335576	12.331863	99.01	12.77806	12.772622
Date	FOR(CI)	CI	Date	FOR(IP)	IP
98.02	12.43808729	12.401047	98.02	4.723066	4.714937
98.03	12.43927556	12.40845	98.03	4.731942	4.71677
98.04	12.43373078	12.416449	98.04	4.736985	4.712181
99.01	12.45267504	12.443221	99.01	4.740673	4.712181
Date	FOR(C)	C			
98.02	14.21554489	14.214613			
98.03	14.22432057	14.226438			
98.04	14.23117226	14.235312			
99.01	14.23836647	14.237956			

TABLE 3

Control forecasts, real model, qcq variations

Date	FOR(D_Y)	D_Y	Date	FOR(D_Q)	D_Q
98.02	2.805367445	2.881586566	98.02	10.86827386	6 400018925
98.03	2.79731668	2.795484905	98.03	10.36496934	2 499983387
98 04	2.3980107	2.294722309	98.04	9.94716993	I 099990228
99.01	2.19110355	1.801738949	99.01	1.055105733	11 10861902
Date	FOR(D_MI)	D_MI	Date	FOR(D_X)	D X
'18.02	7.270170808	7.13161879	98.02	4.606266569	6 200047351
98.03	7.852632191	8.394962234	98.03	·2.166247654	2 499961505
98.04	8.611448348	6.872544632	98.04	−3.865930799	I 099987676
99.01	6.849765643	6.668851101	09.01	−9.078649225	-9 571979018
Date	FOR(D CI)	D_CI	Date	FOR(D_IP)	D_IP
98.02	2.522182745	-1.205799678	98.02	5.991013428	5 132890977
98.03	2.970610188	-0.155093122	98.03	5.189269579	3 605336189
98.04	1.89838971	0.152533492	98.04	4.042842408	1 493919512
99.01	2.038809943	1.078676211	99.01	3.079872694	0 184369752
Dare	FOR(D_C)	D .C			
98.02	2.352771637	2.257434642			
98.03	2.860204087	3.07823459			
98.01	2.792466162	3.2188818			
9'1.01	2.735903769	2.693742582			

Equation 5 (Q) has some problems too: despite the fact that Theil's U's are more or less satisfactory (from 0.933 to 0.629), we can notice a persistent difference between forecasts and actual values of the series. Note that the 4-step-ahead predicted qcq growth rate is very close to zero, whereas its actual value is negative. This could be due to the abnormal evolution of the EMU-11 imports in the last two years of the sample, which has negatively influenced the comparison between predicted and actual values.

Equation 6 (X) has better forecasting properties than the previous one. Theil's U indexes are satisfactory (from 0.783 1 step ahead to 0.479 4 steps ahead). We notice that predicted values are close to actual values, although the forecasts emphasise the decrease of X during the control period. In fact, the predicted qcq growth rates become negative one period earlier than the actual ones. In any case, the exports trend is correctly picked up by the model.

Equation 7 (IP) shows fair properties in terms of Theil's U's (from 0.602 to 0.845) and the control forecasts follow the profile of the series, even if they tend to exaggerate the actual series evolution. The profile of qcq changes is correctly reproduced by forecasts.

4.3. *Forecasting properties of the deflators model*

In table 4 we report 1- to 4-step-ahead Theil's U indexes for all equations of the deflators model. Table 5 contains the control forecasts for the logs of the variables over the period 1998:2 -1991:1, while table 6 contains the control forecasts for the qcq growth rates.

Although this section of the model has in general good performances for almost all equations, there are clearly two different groups of equations: those showing very good forecasting properties, and those with less satisfactory performances. The first equation (DGDP) belongs to the first group. Theil's U's are satisfactory (from 0.634 to 0.423) and the predicted values are close to the actual ones. The second equation (DMI) generates worse forecasts than the previous one. Theil's U's are relatively high (from 0.976 to 0.914) and the control forecasts are not satisfactory at all. In particular, despite the fact that the actual series moves upwards, with a clear decrease in the last sample observation, the forecast series is slightly increasing throughout the whole control period. The DCI deflator equation generates better results than the previous equation, as far as Theil's U's are concerned (from 0.655 to 0.588), and we note that the predicted values are capable of track-

