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INTEGER VALUED AUTOREGRESSIVE PROCESSES WITH GENERALIZED DISCRETE MITTAG-LEFFLER MARGINALS

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1. INTRODUCTION

Integer valued time series are common in practice, yet methods for their analysis have been developed only recently. In the last three decades, there have been a number of imaginative attempts to develop a suitable class of models for time series of counts. Integer valued autoregressive (INAR) models are one way of dealing with count data in time series. Such data may arise from the discretization of continuous variate time series. See (McKenzie, 2003) for a detailed review. The pioneer work on integer valued time series modeling was proposed by McKenzie (1986). A first order autoregressive model with count (or integer valued) data is developed through the binomial thinning operator '$*$' due to Steutel and van Harn (1979). Let X be an Z_+ valued random variable and $\gamma \in [0,1]$, then the thinning operator '$*$' is defined by

$$\gamma * X = \sum_{i=1}^{X} V_i \tag{1}$$

where V_i's are independent and identically distributed (*i.i.d.*) Bernoulli random variables with P (V_i = 1) = 1 – P (V_i = 0) = γ, and are independent of X. If $G_X(s) = \sum_{j=0}^{\infty} P(X=j)s^j = E[s^X]$ represents the probability generating function (*pgf*) of X, then the *pgf* of $\gamma * X$ is obtained as $G_X(1-\gamma+\gamma s)$.

A sequence $\{X_n, n \in Z\}$ of Z_+-valued random variables is said to form an integer-valued first order autoregressive (INAR (1)) process if for any $n \in Z$,

$$X_n = \gamma * X_{n-1} + \varepsilon_n \tag{2}$$

where $\gamma \in [0,1]$ is the first order autocorrelation coefficient of the process and $\{\varepsilon_n\}$ is the innovation process. Under the assumption of strict stationarity, (2) can be rewritten in terms of *pgf* as

$$G_x(s) = G_x(1-\gamma+\gamma s)\ G_\gamma(s),\ -1 \le s \le 1,\ \gamma \in [0,1] \tag{3}$$

where $G_\gamma(s)$ is a proper *pgf*.

McKenzie (1986) introduced a class of discrete valued sequences with negative binomial and geometric marginal distributions obtained as discrete analogues of the standard autoregressive time series models of Lawrance and Lewis (1980), replacing the scalar multiplication by the thinning operation. We consider the alternate probability generating function (*apgf*) defined as $A(s) = G(1\text{-}s) = E[(1\text{-}s)^X]$ instead of the *pgf* in (2), which yields an expression analogous to the Laplace transform for positive valued continuous random variables, so that (3) can be rewritten as

$$A_x(s) = A_x(\gamma s)\ A_\varepsilon(s) \tag{4}$$

for every $\gamma \in [0,1]$. The equation (4) is analogous to the definition of self-decomposability for continuous random variables. For more details see (McKenzie, 2003). INAR models have been discussed by Al-Osh and Alzaid (1987, 1988), Jayakumar (1995), Jin-Guan and Yuan (1991), among others. Pillai (1990) developed Mittag-Leffler functions and related distributions. The more general INAR (p) processes were first introduced by Alzaid and Al-Osh (1990). First order autoregressive semi-alpha-Laplace processes were developed by Jayakumar (1997). Bouzar and Jayakumar (2008) discussed time series with discrete semi-stable marginals. Recently, Lishamol and Jose (2011) introduced geometric normal-Laplace distributions and autoregressive processes.

In recent years, data modelling with heavy-tailed distributions and processes has been an object of great interest. Kotz et al.(2001) have shown that heavy tailed distributions and processes serve as good models for diverse sources of data like network traffic, engineering, financial modeling and risk management. Integer valued time series with heavy tailed marginal distributions have been developed by several authors. Pillai and Jayakumar (1995) developed discrete Mittag-Leffler INAR (1) process. Jayakumar and Davis (2007) introduced INAR models with bivariate geometric marginal distribution. Zheng and Basawa (2007), Zheng et al. (2008) generalized these to random coefficient and observation driven models. Generalised geometric Mittag-Leffler distribution and autoregressive processes were developed by Jose et al. (2010).

