A STATISTICAL PROCEDURE FOR TESTING FINANCIAL CONTAGION

A. Gardini, L. De Angelis

1. INTRODUCTION

The analysis of the relationships among financial markets and the identification of financial contagion episodes are relatively recent in the economic analysis and have experienced a rapid development in the last decade, coinciding with the occurrence of relevant financial crises which had effects that spread outside the geographical areas where they originally started.\(^1\)

The increasing interest in this topic has lead to the definition of different tests for detecting the existence of financial contagion (Corsetti et al., 2001; Forbes and Rigobon, 2001; Dungey et al., 2004; Allen and Gale, 2005; Rodriguez, 2007; Krishnamurthy, 2009; Sugihara, 2010). However, conclusions on both theoretical and statistical analyses of financial contagion are far from unique.

Moreover, there is not even a shared scientific definition of contagion. Pericoli and Sbracia (2001) list five different definitions: i) “contagion is a significant increase in the probability of a crisis in one country conditional on a crisis occurring in another country”; ii) “contagion occurs when volatility spills over from the crisis country to the financial markets of other countries”; iii) Forbes and Rigobon (2002) introduce the concept of shift contagion which is interpreted as change in the intensity of correlations between financial asset prices during a crisis period; iv) some authors (e.g., Masson, 1999a; Claessens et al., 2001) consider contagion as the transmission of global or local shocks among nations derived from irrationality of operators, financial panic, and herd behaviour, known as excess comovement; v) finally, a very broad definition includes in the concept of contagion any form of transmission of global or local shocks among countries (Calvo and Reinhart, 1996; Pristker, 2000). In the latter definition, contagion can occur also without the presence of a financial crisis, being a manifestation of the interdependence.

\(^1\) Economists’ interest in “contagion” surged during the second half of the 1990s, when financial crises spread across emerging countries, affecting nations with apparently healthy fundamentals and whose policies, only a few months earlier, had been praised by market analysts and multilateral institutions (cfr. Masson, 1999a, 1999b; Edwards, 2000).
The changes in the international dynamics of returns, which in the last decades has been characterized by increases in both volatilities and asset price synchronicities in different countries, have raised even further the scientific interest in this topic. In this paper, we propose a new methodology for the evaluation of contagion based on the extent of disequilibria in financial dynamics and, in this framework, we define an innovative test for the detection of contagion which specifically identifies the disequilibrium originated by the international transmission of financial crises and their relationships with the behaviours of market participants. Disequilibria exogenously generated by the spread of the effects of a crisis beyond the dynamic process describing endogenous amplification of volatility from one country to other countries are attributed to contagion phenomena. In this framework, contagion effects are separated from the endogenous transmission processes which have their genesis in both the pricing process system and the investor’s behaviours and which are responsible for the amplification of cross-market financial interdependence.

In Section 2 of the paper, we discuss the theoretical framework underlying our approach. Section 3 illustrates the econometric model and details our three-step procedure for evaluating contagion among countries. In Section 4, we estimate the model and present the results of the analysis. Finally, Section 5 concludes.

2. THEORETICAL FRAMEWORK

One of the most relevant analytic issues in the analysis of the dynamics of financial markets is given by the tendency of prices to amplify the effects of the corresponding changes in the fundamentals, namely risk and return. The phenomenon is not recent: Shiller (1981) discovered and analyzed the excess of price volatility with respect to real future dividends. However, the relevance of these amplifications has strongly increased over the last decade, in conjunction with the innovation of financial products and the globalization of financial markets.

These changes have also affected the structure of the relationship between prices and fundamentals. In the new financial framework, prices, besides reflecting the return expected distribution, are also one of the factors which determine the investment decisions and autonomously influence the probability distribution of returns. This dual role of prices implies that the traditional relationship with the fundamentals is no longer of the casual type as in mainstream asset pricing models, but turns into interdependence (Shin, 2008). Therefore, fundamentals become endogenous variables within the pricing process.

Moreover, the increase in the number of people who invest in financial instruments implies changes in the market structure and increases interdependence. Financial decisions made by a cluster of wider and less professional investors favour herd behaviours (collective accumulations or disinvestments), accentuate irrational fads that become persistent, and generate speculative bubbles, irrational panic, and

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2 In this context, mainstream asset pricing models suffer from evident identification problems.
higher correlations between returns. Furthermore, irrational behaviors of market participants generate increases in volatilities and correlation shifts which configure empirical evidences very similar to the ones due to financial contagion.