TABLE 4

Theil's U's, deflators model

Steps	DGDP	DMI	DCI	DC	DQ	DX	CPI	PPI
1	0.634	0.976	0.655	0.688	0.721	0.568	0.569	0.988
2	0.475	0.964	0.573	0.587	0.709	0.509	0.551	0.958
3	0.433	0.941	0.599	0.503	0.528	0.367	0.492	0.91
4	0.423	0.914	0.588	0.461	0.437	0.295	0.374	0.983

TABLE 5

Control forecasts, deflators model, logs

Date	FOR(DGDP)	DGDP	Date	FOR(DQ)	DQ
98.02	4.781584482	4.78913	98.02	4.572416589	4.602933398
98.03	4.783910344	4.7891	98.03	4.557501237	4.513508034
98.04	4.784495394	4.790672	98.04	4.561345289	4.490957106
99.01	4.787604345	4.789078	99.01	4.547196275	4.511798279
Date	FOR(DMI)	DMI	Date	FOR(DX)	DX
98.02	4.652229374	4.652725	98.02	4.570715399	4.584114792
98.03	4.652307919	4.654618	98.03	4.580770365	4.576154199
98.04	4.652375335	4.654226	98.04	4.606581849	4.593605868
99.01	4.652439073	4.64517	99.01	4.572295111	4.570733758
Date	FOR(DCI)	DCI	Date	FOR(CIP)	CIP
98.02	4.763795079	4.776259	98.02	4.832060255	4.835099
98.03	4.76325439	4.774204	98.03	4.834024567	4.836364
98.04	4.762184495	4.771706	98.04	4.83557879	4.835683
99.01	4.761645235	4.77007	99.01	4.836948082	4.838305
Date	FOR(DC)	DC	Date	FOR(PPI)	PPI
98.02	4.79864376	4.800771	98.02	4.621457043	4.620179327
98.03	4.801335471	4.800732	98.03	4.619313292	4.616149305
98.04	4.802514483	4.813568	98.04	4.617711063	4.606742
99.01	4.799391239	4.802895	99.01	4.615601329	4.600488

TABLE 6

Control forecasts, deflator model, qcq growth rates

Date	FOR(D(DGDP))	D(DGP)	Date	FOR(D(DQ))	D(DQ
38.02	0.750271949	1.513360227	38.02	-3.833894103	-0.853973804
98.03	1.076490641	1.602406294	98.03	-4.106104367	-8.233333333
98.04	0.710848941	1.334825247	98.01	-5.010409417	-11.46666667
99.01	1.042834549	1.191846557	99.01	-4.685059354	-8
Date	FOR(D(DMI)	D(DMI)	Dare	FOR(D(DX))	D(DX)
98.02	-0.480239683	-0.430902859	98.02	-1/806281719	-0.481691005
98.03	-0.504883848	-0.274776353	98.03	0.261758364	-0.2
98.04	-0.570019464	-0.385837464	98.04	0.799523165	-0.5
99.01	0.02991182	-0.694576589	99.01	-0.51145115	-0.666666667
Dare	FOR(D(DCI))	D(DCI)	Dare	FOR(D(CPI))	D(CPI)
98.02	-0.396445291	0.852774431	98.02	1.043533637	1.351046124
98.03	-0.347269945	0.749884318	98.03	0.874128568	1.110393074
98.04	-0.75423198	0.195250199	98.04	0.803216249	0.813721524
99.01	-0.357834715	0.485173172	99.01	0.714239131	0.850992845
Date	FOR(D(DC))	D(DC)	Date	FOR(D(PPI))	D(PPI)
98.02	0.941094433	1.156048897	98.02	0.143481382	0.015608174
98.03	1.159448867	1.098420504	98.03	-0.505262447	-0.819565036
98.04	0.006246754	1.117900569	98.04	-0.80904153	-1.891127788
99.01	0.619132626	0.972296384	99.01	-0.77714251	-2.265455148

ing the evolution and the turning points of the actual series. Equation 4 (DC) has good predictive properties, especially for the 4 step ahead forecasts (Theil's U's decrease from 0.688 to 0.461 as the forecast horizon increases). Taking into consideration both logs and qcq growth rates forecasts, this equation does better 4 steps than 1 step ahead. The same considerations hold for equation 5, which produces forecasts for the import deflator (DQ). The sixth equation (DX) has very good Theil's U's (from 0.568 to 0.295), and the control forecasts are satisfactory, especially those of the logs, from which one can see that the turning points are correctly picked up.