Pakes (1995) introduced non-negative random variables in terms of Laplace-Stieltjes transform of the form,

$$L_{\alpha,c,\beta}(s) = \left(\frac{1}{1+cs^\alpha}\right)^\beta,\ s \ge 0,\ 0 < \alpha \le 1,\ c > 0,\ \beta > 0\,. \tag{5}$$

and referred to them as positive Linnik laws. Christoph and Screiber (1998b) introduced a random variable with *pgf*

$$G(s) = \left[1 + \frac{\lambda(1-s)^{a}}{\beta}\right]^{-\beta}$$

$$= \exp\{\lambda(1-s)^{\alpha}\} \text{ for } \beta = \infty \quad (6)$$

and referred to it as discrete Linnik distributed with characteristic exponent $\alpha \in (0,1]$, scale parameter $\lambda > 0$ and shape parameter $\beta > 0$. If $\beta = 1$, then (6) gives the *pgf* of discrete Mittag-Leffler distribution. Distributional representations for discrete stable distribution, discrete Linnik distribution and Sibuya distribution are available in Devroye (1993). Christoph and Schreiber (1998a, b) considered explicit and asymptotic formulae for the tail probabilities of the discrete stable and Linnik distributions. Bouzar (2002) gives mixture representation for the discrete Mittag-Leffler and Linnik laws. Marshall-Olkin assymmetric Laplace distribution and processes can be found in Jose and Krishna (2011). Jose and Abraham (2011) extend the count models with Mittag-Leffler waiting times. Punathumparambathu (2011) introduced a new family of skewed distributions generated by the normal kernel and discussed its various applications. The purpose of this paper is to introduce and develop autoregressive processes with geometric generalized discrete semi-Mittag-Leffler distributions and study some properties of generalized discrete Mittag-Leffler distributions.

The paper is organized as follows. In Section 2 we introduce Generalized Discrete Mittag-Leffler (GDML) distribution and discuss its various properties. We also develop a first order INAR (1) process with GDML stationary marginal distribution and obtain the joint distribution of adjacent values of the process. Geometric Generalized Discrete Mittag-Leffler (GGDML) distributions and their properties are discussed in Section 3. We also introduce and study geometric generalized discrete Mittag-Leffler processes and INAR (p) processes. A further extension to obtain a more general class called Geometric generalized discrete semi-Mittag-Leffler distributions and processes are developed in Section 4. An application with respect to an empirical data on customer arrivals in a bank counter is discussed in Section 5. The conclusions are given in Section 6.

2. GENERALIZED DISCRETE MITTAG-LEFFLER DISTRIBUTION

Definition 2.1. A random variable X on Z_{+} is said to follow generalized discrete Mittag-Leffler distribution denoted by $GDML(\alpha, c, \beta)$, if it has the probability generating function

$$G(s) = \left\{\frac{1}{1 + c(1-s)^{\alpha}}\right\}^{\beta}; -1 \le s \le 1, 0 < \alpha \le 1, c > 0, \beta > 0. \quad (7)$$

For $\beta = 1$, it reduces to the DML(α). When α = 1, β = 1 it reduces to geometric distribution.

Theorem 2.1. *The GDML (α, c, β) distribution is discrete self-decomposable.*

Proof. From (3) the *pgf* of *GDML (α, c, β)* is

$$\mathrm{G(s)}=\left[\frac{1}{1+\mathrm{c}\gamma^{\alpha}(1-s)^{\alpha}}\right]^{\beta}\left[\gamma^{\alpha}+(1-\gamma^{\alpha})\frac{1}{1+c(1-s)^{\alpha}}\right]=G(1-\gamma+\gamma s)G_{\gamma}(s).$$

Theorem 2.2. *The GDML (α, c, β) distribution is geometrically infinitely divisible and hence infinitely divisible.*

Proof.
Consider the probability generating function of *GDML (α, c, β)* given by,

$$\mathrm{G(s)}=\left[\frac{1}{1+\mathrm{c}(1-s)^{\alpha}}\right]^{\beta}.$$

Replacing 's' by exp $(-\lambda)$, we get the Laplace transform and using the criterion used in Pillai and Sandhya (1990), we see that the *GDML (α, c, β)* is geometrically infinitely divisible.