Therefore, the traditional approach that aims to separate the effects of contagion from those of physiological interdependence between international markets and economies based on the analysis of correlation shifts is unsatisfactory because there are various forms of amplification of the dynamics of asset returns and volatilities which are attributable neither to contagion nor to interdependence. The limits of contagion tests based on the analysis of the shifts in correlation coefficients are thus evident, since there are different situations that generate increasing correlations between returns which follow from the higher level of uncertainty of current financial market structure.

2.1. Uncertainty, risk, and investor psychology in the endogenous processes of financial amplification

The distinction between risk and uncertainty introduced by Knight (1921) defines a clear line of separation between the two concepts: “uncertainty must be taken in a sense radically distinct from the familiar notion of risk [...]. The essential fact is that “risk” is a quantity susceptible of measurement, while uncertainty is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomena depending on which of the two is really present and operating [...]. It will appear that a measurable uncertainty, or “risk” proper, as we shall use the term, is so far different from an unmeasurable one.”

The financial reality observed in the last decades brings about new and relevant uncertainty components that combine with the traditional risk components: the opening of international markets, the diffusion of sophisticated financial engineering and products, and the growing importance of behaviors described by cognitive psychology fuel uncertainty and lead to processes of amplification of the financial dynamics. These processes are endogenous to investment decision-making of market participants and have significantly increased volatilities and correlations between returns within financial markets. Therefore, subjective and objective conditions drive financial markets towards persistent disequilibrium which diverge from the efficient market equilibrium solutions. For this reason, modern analyses have combined traditional models that assume rational agents who maximize their expected utility (Von Neumann and Morgenstern, 1944; Bernoulli, 1954) with models based on cognitive psychology and subjective evaluations of the probability of uncertain events (Kahneman and Tversky, 1979). Thus, also the identification of the cases of cross-country transmission of the effects of specific events in one country requires statistical models able to separate the increased correlation in returns and volatility originated by endogenous uncertainty from those due to exogenous transmission of idiosyncratic shocks among different regions. Focusing on the endogenous or exogenous nature of the processes of amplification we can discriminate contagion phenomena from other correlation shifts.

Three main categories of market participants contribute to create endogenous amplification processes of financial dynamics:
1. Banking and non-banking financial intermediaries;
2. Institutional bank brokers and professional investors;
3. Retail investors.

As a matter of fact, professional risk management based on VaR models used by the financial intermediaries combined with the market to market accounting rule, increases endogenous fluctuations: when stock prices rise, managers are encouraged to commit the raised patrimonial reserve for ensuring the company efficiency. Thus, the rise in stock prices implies an increase in the credit offer of bank money that generates more demand for financial assets and favours a further rise in stock prices sparking a process of growing disequilibrium of prices with respect to fundamentals.

An analogous effect of endogenous correlation shift is generated by the management of collective investment funds based on benchmark: the rise in prices of some stocks improves fund’s performance and increases fund subscription which turns into purchases of less dynamic stocks in order to restore the benchmark (downturns obviously have a symmetric effect of opposite sign) spreading trends among different sectors and regions.\(^3\)

The endogenous shift in financial correlation is primarily fed by the behaviours of retail investors described by behavioural finance: fads, herdings, regret aversion, narrow frame, and overconfidence generate biased probability evaluation and support investment decisions which are independent from the rational expectation of return distribution, producing disequilibria that tend to increase temporal and spatial correlations among returns.

Therefore, both professional and retail decisions are characterized by amplification processes and increasing correlation among returns which are endogenous to financial markets and do not represent contagion episodes.\(^4\) Furthermore, rational behaviours of professional operators and irrational behaviour of retail investors concur in the determination of endogenous disequilibrium situations which are characterized by increased correlations among returns and volatilities.

The phenomena that generate endogenous market disequilibria can thus be ascribed to three classes of factors:

1. Erroneous forecasts of the expected probability distribution of returns;
2. Biased psychological assessments of financial information (perception and cognitive errors);
3. Financial innovation and delays in the regulation of intermediaries and markets.