Equation 7 (CPI) is the most interesting one, since it can be used to forecast inflation. As we can see from table 4.3.1, this equation has very good Theil's U's (from 0.569 to 0.374). The CPI behaviour is correctly predicted, even if there is a slight systematic under-prediction. On the other hand, equation 8 (PPI) has very high Theil's U's values (from 0.988 to 0.983), although the control torecasts show that the tendencies of this indicator are correctly predicted. In any case, the decrease of production prices in the relevant period has been very dramatic and it was a priori hardly foreseeable.

5. A NEW PROPOSAL FOR TREATING STRUCTURAL CHANGE

5.1, *Methodology*

In the BVAR context, a subset of ξ, the hyperparameters vector, is particularly important for the econometric treatment of transition/structural change phenomena. These hyperparameters, which we indicate with ξ_1, determine Ω_i, i = 1, 2, ..., n, the variance covariance matrix of the transition equation error terms for each equation of the VAR. *Coeteris paribus*, if we consider two possible configurations for Ω_i, sap Ω_{i0} and Ω_{i1} with $\Omega_{i1} - \Omega_{i0}$ positive definite, by using Ω_{i1} we have a potentially higher time variability of the parameters than that produced by Ω_{i0}. In general, in the BVAR approach hyperparameters are calibrated and constant for all the sampling period. We believe that, in order to successfully model gradual transition phenomena, it is necessary to use a specification in which hyperparameters define a time varying Ω_i matrix:

$$\Omega_{it} = \Omega_i(\xi_1, t) \tag{5.1}$$

As a simple example, let us assume that the model being used has only one parameter, α_t, with a transition equation affected by the error term η_t. The variance of η_t is $m_{,,}$ which is determined by the following modification of (2.3):

$$\omega_t = [\omega_0 + \theta_1 \cdot s_t \cdot (t - T_0)^{\theta_2} \cdot \exp(-\theta_3 \cdot (t - T_0))] \cdot q_0 \,, \tag{5.2}$$

$$s_t = 0,\ 1,\ \theta_1 > 0,\ \theta_2 > 0,\ \theta_3 > 0,\ T_0 = \text{last period in which } s_{T_0} = 0 \,.$$

We call this kind of model DVI (Dynamic Variability Intensity). Note that in this way we have a discrete state variable, s_t, which we can consider as an indicator variable associated to the state of low ($s_t = 0$) or high ($s_t = 1$) parameters variability. The hyperparameters in the vector $\theta = [\theta_1 \theta_2 \theta_3]'$ define how the potential variability of the parameters is allowed to increase, via a corresponding increase in the transition equation error terms variance, and in which way this variance evolves through time (see figure 1, showing some possible degrees of time evolution of ω_t).

In our view, some aspects deserve special attention. Firstly, it is necessary to establish in which state ($s_t = 0$ or $s_t = 1$) the system is at each sample (or post-sample) observation. This could in principle be achieved in two different ways:

a) it is possible to impose that the system moves from one state to the other on the occurrence of specific events, such as, for example, the transition from a higher to a lower wage indexation scheme. This entails imposing dogmatic priors on the state of the system for the different observations.

h) It is possible to treat s_t as an unobservable variable, with some transition properties (*i.e.* Markovian), and let the model itself decide how to assign each observation to different states, via application of an apt filter (see Hamilton, 1994; Lindgren, 1978), according to the smoothed probabilities

Another problem is that of determining the hyperparameters. We believe that the best way is to treat the model in hierarchical terms, as in Chib and Greenberg (1995), and to verify whether this approach yields good properties for the estimated models.

