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A RATIO-CUM-PRODUCT ESTIMATOR OF POPULATION MEAN IN STRATIFIED RANDOM SAMPLING USING TWO AUXILIARY VARIABLES

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1. INTRODUCTION

A country or state frequently requires estimates of agricultural production to assess status of grain and to make future policies regarding export and import of grains according to need. It requires the estimates of total production, average production and per hectare production of any crop which corresponds to the problem of estimation of population total, population mean and ratio of two population means respectively. This paper discusses the problem of estimation of finite population mean using information on two auxiliary variates. Auxiliary information is often used by researchers in order to improve the efficiencies of estimators. Cochran (1940) used auxiliary information at estimation stage and envisaged ratio method of estimation that provides ratio estimator. Ratio estimator has higher efficiency when study variate and auxiliary variates are positively correlated. Robson (1957) developed product method of estimation that provides product estimator. When study variate and auxiliary variate are negatively correlated, product estimator gives higher efficiency in comparison to simple mean estimator provided correlation coefficient between study variate and auxiliary variate is greater than half of the ratio of coefficient of variation of auxiliary variate and coefficient of variation of study variate. In both, ratio and product methods of estimation, population mean of the auxiliary variate is assumed to be known. Singh (1967) utilized information on two auxiliary variates, one is positively correlated and another is negatively correlated with the study variate and suggested ratio-cum-product estimator of population mean in simple random sampling. Later many authors proposed various ratio and product type estimators in simple random sampling, for instance see Sisodia and Dwivedi (1981), Pandey and Dubey (1988), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh *et al.* (2004), Singh and Tailor (2005), Kadilar and Cingi (2006), Singh *et al.* (2009), Singh *et al.* (2011), etc.. Hansen *et al.* (1946) defined combined ratio estimator using auxiliary information in stratified random sampling. Many authors including Kadilar and Cingi (2003, 2005, 2006) and Singh *et al.* (2008) worked out ratio type estimators in stratified random sampling.

Simple random sampling technique has some shortcomings like less representative of different sections of the population, administrative inconvenience and less efficiency in case of heterogeneous population. Literature reveals that ratio-cum-product estimator performs better than ratio and product type estimators in simple random sampling under certain conditions.

This motivates authors to work out Singh (1967) ratio-cum-product estimator in stratified random sampling and study its properties.

Consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of size N and it is divided into L strata of size $N_h \, (h = 1, 2, ..., L)$. Let Y be the study variate and X and Z be two auxiliary variates taking values y_{hi}, x_{hi} and z_{hi} $(h = 1, 2, ..., L;\; i = 1, 2, ..., N_h)$ on i^{th} unit of the h^{th} stratum. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n = \sum_{h=1}^{L} n_h$ and we define:

$$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi} : h^{th} \text{ stratum mean for the study variate } Y,$$

$$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi} : h^{th} \text{ stratum mean for the auxiliary variate } X,$$

$$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi} : h^{th} \text{ stratum mean for the auxiliary variate } Z,$$

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h : \text{population mean of the study variate } Y,$$

$$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{X}_h : \text{population mean of the auxiliary variate } X,$$

$$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{Z}_h : \text{population mean of the auxiliary variate } Z,$$

$$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi} : \text{sample mean of the study variate } Y \text{ for } h^{th} \text{ stratum},$$

$$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi} : \text{sample mean of the auxiliary variate } X \text{ for } h^{th} \text{ stratum},$$

$\bar{z}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$: sample mean of the auxiliary variate Z for h^{th} stratum,

$W_h = \frac{N_h}{N}$: stratum weight of h^{th} stratum.

Usual unbiased estimators of population means $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ in stratified random sampling are defined respectively as

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \ , \tag{1}$$

$$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h \ , \tag{2}$$

$$\bar{z}_{st} = \sum_{h=1}^{L} W_h \bar{z}_h \ . \tag{3}$$

In the line of Cochran (1940) ratio estimator, Hansen et al. (1946) utilized known value of population mean $\bar{X}$ of auxiliary variate X and defined combined ratio estimator for population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right). \tag{4}$$

Here it is assumed that the study variate Y and the auxiliary variate X are positively correlated.