The first class of factors generates disequilibria because operators do not know the “true” parameters of the probability distribution of returns which are approximated and updated evaluating the available information. Therefore, their choices will not always be on the efficient frontier. By specifying the learning processes through which market participants approximate the expected distribu-

\(^3\) The benchmark induces the fund manager to sell precisely the best performing stocks because, as effect of price variations, their weights in the portfolio exceed the desired level and vice versa in opposite case.

\(^4\) On the endogenous nature of these disequilibrium processes see Shin (2008), Chapters 1 and 3.
tion of returns, we can supplement the rational model (in the sense of classical finance) with learning mechanisms and thus explain market disequilibria\(^5\).

The second class of factors refers to behavioural finance based on the results of cognitive psychology which has shown the differences between the rational expectation psychological assumption of classical financial models and the actual psychological behaviour of investors affected by biased evaluation of return probability distribution. In this context, news regarding specific assets or sectors may irrationally spread to other financial instruments and sectors (perception errors) generating speculative bubbles, financial crises and excess of volatility with respect to the dynamics of fundamentals (irrational exuberance). These situations are empirically similar to disequilibria caused by financial contagion phenomena\(^6\).

Finally, also the third class of factors brings about endogenous amplification processes in financial markets: financial intermediaries decisions driven by deficient regulations and the enormous innovation in financial products\(^7\) which has taken place in the past two decades amplify market disequilibria and increase return correlation.

2.2. Exogenous shocks: financial contagion

The transmission of idiosyncratic shocks across countries (contagion) configures situations empirically similar but logically different from the ones described in Section 2.1.

Correlation shift is the criterion generally chosen in contagion literature to separate “normal” from contagious periods (e.g., Corsetti et al., 2001, 2005; Forbes and Rigobon, 2002). However, in periods characterized by increase in the number of market participants, financial innovation, and reform of financial regulations, this approach does not seem adequate since it does not take into consideration the correlation endogenously and independently generated by phenomena different from contagion, as described in Section 2.1. Hence, in the current financial economy, amplification processes, spillover effects, and correlation shifts can be either effects of financial contagion or processes endogenously generated inside the markets.

Rigorous computation of the reference values for the measurement of the shifts requires the separation of the correlation endogenously generated from that

\(^5\) Hansen and Sargent (2006) specify a learning process based on the ratio between the likelihood of the density conditional to the pessimism and the normal likelihood. Other authors have formulated mechanisms of Bayesian learning processes computed as interaction between the likelihood and the a priori probability distribution of returns and risks (e.g., Kurtz and Beltratti, 1997). Furthermore, also Timmerman (1996) and Bossaerts (2002) refer to the predictive probability distribution in order to specify the learning process.

\(^6\) Learning processes can be related to traditional asset pricing models, whereas cognitive psychology errors are not consistent rational expectation models: in rational expectation models, agents know the true expected probability distribution of returns and learning processes express knowledge modalities, whereas the assumptions of cognitive psychology and behavioural finance describe financial behaviours alternative to the ones with rational expectations.

\(^7\) Such as asset backed securities, warrants, covered warrants, interest rate swap, commodity swap, currency swap, credit risk swap, certificates, leverage certificates, bull leverage certificates, bear leverage certificates, and ETFs, just to mention a few.
ascrivable to the transmission of exogenous shocks and the evaluation of the dynamics of these correlations during periods of normality (absence of contagion). Limiting the notion of contagion to disequilibrium amplification processes having exogenous nature, we are able to separate more clearly the two classes of phenomena and to define more powerful tests of financial contagion. The specific characteristic of contagion is the spread of effects of exogenous shocks occurred in country $i$ to the financial dynamics in country $j$, therefore, a distinctive feature of contagion is the exogenous cause of the amplification process. Hence, the rigorous detection of contagion episodes requires statistical techniques which model simultaneously the whole processes of amplification of financial trends and discriminate between changes in correlation due to contagion and other changes.

2.3. Financial market disequilibrium, risk premia volatility, and contagion

In an environment populated by rational agents who optimize their preferences, prices reflect the effects of core economic data on the probability distribution of returns and determine supply and demand of each asset within financial market. Vice versa, an environment characterized by irrational behaviours, cognitive errors, and benchmark portfolio management shows dynamics where prices are not restricted to reflect fundamentals but are themselves the main causes influencing market dynamics. In the former environment, prices are determined through the casual relation of the (exogenous) expected distribution of returns, whereas in the latter, prices perform a double function: in addition to reflecting the economic fundamentals of the real economy, they interact with the investment choices and, therefore, influence the expected distribution of returns.