Theorem 2.3. Let G(s) be the pgf of a GDML distribution with $\gamma\in(0,1], c>0, \beta>0, 0<\alpha\le 1, -1\le s\le 1$. *Then there exists a strictly stationary INAR (1) process* $\{X_n, n\in \mathrm{Z}\}$ *having structure given by (2) with G(s) as the pgf of its marginal distribution. Also the marginal distribution of the innovation sequences* $\{\varepsilon_n, n\in \mathrm{Z}\}$ *has apgf* $\mathrm{A}_{\varepsilon}(\mathrm{s})$ *given by*

$$A_{\varepsilon}(s)=\left\{\frac{1+c\gamma^{\alpha}s^{\alpha}}{1+cs^{\alpha}}\right\}^{\beta} \tag{8}$$

Proof.
In terms of *apgf*, the INAR(1) model defined in (2) can be rewritten as

$$A_{X_n}(s)=A_{X_{n-1}}(\gamma s)A_{\varepsilon_n}(s)$$

Under strict stationarity assumption, it reduces to

$$A_X(s)=A_X(\gamma s)A_{\varepsilon}(s)$$

Hence we have,

$$A_\varepsilon(s) = \frac{A_X(s)}{A_X(\gamma s)}$$

The INAR(1) with GDML marginals is defined, only if there exists an innovation sequence ε_n such that $A_\varepsilon(s)$ is an *apg f.*

From (7), we have

$$A_X(s) = \left\{\frac{1}{1+cs^\alpha}\right\}^\beta .$$

Then we have,

$$A_\varepsilon(s) = \left\{\frac{1+c\gamma^\alpha s^\alpha}{1+cs^\alpha}\right\}^\beta$$

$$= \left\{\gamma^\alpha + (1-\gamma^\alpha)\frac{1}{1+cs^\alpha}\right\}^\beta .$$

Therefore, the innovations ε_n are β -fold convolutions of DML (α , c, β).

2.1 *Joint distribution of* (X_{n-1}, X_n)

The joint pgf of (X_{n-1}, X_n) is given by

$$G_{X_{n-1},X_n}(s_1, s_2) = E({s_1}^{X_{n-1}} {s_2}^{\gamma \odot X_{n-1}+\varepsilon_n})$$

$$= G_{\varepsilon_n}(s_2) G_{X_{n-1}}[s_1(1-\gamma+\gamma s_2)]$$

$$= \left[\frac{1+c\gamma^\alpha {s_2}^\alpha}{1+c{s_2}^\alpha}\right]^\beta \left[\frac{1}{1+c\{s_1(1-\gamma+\gamma s_2)\}}\right]^\beta .$$

By inverting this expression the joint distribution can be obtained. The above expression is not symmetric in s_1 and s_2 and hence the process is not time reversible.

3. GEOMETRIC GENERALIZED DISCRETE MITTAG-LEFFLER DISTRIBUTION

Jose *et al.* (2010) introduced and studied the geometric generalized Mittag-Leffler distribution and its properties. Now we shall introduce its discrete analogue as follows.