When the study variate Y and the auxiliary variate Z are negatively correlated, assuming that the population mean $\bar{Z}$ of auxiliary variate Z is known, combined product estimator is defined as

$$\hat{\bar{Y}}_{PC} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right). \tag{5}$$

The bias and mean squared error of $\hat{\bar{Y}}_{RC}$ and $\hat{\bar{Y}}_{PC}$, up to the first degree of approximation, are obtained as

$$B(\hat{\bar{Y}}_{RC}) = \frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2 \gamma_h (\mathrm{R}_1 S_{xh}^2 - S_{yxh}) \ , \tag{6}$$

$$B(\hat{\bar{Y}}_{PC}) = \frac{1}{\bar{Z}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yzh} \ , \tag{7}$$

$$MSE(\hat{\bar{Y}}_{RC}) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}) \, , \tag{8}$$

$$MSE(\hat{\bar{Y}}_{PC}) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}) \tag{9}$$

where $S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2$, $S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2$,

$S_{zh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (z_{hi} - \bar{Z}_h)^2$, $S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h)$,

$S_{yzh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(z_{hi} - \bar{Z}_h)$, $S_{xzh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)(z_{hi} - \bar{Z}_h)$,

$R_1 = \frac{\bar{Y}}{\bar{X}}$, $R_2 = \frac{\bar{Y}}{\bar{Z}}$ and $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h} \right)$.

2. PROPOSED ESTIMATOR

Assuming that the population means of auxiliary variates $\bar{X}$ and $\bar{Z}$ are known, Singh (1967) proposed a ratio-cum-product estimator for population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{RP} = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right) \left(\frac{\bar{z}}{\bar{Z}} \right) \tag{10}$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ are unbiased estimates of population means $\bar{Y}$ and $\bar{X}$ in simple random sampling without replacement.

We propose Singh (1967) ratio-cum- product estimator $\hat{\bar{Y}}_{RP}$ in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\right)\left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h}{\sum_{h=1}^{L} W_h \bar{Z}_h}\right). \tag{11}$$

To compare the efficiency of the proposed estimator in comparison to other estimators, bias and mean squared error of the proposed estimator are obtained. To obtain the bias and mean squared error expressions of the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$, we write

$$\bar{y}_h = \bar{Y}_h(1+e_{0h}),\ \bar{x}_h = \bar{X}_h(1+e_{1h}) \text{ and } \bar{z}_h = \bar{Z}_h(1+e_{2h})$$

such that $E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0$ and

$$E(e_{0h}^2) = \gamma_h C_{yh}^2,\ E(e_{1h}^2) = \gamma_h C_{xh}^2,\ E(e_{2h}^2) = \gamma_h C_{zh}^2,$$

$$E(e_{0h}e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh},\ E(e_{0h}e_{2h}) = \gamma_h \rho_{yzh} C_{yh} C_{zh}$$

$$E(e_{1h}e_{2h}) = \gamma_h \rho_{xzh} C_{xh} C_{zh}.$$

Expressing (11) in terms of e_i's, we have

$$\hat{\bar{Y}}_{RP}^{ST} = \sum_{h=1}^{L} W_h \bar{Y}_h (1+e_{0h}) \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \bar{X}_h (1+e_{1h})}\right)\left(\frac{\sum_{h=1}^{L} W_h \bar{Z}_h (1+e_{2h})}{\sum_{h=1}^{L} W_h \bar{Z}_h}\right)$$

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{Y}(1+e_0)(1+e_1)(1+e_2)^{-1}, \tag{12}$$

where $e_0 = \dfrac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}}{\bar{Y}}$, $e_1 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h e_{1h}}{\bar{X}}$ and $e_2 = \dfrac{\sum_{h=1}^{L} W_h \bar{Z}_h e_{2h}}{\bar{Z}}$ such that

$$E(e_0) = E(e_1) = E(e_2) = 0 \text{ and } E(e_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2,$$

$$E(e_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2,\ E(e_2^2) = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{zh}^2,$$

$E(e_0 e_1) = \frac{1}{\bar{Y}\,\bar{X}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh}$, $E(e_0 e_2) = \frac{1}{\bar{Y}\,\bar{Z}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yzh}$ and

$E(e_1 e_2) = \frac{1}{\bar{X}\,\bar{Z}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{xzh}$.

After solving (12) we get the bias and mean squared error of proposed estimator upto the first degree of approximation as

$$B(\hat{\bar{Y}}_{RP}^{ST}) = \bar{Y} \sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{xh}^2}{\bar{X}^2} - \frac{S_{yxh}}{\bar{Y}\,\bar{X}} - \frac{S_{xzh}}{\bar{X}\,\bar{Z}} + \frac{S_{yzh}}{\bar{Y}\,\bar{Z}} \right), \tag{13}$$

$$MSE(\hat{\bar{Y}}_{RP}^{ST}) = \sum_{h=1}^{L} W_h^2 \gamma_h \{ S_{yh}^2 + R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2(R_1 S_{yxh} - R_2 S_{yzh} + R_1 R_2 S_{xzh}) \} . \tag{14}$$

3. EFFICIENCY COMPARISONS

To see the efficiency of the proposed estimator in comparison to other considered estimators, we compare the mean squared error of the proposed estimator with variance or mean squared errors of other estimators. Variance of usual unbiased estimator in stratified random sampling $\bar{y}_{st}$ is