This double role of prices endogenously amplifies financial dynamics and circular processes derived from the double role of prices may generate cumulated disequilibria endogenous to the system, which manifest themselves in forms of bubbles, irrational crashes, or systemic crises. Moreover, the spread of cross-

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8 The phenomena of amplification observed in the recent financial dynamics have similar characteristics to physical phenomena in which the exogenous stability blows up introducing the endogenous effects of the anthropic component. The cases of Tacoma (see http://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge) and London bridges are well-known. The stability of London Millennium Bridge, ensured by the static analysis has been resoundingly contradicted by experience of people walking on the bridge. Pedestrians crossing the Thames endogenously interact with the “bridge system” and influence the oscillations of the bridge, endogenously generating increasing oscillations which compromise stability. Thereby, the bridge was closed. In the Tacoma case, having omitted endogenous process, the bridge collapsed.

9 Hence, financial disequilibria present similar characteristics to the ones that determined the closure of the Millenium Bridge (http://www.arup.com/millenniumbridge): if we assume that the footsteps of each pedestrian who crosses the bridge are independent of possible oscillations (exogenously determined), then the stability of the bridge is sure. On the contrary, if we acknowledge that oscillations of the bridge influence the behaviours of pedestrians, since each one tends to naturally follow these oscillations and thus developing a cumulative effect, then we generate an explosive process (which has led to the closure of the bridge in order to avoid collapse). Analogously, in financial markets, if the equilibrium positions of prices are influenced by prices themselves, disequilibria may become bubbles or crashes.
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country volatility arising from common shocks and economic interdependence raises return correlation. However, since it is rationally justified by the evolution of the fundamentals in the concerned countries, this spread does not constitute cases of contagion.

Risk premia are very sensitive indicators of return amplification and financial markets disequilibrium which can be used to distinguish the situations of exogenous amplification derived from contagion from those generated by endogenous processes.

Disequilibria are identified comparing equilibrium risk premia, determined by preference parameters and the consumption dynamics, with empirical risk premia: the distance between the equilibrium level and the risk premia actually observed measures financial market disequilibrium.

The measure of risk premia achieved within the consumption-based asset pricing model automatically reflects the dynamics of fundamentals influenced by endogenous processes of amplification and, therefore, the comparison between the risk premium empirically measured and the one inferable from the model provides a measurement of disequilibrium which is feasible to test contagion. By modelling dynamic conditional correlations in risk premia disequilibrium, we define a statistical procedure which discriminates between the processes endogenously generated and the exogenous ones, attributable to financial contagion. The asymmetric nature of this relation removes any risk of mixing contagion episodes with business cycle comovements which are symmetric. Moreover, the two situations are empirically distinct, since the latter is driven by productivity (Backus and Kehoe, 1992; Kose et al., 2008; Artis and Okubo, 2009; Mumtaz et al., 2011; Crucini et al., 2011), whereas the former is driven by monetary and financial factors.

In order to test the existence of exogenous shocks which generate contagion, we analyze the dynamics of conditional correlation coefficients including a deterministic variable which expresses the idiosyncratic shock. Hence, we verify the possible impact that such shock had in other regions than the ones in which it occurred. The statistical instrument defined in this way allows us to test the presence of contagion in the forms which are rigorously consistent with the characteristics of financial markets in the current phase. As a matter of fact, the stochastic process represents all the forms of amplification of volatility and correla-

10 The analysis of conditional variance and dynamic conditional correlation reflects the logical requirement for evaluating shifts due to contagion effects depurated from structural factors different from contagion and thus developing rigorous tests. Multivariate GARCH models with dynamic conditional correlation allow us to evaluate these shifts and to determine whether a significant increase in the conditional correlation between two countries occurred as consequence of a crisis in one of the two (evidence of contagion).

11 Therefore, with respect to the five notions of contagion reported in Section 1, the definition we propose excludes both manifestations of interdependence related to commercial, industrial, and financial linkages (spillover effect) and the effects of global shocks (monsoonal effect), and it is associated to the literature on “crisis of confidence, irrationality, financial panic, herd behaviour” (Masson, 1998; Calvo and Reinhart, 1996; Pristker, 2000; Cipriani and Guarino, 2003; Masson, 2007; Sugihara, 2010).
tion shifts, whereas the deterministic variable may capture the possible amplifications derived from the transmission of shocks across countries. In periods when there are no significant idiosyncratic shocks, the dynamics of returns and volatilities are influenced by the endogenous phenomena described by the stochastic process, whereas in presence of shocks (the so-called fault lines; Rajan, 2010), an additional differential spread of volatility and shift in correlation may manifest in geographical areas that are different from the ones in which the shock originally occurred.