5.2. *Choice* among competing models

This new proposal of tuning the parameters time variability intensity has to be compared with the traditional BVAK methodology. In Bayesian terms, the choice between two competing models, M_1 and M_2, is made by constructing the posterior odds ratio (POR):

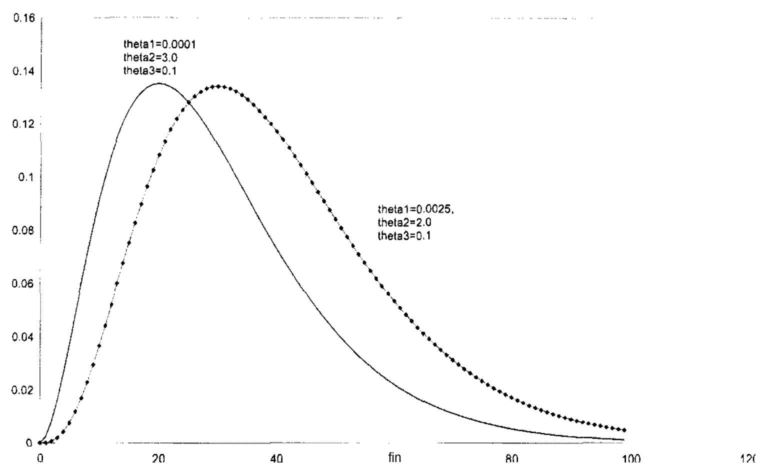

Figure 1 - Values of ω_t corrisponding to different configurations of θ_1, θ_2 and θ_3.

$$POR_{1|2} = \frac{p(M_1|\mathrm{y})}{p(M_2|\mathrm{y})} = \frac{p(M_1)}{p(M_2)} \cdot BF_{1|2}, BF_{1|2} = \frac{p(\mathrm{y}|M_1)}{p(\mathrm{y}|M_2)}, \tag{5.3}$$

In this case, we compare two models: (1) M_1 = HVAK model with DVI; (2) M_2 = standard BVAR model. We choose M, if $POR_{1|2}$ is higher than one. Note that M_2 is nested within M,, given that M_2 is obtained from M, just by imposing that the hyperparameters vector θ (controlling DVI) is equal to a vector of zeros. If we assign to each model equal prior probabilities, the *POR* coincides with the Bayes factor RF, $_{|2}$. The evaluation of BFs is generally very difficult in most applications, (see Geweke, 1999). For this reason, we resort to the asymptotic approximation described by Bernardo and Smith (1994, p. 487), and choose the model with the minimum BIC criterion:

$$BIC_i = -2 \cdot \ln (m(\mathrm{y}\,|\,M_1, \lambda_1^*)) + l_i \cdot \ln (T) \tag{5.4}$$

The properties of the approximation used in this context are unknown; in future research, we intend to use exact simulation techniques in order to directly evaluate the *POR*.

5.3. *Results*

In this subsection we show the results of some applicationc, used as an example to verify the applied properties of our proposal. In order to simplify computations, we decided to work with a more "parsimonious" prior distribution, *i e* , taking into conrideration (2.1), we specify the prior distribution symmetrically across all the equations of the VAR, in the following way:

$$\pi_j^i = \pi_j, \qquad i = 1,2,\ldots,m, \qquad j = 1,3,4,5,6,7,8 \tag{5.5}$$
$$\pi_2^{ij} = \begin{cases} \pi_2 \forall i \neq j \\ 1 \forall i = j \end{cases}$$

Hence, we have a very small set of hyperparameters which can be easily dealt with. 'She complete ξ vector is:

$$\xi = \begin{bmatrix} \pi \\ \theta \end{bmatrix} = [\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4 \quad \pi_5 \quad \pi_6 \quad \pi_7 \quad \pi_8 \quad \theta_1 \quad \theta_2 \quad \theta_3]' \tag{5.6}$$

Moreover, for the sake of simplicity, we have set $\pi_8 = 1$ and $\pi_4 = \pi_5$.

The properties of the method that we propose have been assessed by running two different applications. The first one is based on a simulated data set, whereas the second one uses a small subset of the EMU·11 real variables model.