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \ . \tag{15}$$

Comparison of (14) and (15) shows that the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would be more efficient than usual unbiased estimator $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \{ R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2(R_1 S_{yxh} - R_2 S_{yzh} + R_1 R_2 S_{xzh}) \} < 0 \ . \tag{16}$$

From (8) and (14), it is observed that the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would be more efficient than combined ratio estimator $\hat{\bar{Y}}_{RC}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \{ R_2^2 S_{zh}^2 + 2R_2 (S_{yzh} - R_1 S_{xzh}) \} < 0 \ . \tag{17}$$

Comparison of (9) and (14) reveals that the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would be more efficient than combined product estimator $\hat{\bar{Y}}_{PC}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \{R_1^2 S_{xh}^2 - 2R_1 (S_{yxh} + R_2 S_{xzh})\} < 0 \,. \tag{18}$$

Expressions (16), (17) and (18) provide the conditions under which the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would have less mean squared error in comparison to mean squared error of usual unbiased estimator $\bar{y}_{st}$, combined ratio estimator $\hat{\bar{Y}}_{RC}$ and combined product estimator $\hat{\bar{Y}}_{PC}$.

4. EFFICIENCY COMPARISONS IN CASE OF PROPORTIONAL ALLOCATION

When the units from the h^{th} stratum are selected according to proportional allocation i.e. $n_h \propto N_h$ then, $\frac{n_h}{N_h} = \frac{n}{N}$.

In case of proportional allocation, variance of unbiased estimator $\bar{y}_{st}$, mean squared error of combined ratio estimator $\hat{\bar{Y}}_{RC}$, combined product estimator $\hat{\bar{Y}}_{PC}$ and ratio-cum-product estimator $\hat{\bar{Y}}_{RP}^{ST}$ are obtained as

$$V(\bar{y}_{st})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yh}^2 \,, \tag{19}$$

$$MSE(\hat{\bar{Y}}_{RC})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h (S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}) \,, \tag{20}$$

$$MSE(\hat{\bar{Y}}_{PC})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h (S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}) \,, \tag{21}$$

$$MSE(\hat{\bar{Y}}_{RP}^{ST})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h \{S_{yh}^2 + R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2(R_1 S_{yxh} - R_2 S_{yzh} + R_1 R_2 S_{xzh})\} \,. \tag{22}$$

From (19), (20), (21) and (22), it is observed that in case of proportional allocation, proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h \{R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2(R_1 S_{yxh} - R_2 S_{yzh} + R_1 R_2 S_{xzh})\} < 0 \,, \tag{23}$$

(ii) $\hat{\bar{Y}}_{RC}$ if

$$\sum_{h=1}^{L} W_h \{R_2^2 S_{zh}^2 + 2R_2(S_{yzh} - R_1 S_{xzh})\} < 0 \ , \tag{24}$$

(iii) $\hat{\bar{Y}}_{PC}$ if

$$\sum_{h=1}^{L} W_h \{R_1^2 S_{xh}^2 - 2R_1(S_{yxh} + R_2 S_{xzh})\} < 0 \ . \tag{25}$$

Expressions (23), (24) and (25) provides the conditions under which proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ would have less mean squared error in comparison to usual unbiased estimator $\bar{y}_{st}$, combined ratio estimator $\hat{\bar{Y}}_{RC}$ and combined product estimator $\hat{\bar{Y}}_{PC}$ in case of proportional allocation.

5. EMPERICAL STUDY

To see the performance of the proposed estimator empirically in comparison to other estimators, we consider two natural population data sets. Description of the populations are given below.

Population I [Source: Murthy (1967)]
Y : Output,
X : Fixed capital,
Z : Number of workers.

	$n_1 = 2$	$n_2 = 3$	$N_1 = 5$	$N_2 = 5$
	$\bar{Y}_1 = 1925.80$	$\bar{Y}_2 = 315.60$	$\bar{X}_1 = 214.40$	$\bar{X}_2 = 333.80$
N=10	$\bar{Z}_1 = 51.80$	$\bar{Z}_2 = 60.60$	$S_{y_1} = 615.92$	$S_{y_2} = 340.38$
n=5	$S_{x_1} = 74.87$	$S_{x_2} = 66.35$	$S_{z_1} = 0.75$	$S_{z_2} = 4.84$
	$S_{yx_1} = 39360.68$	$S_{yx_2} = 22356.50$	$S_{yz_1} = 411.16$	$S_{yz_2} = 1536.24$
	$S_{zx_1} = 38.08$	$S_{zx_2} = 287.92$		

Population II [Source: National Horticulture Board (2010)]
Y : Productivity (MT/Hectare),
X : Production in '000 Tons,
Z : Area in '000 Hectare.