3. THE MODEL

The behavioural model for measuring the equilibrium risk premium is assumed to be the well-known function with separable and isoelastic preferences (Mehra and Prescott, 1985):

\[ U = E \left\{ \sum_{t=0}^{\infty} \delta^t c_t^{1-\gamma} \frac{1}{1-\gamma} \right\} \]

where \( c_t \) denotes the consumption, \( \delta \) indicates the subjective time discount factor, \( \gamma \) is the relative risk aversion coefficient, and operator \( E \) indicates the subjective expected value for the representative agent.

The rational expectations model is based on the hypothesis that agents know the “true” predictive probability density function of the returns one time ahead: \( f(r_{t+1} | r_t, p_t) \). If we suppose that market participants do not know the true density function, but they update the posterior probability distribution combining their “a priori” density \( \pi(\delta, \gamma) \) with the likelihood \( g(r_t | \delta, \gamma) \) according to the Bayes’ rule, they achieve:

\[ f(r_{t+1} | r_t, p_t) = \int g(r_t | \delta, \gamma) \pi(\delta, \gamma | r_t, p_t) \]

\[ f(r_{t+1} | r_t, p_t) \propto g(r_t | \delta, \gamma) \]

where \( \pi(\delta, \gamma | r_t, p_t) \) is the posterior density function.

The evaluation of the predictive probability distribution by financial market participants (Corradi and Swanson, 2011) is influenced by possible informative asymmetries, imperfect information, and cognitive errors which favour wrong beliefs causing a bias in both the expectations and pricing (Cecchetti et al., 2000). These biases are reflected in empirical risk premium which measures the final effects of both endogenous and exogenous amplification processes. Therefore, some temporary gaps between actual and equilibrium risk premium do not imply misspecification of the behavioral model; empirical disequilibrium data are not at odds with the behavioral model because they are routed in the process of learning through financial information, carried out by market participants.
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The procedure specified for testing the hypothesis of amplification deriving from cross-country contagion phenomena can be summarized in three steps. The first step consists in determining the risk premia disequilibria for each country defined by the difference between empirical risk premia and risk premia predicted by the estimated consumption capital asset pricing models (CCAPM). In the second step, we model risk premia disequilibria through a DCC multivariate GARCH model. In the third step, we test the presence of contagion between countries by analyzing the exogenous shifts in conditional correlation coefficients, estimated in the second step. Specifically, the test is defined modelling the estimated conditional correlation coefficients with autoregressive models including dummy variables corresponding to crisis periods. In this framework, the dummy coefficients measure possible correlation shifts due to the transmission of idiosyncratic shocks from one country to another, which denotes contagion phenomena.

3.1. Step 1: Countries disequilibrium risk premia

In the first step, we estimate the preference parameters (risk aversion and intertemporal substitution rate) of agents who rationally maximize their expectations using a power utility function. The parameters are estimated by the first order conditions through the generalized method of moments (GMM).

In this framework, the estimation of the model allows the evaluation of the consumer-investors' behaviour in each country with respect to macroeconomic fundamentals. In particular, specifying the utility function in the constant relative risk aversion (CRRA) form

\[ U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \]

where parameter \( \gamma > 0 \) measures the investor's risk aversion, i.e. determines the concaveness of the utility function

\[ \gamma = -\epsilon_t \frac{U''(c_t)}{U'(c_t)}, \]

and thus represents the relative risk aversion coefficient (see, e.g., Cochrane, 2001). In this context, the Euler equations which allow the estimation of parameters \( \delta \) and \( \gamma \) using GMM are given by (Hansen and Singleton, 1982)

\[ E \left[ \delta \left( \frac{\epsilon_{t+1}}{\epsilon_t} \right)^{-\gamma} \left[ r_{t+1} - 1 \right] \mid Z_t \right] = 0 \]

where \( r_{t+1} = [r_{m,t+1}, r_{f,t+1}]' \) is the vector of asset returns and \( 1 = [1, 1]' \). The model is thus specified as a system of equations which, in addition to the consumption
growth rate \( w_t = \epsilon_t / \epsilon_{t-1} \), considers the stock market return \( r_m \) in the first equation, and the risk-free asset return \( r_f \) (approximated by the interest rate of Treasury Bills) in the second equation:

\[
\begin{align*}
E(\delta(w_{t+1})^\gamma r_{m,t+1} - 1) &= 0 \\
E(\delta(w_{t+1})^\gamma r_{f,t+1} - 1) &= 0
\end{align*}
\]