5.3.1. *Application on a simulated dataset*

The dataset being analysed has sample size equal to 300. The data have been generated by a time varying parameter VAR(1) with two equations, a stationary exogenous variable and an intercept term. The DGP parameters evolve over time according to (2 3) for the first 250 observation, with hyperparameters values corrisponding to our default configuration; for the remaining 50 observations, we use a DVI scheme, with a known initial time ('l;,= 251), and the values $\theta_1 = 1.0e - 006$, $\theta_2 = 4.0$, $\theta_3 = 0.2$. In that way, the peak of the DVI profile occurs at observation 269, with a value of 0.0033

We have used this generated dataset to estimate 2 different BVAR models: the first one (Model 1) reproduces the original DGP, whereas the second (Model 2) associates a uniform degree of parameter variability to the whole dataset, ignoring any DVI phenomenon (θ_1 is constrained to zero). For both models, we have numerically optimised the hyperparameters configuration. The objective function is the sum of the sample pseudo-likelihoods (see Doan *et al.*, 1984). To compare the two models, we have used Schwartz's BIC criterion (described in (5.4)), which takes into consideration the modal pseudo-lilteliliood function and the dimensionality ot the hyperaparameters vector

As one can easily see in table 7, the performance of Model 1 is clearly superioı to those of Model 2, since its BIC value (–2122.26)is much smaller than that of the no-DVI model (-2101.06). The values of Theil's U indexes (table 5.3.1) reveal the superiority of Model 1 throughout the forecasting horizon (1 to 20 steps): for the first equation, Theil's U's are below those of Model 2 in 18 cases out of 20 (the exceptions are 1 and 7 step ahead forecasts), whereas they show the superiority of Model 2 in all cases for the second equation. Finally, also on the grounds of control forecasts (figures 2 and 31, Model 1 seems to exhibit the best behaviour In

TABLE 7

Simulated data: BIC and Theil's U's

Mod.1 (DVI) BIC = –2322.26 Mod.2 (NO DVI) BIC = –2101.06

Steps	M1 (DVI) equation 1	M2 (NO DVI) equation 1	M1 (DVI) equation 2	M2 (No DVI) equation 2
1	0.0597685	0.0579542	0.0579805	0.0584998
2	0.1322360	0.1353727	0.1361636	0.1446297
3	0.1380773	0.1414824	0.1405733	0.1550932
4	0.137922	0.1431385	0.1465929	0.1658241
5	0.1167538	0.1225330	0.1274062	0.1504076
6	0.1259352	0.1299560	0.1452906	0.1653852
7	0.1416791	0.1404957	0.1725056	0.1893234
8	0.1806920	0.1847262	0.2248470	0.2378791
9	0.1900664	0.1977490	0.2454273	0.2563621
10	0.2420384	0.2523690	0.3045234	0.3127318
11	0.2317554	0.2435137	0.3015658	0.3083606
12	0.2413997	0.2571473	0.3182382	0.3245772
13	0.3034268	0.3244331	0.3800419	0.3907775
14	0.2850861	0.3102414	0.3684890	0.3813072
15	0.3037659	0.3338182	0.3925080	0.4094044
16	0.3456054	0.3821593	0.4391542	0.4622514
17	0.4038612	0.4477714	0.5051759	0.5358435
18	0.4594045	0.5125711	0.5731472	0.6128862
19	0.5085531	0.5722483	0.6355282	0.6845454
20	0.5018137	0.5774816	0.6452087	0.7053765

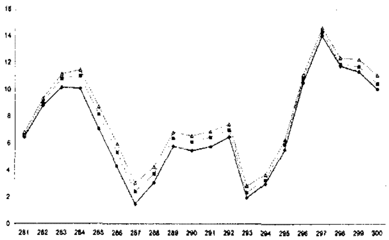

Figure 2 - Control forecasts, eq. 1.

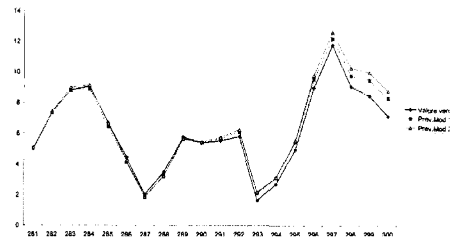

Figure 3 - Control forecasts, eq. 2

particular, for both equations of Model 1 the means and standard deviations of the 1 to 20 steps forecasting errors are always smaller than their counterparts obtained by using Model 2.