	n_1 =3	n_2 =4	N_1 =10	N_2 =10
	$\overline{Y}_1$ = 1.70	$\overline{Y}_2$ =3.67	$\overline{X}_1$ =10.41	$\overline{X}_2$ =309.14
N=20	$\overline{Z}_1$ =6.20	$\overline{Z}_2$ =80.67	S_{y_1} =0.54	S_{y_2} =1.41
n=7	S_{x_1} =3.53	S_{x_2} =80,54	S_{z_1} =1.19	S_{z_2} =10.81
	S_{yx_1} =1.60	S_{yx_2} =83.47	S_{yz_1} =-0.02	S_{yz_2} =-7.06
	S_{zx_1} =1.75	S_{zx_2} =68.57		

TABLE 1

Empirical presentation of conditions (16), (17) and (18) under which the suggested estimator $\hat{\bar{Y}}_{RP}^{ST}$ is more efficient than $\overline{y}_{st}$, $\hat{\bar{Y}}_{RC}$ and $\hat{\bar{Y}}_{PC}$

Estimators	$\overline{y}_{st}$	$\hat{\bar{Y}}_{RC}$	$\hat{\bar{Y}}_{PC}$
Population 1	-22913.00 < 0	-30309.70 < 0	-1257.22 < 0
Population 2	-32433.30 < 0	-14987.40 < 0	-37854.90 < 0

TABLE 2

Percent relative efficiencies of $\overline{y}_{st}$, $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$ and $\hat{\bar{Y}}_{RP}^{ST}$ with respect to $\overline{y}_{st}$

Estimators	$\overline{y}_{st}$	$\hat{\bar{Y}}_{RC}$	$\hat{\bar{Y}}_{PC}$	$\hat{\bar{Y}}_{RP}^{ST}$
Population 1	100.00	263.19	78.18	293.21
Population 2	100.00	179.76	121.32	311.30

TABLE 3

Percent relative efficiencies of $\overline{y}_{st}$, $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$ and $\hat{\bar{Y}}_{RP}^{ST}$ with respect to $\overline{y}_{st}$
(in case of proportional allocation)

Estimators	$\overline{y}_{st}$	$\hat{\bar{Y}}_{RC}$	$\hat{\bar{Y}}_{PC}$	$\hat{\bar{Y}}_{RP}^{ST}$
Population 1	100.00	239.88	68.95	308.58
Population 2	100.00	184.86	123.06	343.16

5. CONCLUSION

Section 3 provides the conditions under which the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ has less mean squared error in comparison to usual unbiased estimator $\overline{y}_{st}$, combined ratio estimator $\hat{\bar{Y}}_{RC}$ and combined product estimator $\hat{\bar{Y}}_{PC}$. Section 4 that deals with the efficiency comparisons in case of proportional allocation, provides the conditions (23), (24) and (25) under which proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ is more efficient than usual unbiased estimator, combined ratio estimator and combined product estimator in case of proportional allocation.

The conditions given in section 3 have been checked empirically and given in

Table 1, which shows that all conditions are satisfied for both the populations. Table 2 exhibits that the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ has highest percent relative efficiency in comparison to $\bar{y}_{st}$, $\hat{\bar{Y}}_{RC}$ and $\hat{\bar{Y}}_{PC}$. Therefore, it is concluded that proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ is more efficient than $\bar{y}_{st}$, $\hat{\bar{Y}}_{RC}$ and $\hat{\bar{Y}}_{PC}$ provided that conditions (16), (17) and (18) are satisfied

Table 3 shows that in case of proportional allocation, the proposed estimator has highest percent relative efficiency in comparison to other considered estimators in both populations. It is important to note that proportional allocation provides higher percent relative efficiency as compared to non-proportional case.

Thus the proposed estimator $\hat{\bar{Y}}_{RP}^{ST}$ is recommended for use in practice instead of other conventional estimators when conditions given in section 3 and 4 are satisfied.

This study uses information on the population mean of two auxiliary variates. The same study may be extended using various known parameters of auxiliary variates such as coefficient of variation, coefficient of kurtosis, correlation coefficient between two auxiliary variates etc. see Singh and Tailor (2005). Using simulation study impact of use of various parameters of auxiliary variates can also be studied in the estimation of population parameters.

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SUMMARY

A ratio-cum-product estimator of population mean in stratified random sampling using two auxiliary variables

This paper proposes a ratio-cum-product estimator of finite population mean in stratified random sampling using information on population means of two auxiliary variables. The bias and mean squared error expressions are derived under large sample approximations. Proposed estimator has been compared with usual unbiased estimator in stratified sampling, combined ratio estimator and combined product estimator theoretically as well as empirically.