Furthermore, we also evaluate the information set that the representative consumer-investor has at time \( t \). This set of instrumental variables is collected in the information matrix \( Z_t \) and is included in the first order conditions:

\[
\begin{align*}
E(\delta(w_{t+1})^\gamma r_{m,t+1} - 1 | Z_t) &= 0 \\
E(\delta(w_{t+1})^\gamma r_{f,t+1} - 1 | Z_t) &= 0
\end{align*}
\]

The parameter vector \( \theta = [\delta, \gamma]' \) is estimated by means of a two-stage GMM procedure, where the first estimation stage consists in minimizing the quadratic form

\[
\hat{\theta}_{(1)} = \text{arg min} \ g(\theta)' V g(\theta)
\]

where \( g(\theta) = T^{-1/2} \sum_{t=1}^{T} \{ \delta(w_{t+1})^\gamma r_{t+1} - 1 \} \otimes Z_t \) and \( V \) is an arbitrary weighting matrix (usually, \( V = I \)). In the second stage of the GMM procedure, it is possible to estimate the “optimal” covariance matrix \( S \) using the parameter values estimated in the first stage and obtaining a new parameter estimation by replacing weighting matrix \( V \) in Equation (1) with \( \hat{S} \): \( \hat{\theta}_{(2)} = \text{arg min} \ g(\hat{\theta})' \hat{S}^{-1} g(\hat{\theta}) \) (see Hansen and Singleton, 1982). The estimates achieved by the first stage procedure are consistent and asymptotically normal, whereas using also the second stage the additional asymptotic property of efficiency is achieved.

In our approach, the inclusion of instruments in the model specification provides three fundamental tasks: matrix \( Z_t \) allows (i) more accurate and reliable parameter estimation, (ii) the assessment of the macroeconomic fundamentals which influence the decisions of consumption and investment, and (iii) the inclusion of financial prices among the drivers of economic fundamentals which accounts for both the dual role of prices and the endogenous factors which cause disequilibria in financial markets\(^{12}\). In order to address the weak identification problem (Stock and Wright, 2000), we define matrices \( Z_t \) including a wide set of

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\(^{12}\) It is well-know that estimates from GMM always depend on the choice of instrumental variables. With different sets of instruments, the estimated results may differ significantly.
lagged macroeconomic variables\textsuperscript{13}. Furthermore, with the attempt of ensuring a certain level of homogeneity to our analysis, we consider the same variables for all the countries under investigation.

The CCAPM parameter estimates allow the computation of the equilibrium risk premium consistent with the estimated preferences. Furthermore, the model estimation enables the measurement of gaps between estimated and actual risk premia and the evaluation of irrational amplification generating disequilibrium.

For country \( i \), the estimated risk aversion rate \( \hat{\gamma}_i \) for the representative consumer-investor allows the measurement of the equilibrium risk premium (ERP) which depends on the dynamics of the consumption growth rate:

\[
ERP_{i,t} = \hat{\gamma}_i w_{i,t}.
\] (2)

Therefore, the CCAPM estimation allows us to rigorously measure the evolution of risk premia by considering the dynamic pattern of consumption as main reference of macroeconomic fundamentals.

The distance between the risk premium empirically measured for each country as excess return with respect to the risk-free assets and the (theoretical) equilibrium risk premium obtained on the basis of the preferences estimated by the model measures the disequilibrium of each financial market.

Risk premium reflects market participants’ assessment about the amount of risk they see in each financial market and the price they attach to those risks. However, remembering that only the risk added to a well-diversified portfolio should be priced, equity risk premium primarily reflects the price of the average risk in the equity market rather than the amount of risk (Damodaran, 2010). Therefore, the main driver of (equity) risk premium is the price of risk and the measured gaps reflect its dynamics.