Definition 3.1 A random variable X on Z_+ is said to follow geometric generalized discrete Mittag-Leffler (GGDML) distribution and we write X ~ GGDML (α, c, β), if it has the alternate probability generating function

$$A(s)=\frac{1}{1+\beta\ln[1+cs^{\alpha}]};-1\le s\le 1,0<\alpha\le 1,c>0,\beta>0\,. \tag{9}$$

Remark 1. The *GGDML distribution is geometrically infinitely divisible.*

Theorem 3.1. *Let $X_1, X_2,...$ are independently and identically distributed GGDML random variables where $Y=X_1+X_2+...+X_{N(p)}$ and N(p) be geometrically distributed with mean $\frac{1}{p}$, $P[N(p)]=p(1-p)^{k-1}, k=1,2,...,0<p<1$. Then Y~GGDML ($\alpha$, c, β).*

Proof.
Taking the *apgf* of Y we have,

$$A_Y(s)=\sum_{k=1}^{\infty}\{A_X(s)\}^k p(1-p)^{k-1}$$

$$=\frac{1}{1+\frac{\beta}{p}\ln[1+cs^{\alpha}]}.$$

3.1 *Geometric generalized discrete Mittag-Leffler processes*

In this section, we develop a first order new autoregressive process with geometric generalized discrete Mittag-Leffler marginal distribution.

Theorem 3.2. *Let $\{X_n, n\ge 1\}$ be defined as*

$$X_n=\begin{cases}\varepsilon_n & \text{with probability} \quad p\\ X_{n-1}+\varepsilon_n & \text{with probability}\ (1-p)\end{cases} \tag{10}$$

where $\{\varepsilon_n\}$ is a sequence of i.i.d. random variables. A necessary and sufficient condition that $\{X_n\}$ is a strictly stationary Markov process with GGDML (a, c, ß) marginals is that ε_n are distributed as geometric Mittag-Leffler provided X_0 is distributed as geometric generalized discrete Mittag-Leffler.

Proof.
Rewriting (10) in terms of *apgf* we have,

$$A_{X_n}(s) = pA_{\varepsilon_n}(s) + (1-p)A_{X_{n-1}}(s)A_{\varepsilon_n}(s)$$

Assuming strict stationarity, it becomes,

$$A_X(s) = A_\varepsilon(s)\{p + (1-p)A_X(s)\}$$

That is,

$$A_\varepsilon(s) = \frac{A_X(s)}{p + (1-p)A_X(s)}$$

where

$$A_X(s) = \frac{1}{1 + \beta \ln[1 + cs^\alpha]}$$

On simplification we get,

$$A_\varepsilon(s) = \frac{1}{1 + p\beta \ln[1 + cs^\alpha]}$$

and hence ε_n ~GGDML(α, c, pβ).

The converse part can be proved by the method of mathematical induction as follows. Now assume that X_{n-1} ~GGDML (α , c, β). Then

$$A_{X_{n-1}}(s) = A_{\varepsilon_n}(s)\{p + (1-p)A_{X_{n-2}}(s)\}$$

$$= \frac{1}{1 + p\beta \ln[1 + cs^\alpha]}\left[p + (1-p)\left\{\frac{1}{1 + \beta \ln[1 + cs^\alpha]}\right\}\right]$$

$$= \frac{1}{1 + \beta \ln[1 + cs^\alpha]}$$

The rest follows similarly.

3.2 *INAR(p) process with GGDML marginal distribution*

Now we consider a p^{th} order integer valued autoregressive (INAR (p)) process with probability structure,

$$X_n = \begin{cases} \gamma_1 * X_{n-1} + \varepsilon_n & \textit{with} \quad \textit{probabilty} \quad \delta_1 \\ \gamma_2 * X_{n-2} + \varepsilon_n & \textit{with} \quad \textit{probabilty} \quad \delta_2 \\ \cdots & \cdots \quad \cdots \quad \cdots \\ \cdots & \cdots \quad \cdots \quad \cdots \\ \gamma_p * X_{n-p} + \varepsilon_n & \textit{with} \quad \textit{probabilty} \quad \delta_p \end{cases} \tag{11}$$

where $0 < \gamma_i, \delta_i \leq 1, i = 1, 2, \ldots p; \sum_{i=1}^{p} \delta_i = 1$.