The capability of tracking the true DGP DVI profile is very good; the estimated values of θ correspond to an estimated peak at observation 270 (the true maximum is at observation 269), and the estimated maximum value of DVI is 0.00293 (whereas the true value is 0.0033). More generally, the entire estimated profile of DVI is very similar to the true DGP profile, as is evident from figure 4.

These encouraging results are fairly robust with respect to simulated datasets with different DGPs, with different sample sizes, and different DVI profiles. We have encountered some difficulties in the estimation and the forecasting steps, when the degree of DVI is particularly high, far from the ones that we consider reasonable and appropriate for real world phenomena.

5.3.2. A reduced *EMU-11 model*

Our second application is on real data: we use a simplified version of the constant price section of the EMU-11 model, in which we have eliminated the external trade flows. It is, in fact, a HVAK model with 3 equations, GDP (Y), total investments (I) and private consumption (C). We inserted only one exogenous variable, the terms of trade variable (ToT). The sample size is the same as in the original model (72 observations, from 1990:1 to 1999:1), and the last four observations have been put aside to generate the usual set of control forecasts.

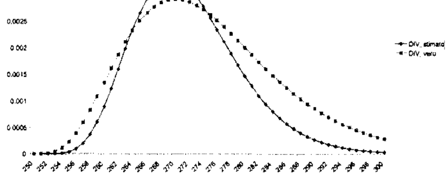

Figure **3** - Estimated and true DVI profiles.

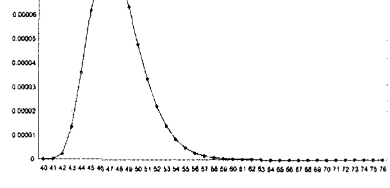

Figure 5 - Estimated DVI profile for the reduced EMU-11 model.

TABLE 8

Reduced EMU-11 model, BIC values

	Model 1 (DVI)	Model 2 (NO DVI)
BIC	-2089.5	-2033.08

As in the simulated data set example, we specify two different models, Model 1 and Model 2. Model 2 has 8, constrained to zero (no DVI), while Model 1 allows for DVI. In the case of Model 1, the initialisation for the vector 8 reflects a priori beliefs on the transition process; our beliefs in this respect are that the transition started in 1990:1 (observation 41), with the German unification process, and reached its maximum intensity at the end of 1993 (observation 56), after the big currency crises that hit the European economies in 1992 and 1993. These hypotheses are accommodated by initialising 8, = 0.0000001, $\theta_2 = 1.6$, 8, = 0.1. The estimated DVI profile is reported in figure 5.

The advantages of dealing with a DVI model are evident by looking at the RIC criterion values for the two models (table 8): Model 1 BIC is −2089.5, while Model 2 has −2033.08.

In general, the evidence gathered with this exercise leads us to make the following observations.

1. In the gradual convergence towards EMU, the area-wide aggregates and their interrelations have been characterised by some major changes.

2. We note that the traditional time varying parameter BVAR mechanism is not fully capable of modelling these transition phenomena. This fact can be used to explain some of the less than fully satisiactory properties of the EMU-11 models presented in the previous sections of this paper

3. The empirical evidence confirms that our procedure can be seen as a sensible solution to the problem.

4. Our a priori beliefs concerning the DVI process are substantially modified by the data. In fact, the θ hyperparameters obtained by numerical optimisation (8,= 1.3e - 007; $\theta_2 = 6.97$; 8, = 1.01) locate the DVI peak at 1991:3 (observation 47), whereas we originally thought that the maximum would be at observation 56 (1993:4). From 1991:3 on, the phenomenon gradually decreases, and it vanishes completely after the first half of 1994 (observation 58).

Summing up, the two experiments we have run confirm the usefulness of modelling gradual transition processes via time varying parameter schemes which are more articulated than the one of the standard BVAR approach. Our proposal in this respect, which is characterised by a DVI mechanism governed by a small set of hyperparameters, is preferable to the traditional BVAR parameters evolution scheme on a closed economy, stripped down version of the KVS section of the EMU-11 model.

Considering each endogenous variable, we note that the Y and I forecasts are the ones that show the biggest improvements with respect to the no-DVI model. In our view, this is quite interesting since we have seen in section (4) that the investment equations had the worst properties in the fully fledged, no DVI, EMU-11 model.