Disequilibria are thus obtained as the distance between the series of the equilibrium risk premia in Equation (2) and the empirical (observed) risk premia (ORP) obtained as the excess returns\textsuperscript{14}:

\[
ORP_{i,t} = r_{m,i,t} - r_{f,i,t}.
\]

The series denoted as \( X_i \) and achieved as

\[
X_{i,t} = ORP_{i,t} - ERP_{i,t}.
\] (3)

\textsuperscript{13} We are aware that some instruments included in the analysis might be only weakly correlated with the endogenous variables. However, we believe that the instruments we have considered are somewhat adequate since our parameter estimates are quite stable if we omit one or more lags of the instruments. This result should overcome Stock and Wright (2000) critique: nevertheless, we plan to re-estimate the CCAPM models using Stock and Wright’s inference method as future development.

\textsuperscript{14} We discarded the survey and the implied methods for the estimation of equity risk premium and choose the standard method because of its plainness and its consistency with our research goals.
represents the variable of interest in our analysis since it measures the differential between the risk premium of rational equilibrium and the one actually observed for country $i$. These measures of disequilibria which include both endogenous and exogenous factors are investigated in the second step of our analysis.

3.2. Step 2: Measurement of disequilibria amplification

The methodology of sorting episodes of contagion (exogenous amplification processes) from other situations in which the financial dynamics is (endogenously) amplified with respect to the fundamentals can be based on the investigation of the disequilibria in Equation (3) following two directions:

(i) The relevance that financial crises exert in cross-country transmission of negative tendencies which are not justified by fundamentals in each country;

(ii) The persistence of these irrational processes which cause the transmission and the amplification of the dynamics during a financial crisis.

We address both aspects modelling the irrational amplifications identified by series $X_i$ using a multivariate ARCH-type model able to evaluate the volatility processes of risk premia both in level and in persistence, and their interrelations. Thus, the statistical test for assessing cross-country contagion effect is based on the analysis of the dynamic conditional correlations between two countries, one of which is assumed to be the originator of the crisis. Therefore, we evaluate whether an (exogenous) shift in the correlation coefficient value is due to the spillover effect of a crisis in the other country. This analysis is achieved using the multivariate GARCH model with dynamic conditional correlations (DCC MV-GARCH) proposed by Engle (2002) which allows the estimation of the correlation coefficients between pairs of countries, combined with an autoregressive process in the conditional correlation dynamics.

Many contributions in the econometric literature show that correlation between financial markets tends to increase during periods of financial turmoil (Ang and Bekart, 2002; Longin and Solnik, 1995, 2001). Furthermore, if we define contagion as significant increase in cross-country co-movements, whereas high levels of correlation protracted in time are considered as evidence of financial interdependence (Forbes and Rigobon, 2002), then the existence of contagion must be sought in an exogenous increase in correlation coefficients.

A limitation which virtually affects all the tests developed in literature for evaluating the presence of contagion is that such tests suffer from the arbitrary choice of two fundamental factors: (i) which country has to be considered as the originator of the crisis and (ii) what should be the time window length for the crisis periods (Billio and Pelizzon, 2003). Moreover, the definition of sub-samples on the basis of high and low levels of volatility is a further arbitrary process subjected to a selection bias (Boyer et al., 1999). The DCC MV-GARCH model is particularly suitable for overcoming these limitations. In particular, this methodology allows us to face the heteroskedasticity problem raised by Forbes and Rigobon (2002) without arbitrarily splitting the time series in two sub-samples ac-
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according to its volatility levels. The main contribution of the DCC MV-GARCH model consists in providing a mechanism to identify and measure the time-varying correlation coefficients for risk premium disequilibria between countries.

Our approach continues in the third step where we model the temporal dynamics of estimated conditional correlation coefficients with an autoregressive process including deterministic variables which allows us to test the existence of exogenous shifts in those coefficients during or near financial crises.