In terms of *apgf,* the above equation can be written as

$$A_{X_n}(s) = A_{\varepsilon_n}(s) \sum_{i=1}^{p} \delta_i A_{X_{n-i}}(\gamma_i s).$$

Assuming strict stationarity it reduces to

$$A_X(s) = A_\varepsilon(s) \sum_{i=1}^{p} \delta_i A_X(\gamma_i s).$$

Hence

$$A_\varepsilon(s) = \frac{A_X(s)}{\sum_{i=1}^{p} \delta_i A_X(\gamma_i s)}.$$

For the GGDML marginals, the innovation sequence of the process has *apgf given by,*

$$A_\varepsilon(s) = \frac{[1 + \beta \ln(1 + cs^\alpha)]^{-1}}{\sum_{i=1}^{p} \delta_i [1 + \beta \ln(1 + c\gamma_i^{\ \alpha} s^\alpha)]^{-1}}. \tag{12}$$

For the particular case of $\gamma_i = \lambda$, for i = 1, 2, ..., p, (12) yields a similar pattern of *apgf* as in (8). Hence with an error sequence ε_n following GGDML distribution, the p[th] order GGDML autoregressive processes are properly defined.

4. FURTHER EXTENSIONS OF GDML AND GGDML DISTRIBUTIONS

In this section we extend the GDML distribution to obtain a more general class of distributions called generalized discrete semi- Mittag-Leffler (GDSML) distribution and study its properties.

Definition 4.1 A random variable X on Z_+ *is said to follow generalized discrete semi- Mittag-Leffler distribution and write X ~ GDSML (a, c, β), if it has the pgf given by*

$$G(s)=\left\{\frac{1}{1+\psi(1-s)}\right\}^{\beta} \tag{13}$$

where $\psi(s)$ *satisfies the functional equation* $a\psi(s)=\psi(a^{\alpha}s)$ *for all* $0 < s < 1$.

Remark 2. The solution of the functional equation is given by $\psi(s)=s^{\alpha}h(s)$ *where h(s) is a periodic function in* $\ln s$ *with period* $\frac{-2\pi\alpha}{\ln a}$. *This is a special case of the general equation given in pp. 310 in Aczel (1966). For more details see Jayakumar (1997) and Kagan et al. (1973).*

In a similar we can define a geometric generalized discrete semi-Mittag-Leffler distribution and write X~GGDSML (α, c, β *), if it has the pgf,*

$$G(s)=\frac{1}{1+\beta\ln[1+\psi(1-s)]} \tag{14}$$

where $\psi(.)$ *satisfies the above conditions. It can also be verified that the GGDSML distribution is geometrically infinitely divisible.*

Theorem 4.1 If $X_1, X_2, \ldots$ *are independently and identically distributed geometric generalized discrete semi Mittag-Leffler random variables with parameters* α *and* β *where* $Y = X_1 + X_2 + \ldots + X_{N(p)}$ *and N (p) be distributed as geometric with mean* $\frac{1}{p}$, *then* $Y \sim$ *GGDSM L (a, c,* $\frac{\beta}{p}$ *).*

Proof.

In terms of the *apgf* of *Y*, we have

$$A_Y(s)=\sum_{k=1}^{\infty}\{A_X(s)\}^k p(1-p)^{k-1}$$

$$=\frac{1}{1+\frac{\beta}{p}\ln[1+\psi(s)]}.$$

Theorem 4.2 Geometric *Generalized Discrete Semi Mittag-Leffler (GDSML) distribution is the limit distribution of geometric sum of* GDSML $\left(\alpha, \frac{\beta}{n}\right)$ *random variables.*