6. CONCLUSION

In this paper, we present a quarterly forecasting model for the the EMU-11 economies considered as a single country. The model has two sections: a constant price variables section and a section dealing with deflators. The model is a time varying parameter BVAR model which shows generally good forecasting performances and a good ability of tracking turning points without delays. Its forecasting performances, as measured by Theil's U indexes are largely superior to those of a non-Bayesian VAR model. In particular, on some crucial variables, such as GDP, consumption, exports, and inflation, the forecasts are particularly satisfactory.

The estimated models present some problems: the forecasting performances are not particularly brilliant for certain variables, such as construction investments and PPI. Moreover, the forecasting properties are very sensitive to the calibration of certain hyperparameters, and not robust with respect to the addition of new observations. Our interpretation of this problem is that the European economies have entered into a gradual transition process that has been revealed by a sudden worsening of the forecasting performances of the models based on the traditional time varying parameter approach.

Taking all these things into consideration, in the second part of this paper we present an innovative method for handling parameters variability in a KVAR approach. We show that our procedure has good properties, by using two different applications. The first one is based on simulated data, and the second one on a stripped down version of the EMU-11 model.

Some lines of research are at the top of our agenda. As for the EMU-11 model, we need to carefully assess the sensitivity of the model with respect to different scenarios by producing real out-of-sample forecasts. As regards our DVI proposal, we still have to apply it to the fully fledged version of the EMU-11 model, and see how it works for that application. Moreover, we have to investigate the possibility of modelling the DVI with different functional forms.

A further evolution is that of moving to a fully Hayesian approach, based on a hierarchical structure, to be analysed by means of MCMC techniques, as in Amisano and Serati (2000).

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RIASSUNTO

Modelli BVAR e previsione: un modello trimestrale per l'UME a 11 paesi

Questo lavoro è dedicato alla costruzione e alla ralutazione di un modello previsivo trimestrale, appartenente alla famiglia dei VAR bayesiani (RVAR), per il gruppo degli 11 paesi aderenti all'Unione Monetaria Europea (UME) trattati come un unico paese. In questa fase iniziale e transitoria del processo di completamento dell'UME, l'evoluzione di molte variabili economiche è caratterizzata da turbolenze e numerose relazioni macroeconmiche sono afflitte da instabilità strutturale. Per questi motivi, i modelli utilizzati in questo lavoro sono modelli RVAR a parametri variabili. Ad ogni modo, a fronte delle buone proprietà previsive di questi modelli, rimangono ancora segnali di una qualche loro parziale inadeguatezza. Alla luce di tali segnali, nella seconda parte del lavoro presentiamo un approccio innovativo sulla base del quale la tradizionale metodologia BVAR a parametri variabili viene estesa e modificata: l'intensità di variazione dei parametri viene governata per mezzo di una matrice di varianza e covarianza dei termini d'errore dell'equazione di stato anch'essa variabile nel

tempo. Ciò è possihile ampliando (in misura minima) la dimensione dello spazio iperparametrico. L'evidenza empirica, prodotta sia sulla base di dati simulati, sia nell'ambito di una version6 ristretta del modello sull'UME a 11 paesi, seppur preliminare, appare incoraggiante per quanto riguarda l'efficacia della nostra proposta.

SUMMARY

BVAR models and forecasting: a quarterly model for the EMU-11

This paper deals with the costruction and evaluation of a quarterly forecasting BVAR model for the EMU-11 countries treated as a single country. In the current stage of EMU completion, most variables are affected by turbulences, and many macroeconomic relationships are characterised by structural instability. For this reason, the forecasting models used in this paper are time varying RVAR models. There are still signs that the models me have estimated are affected by some limitations, in spite of their good forecasting properties. In the light of this, in the second part of this paper we present an innovative approach in which we extend the RVAR time varying parameter methodology: the intensity of parameter variation is governed by a time varying variance covariance matrix of the state equation error terms. This is achieved by slightly increasing the dimensionality of the hyperparameter space. We show some preliminary, encouraging evidence on how this proposal works, based on simulated data and on a restricted version of the EMU-11 model.