Proof. We have,

$$[1+\psi(s)]^{-\beta} = \left\{1+[1+\psi(s)]^{\frac{\beta}{n}} - 1\right\}^{-n}$$

is the apgf of a probability distribution since generalized discrete semi Mittag-Leffler distribution is infinitely divisible. Hence by lemma 3.2 of Pillai (1990),

$$A_n(s) = \left\{1+n[1+\psi(s)]^{\frac{\beta}{n}} - 1\right\}^{-n}$$

is the apgf of a geometric sum of independently and identically distributed discrete semi Mittag-Leffler random variables. Taking limit as $n \to \infty$

$$A(s) = \lim_{n\to\infty} A_n(s)$$

$$= \left\{1+\lim(n[1+\psi(s)]^{\frac{\beta}{n}} - 1)\right\}^{-1}$$

$$= \{1+\beta \ln[1+\psi(s)]-1\}^{-1}.$$

4.1 G*eometric generalized discrete semi-Mittag-Leffler processes*

Here we develop a first order new autoregressive process with geometric generalized discrete semi Mittag-Leffler marginals.

Theorem 4.3. *Let* $\{X_{n,} n \geq 1\}$ *be defined as*

$$X_n = \begin{cases} \varepsilon_n & \textit{with} \quad \textit{probability} \quad p \\ X_{n-1}+\varepsilon_n & \textit{with} \quad \textit{probability} \quad 1-p \end{cases} \tag{15}$$

where $\{\varepsilon_n\}$ *is a sequence of i.i.d. random variables. A necessary and sufficient condition that* $\{X_n\}$ *is a strictly stationary Markov process with* $GGDSML(\alpha, c, \beta)$ *marginals is that* ε_n *are distributed as geometric generalized discrete semi-Mittag-Leffler.*

Proof.
Rewriting in terms of the *apgf,* the equation (15) reduces to

$$A_{X_n}(s) = pA_{\varepsilon_n}(s) + (1-p)A_{X_{n-1}}(s)A_{\varepsilon_n}(s)$$

$$= A_{\varepsilon_n}(s)\{p + (1-p)A_{X_{n-1}}(s)\} \tag{16}$$

When X_n is weak stationary, we have

$$A_X(s) = A_\varepsilon(s)\{p + (1-p)A_X(s)\}$$

This gives,

$$A_\varepsilon(s) = \frac{A_X(s)}{p + (1-p)A_X(s)}$$

where $A_X(s) = \dfrac{1}{1+\beta\ln[1+\psi(s)]}$

On simplification we get,

$$A_\varepsilon(s) = \frac{1}{1+p\beta\ln[1+\psi(s)]}$$

so that ε_n ~GGDSML $(\alpha, c, p\beta)$.

The converse can be proved by the method of mathematical induction as follows. We assume that X_n ~GGDSML(α, c, β). Then we have,

$$A_{X_{n-1}}(s) = A_{\varepsilon_n}(s)\{p + (1-p)A_{X_{n-2}}(s)\}$$

$$= \frac{1}{1+p\beta\ln[1+\psi(s)]}\left[p + (1-p)\left\{\frac{1}{1+\beta\ln[1+\psi(s)]}\right\}\right]$$

$$= \frac{1}{1+\beta\ln[1+\psi(s)]}$$

The rest follows easily.

Remark 3. An INAR(p) process having structure of the form (11) with GGDSML(α, c, β) stationary marginal distribution can also be easily developed following similar steps as in section 3.2.

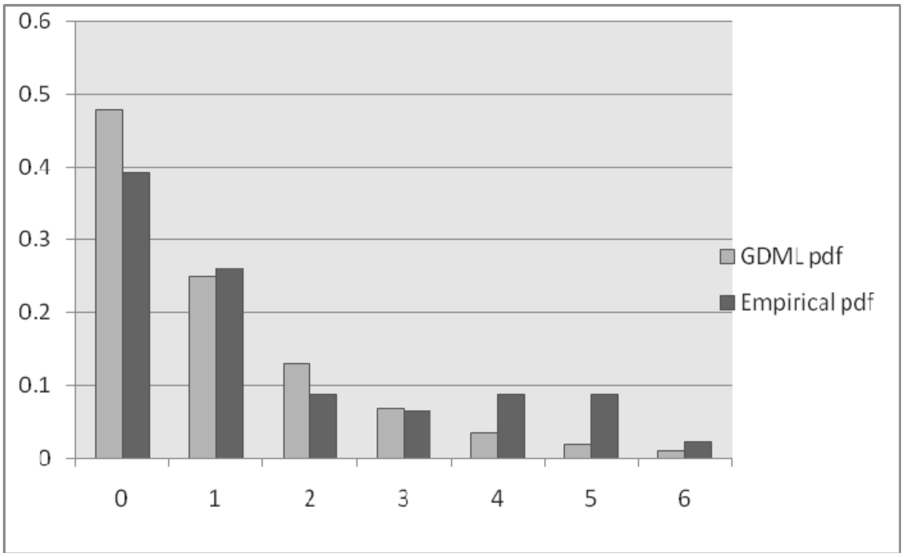

Figure 1 – The empirical pdf and theoretical pdf of GDML (0.99, 0.91, 1) distribution.

5. APPLICATION TO AN EMPIRICAL DATA

In this section we apply the model to a data on the interarrival times of customers in a bank counter measured in terms of number of months from January 1994 to October 2003, which is taken from the file bank.arrivals.xlsx available in the website www.westminstercollege.edu. The empirical pdf shows a decreasing trend in the probabilities. Figure 1 gives the empirical pdf and theoretical pdf of $\text{GDML}(\alpha, c, \beta)$.

The mean, variance, coefficient of skewness and kurtosis measure for the data are respectively 1.5435, 3.2314, 0.9845 and 2.6714. The Durbin-Watson test confirms strong autocorrelation in the data with first order autocorrelation coefficient 0. 92 so that INAR models are needed to explore the future behaviour of the data. Since the mean is less than variance the geometric distribution is a possible probability model. Since geometric distribution is a special case of $\text{GDML}(\alpha, c, \beta)$, we shall examine whether it is a suitable model to the above data. We obtain the estimates of the parameters as $\alpha = 0.99$, $\beta = 1$ and c = 0.91.

Now we use the Kolmogorov-Smirnov [K.S.] test for testing H_0: GDML distribution with parameters $\alpha = 0.99$, $\beta = 1$ and c = 0.91 is a good fit for the given data. Since the computed value of the K.S. test statistic is obtained as 0.1212 and the critical value corresponding to the significance level 0.01 is 0.2403, the GDML assumption for interarrival times is justified. Using this we can obtain the probabilities associated with the stationary distribution of the INAR(1) model as well as predict the future values of the process. This will help in developing optimal service policies for ensuring customer satisfaction.

6. CONCLUSIONS

In this paper we have considered GDML distributions and introduced a new family of distributions called GGDML distributions and developed integer valued time series models. We also developed various generalizations such as GGDSML and INAR (p) processes. The use of the model is illustrated by fitting it to an empirical data on customer arrivals in a bank counter and the goodness of fit is established. The processes developed in this paper can be used for modeling time series data on counts of events, objects or individuals at consecutive points in time such as the number of accidents, number of breakdowns in manufacturing plants, number of busy lines in a telephone network, number of patients admitted in a hospital, number of claims in an insurance company, number of persons unemployed in a particular year, number of aero planes waiting for take-off, number of vehicles in a queue, etc. Thus the models have applications in various contexts like studies relating to human resource development, insect growth, epidemic modeling, industrial risk modeling, insurance and actuaries, town planning etc.

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SUMMARY

Integer valued autoregressive processes with generalized discrete Mittag-Leffler marginals

In this paper we consider a generalization of discrete Mittag-Leffler distributions. We introduce and study the properties of a new distribution called geometric generalized discrete Mittag-Leffler distribution. Autoregressive processes with geometric generalized discrete Mittag-Leffler distributions are developed and studied. The distributions are further extended to develop a more general class of geometric generalized discrete semi-Mittag-Leffler distributions. The processes are extended to higher orders also. An application with respect to an empirical data on customer arrivals in a bank counter is also given. Various areas of potential applications like human resource development, insect growth, epidemic modeling, industrial risk modeling, insurance and actuaries, town planning etc are also discussed.