The DCC MV-GARCH model is an extension of the constant conditional correlation (CCC) MV-GARCH specification proposed by Bollerslev (1990) and assumes that random variables $X_{i,t}$ are distributed as conditional multivariate normals with zero means and covariance matrix $\Sigma_t$: $X_t | I_{t-1} \sim N(0, \Sigma_t)$\(^{15}\). In this specification, the covariance matrix is decomposed in three matrices, namely $\Sigma_t = D_t R_t D_t$, where $D_t$ is the diagonal matrix of conditional standard deviations and $R_t$ is the time-varying conditional correlation matrix, and is estimated using a two-stage procedure. In the first stage, each conditional variance included in matrix $D_t$ is estimated using univariate GARCH\((p,q)\) processes,

$$\sigma_{i,t}^2 = \omega_i + \sum_{p=1}^{p} \alpha_{i,p} X_{i,t-p}^2 + \sum_{q=1}^{Q} \beta_{i,q} \sigma_{i,t-q}^2 .$$

Matrix $R_t = [\rho_{j,t}]_{x,j}$ is computed in the second stage using maximum likelihood estimator for the dynamic correlation structure

$$Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n) \bar{Q} + \sum_{m=1}^{M} a_m (e_{t-m} e'_{t-m}) + \sum_{n=1}^{N} b_n Q_{t-n}$$

and

$$R_t = Q_t^{-1} Q_t^* Q_t^{-1}$$

where $\bar{Q} = \frac{1}{T} \sum_{m=1}^{T} e_{t-m} e'_{t-m}$ is the unconditional covariance of standardized residuals obtained in the first stage of the estimation procedure and $Q_t^*$ is a diagonal matrix containing the square roots of elements on the diagonal of matrix $Q_t$, namely $Q_t^* = \text{diag} \{\sqrt{q_{t,i}}\}$. Thus, the general element in matrix $R_t$ is given by $\rho_{j,t} = q_{j,t} / \sqrt{q_{i,t} q_{j,t}}$, for $i \neq j$, and the expression of the correlation coefficient in the bivariate case is given by:

\(^{15}\) However, the multivariate normality assumption is not needed for achieving consistency and asymptotic normality of the estimated parameters. When variables have non-Gaussian innovations, DCC estimator can be interpreted as quasi-maximum likelihood estimator (see Engle and Sheppard, 2001).
These estimates are modelled to test for the existence of contagion between pair of countries.

3.3. Step 3: Test for contagion between countries

Once we have estimated the series of dynamic conditional correlations between different countries, we evaluate the existence of contagion between countries $i$ and $j$ by investigating the temporal pattern of the estimated $\rho_{ij,t}$. These correlations reflect both endogenous and exogenous disequilibrium factors, therefore we introduce a time dummy variable for the financial turmoil periods which are defined on the basis of the variables $X_{ij,t}$ estimated in Step 1 of our analysis. This framework allows the identification of the shift in correlation coefficients ascribed to contagion phenomena having controlled for all the endogenous amplification factors in the previous steps. Thus, we evaluate the estimated coefficients of the following (autoregressive) regression model

$$
\rho_{ij,t} = c + \phi_1 \rho_{ij,t-1} + \phi_2 \rho_{ij,t-2} + \ldots + \phi_p \rho_{ij,t-p} + \delta_l DM_{ij,t} + \epsilon_{ij,t}
$$

where $\rho_{ij,t} = \frac{1}{2} \ln \left( \frac{1 + \rho_{ij,t}}{1 - \rho_{ij,t}} \right)$ is the Fisher transformation of the dynamic correlation coefficient between countries $i$ and $j$, and $DM_{ij,t}$ denotes the dummy variable for crisis $l$, with $l = 1, \ldots, C$. For each pair-wise correlation coefficient, the order $P$ of the autoregressive component is identified according to the Akaike, Schwarz, and Hannan-Quinn information criteria (AIC, BIC, and HQC).

In our framework, the model in Equation (4) implies that the significance of the estimated coefficients for the dummy variables indicate structural breaks in the correlation coefficients during financial crises.

Thus, the test for evaluating contagion between two countries with respect to crisis $l$ is based on the null hypothesis $H_0 : \delta_l = 0$ which assumes the absence of contagion effects against the alternative $H_1 : \delta_l \neq 0$ of presence of contagion.

A similar approach has been proposed by Chiang et al. (2007). In their work, the Authors estimate time-varying conditional correlations via DCC MV-GARCH for daily stock market returns and then analyze their dynamics using a

\[16\] Fisher transformation allows us to transform a random variable with support in the interval $[-1, 1]$ such as the correlation coefficient to a normal random variable that may assume any real value. This transformation facilitates the use of autoregressive models as the one we propose in this paper.

\[17\] Another recent work which uses a similar procedure is Syllignakisa and Kouretas (2011).
A statistical procedure for testing financial contagion

GARCH model with dummy variables for account for heteroskedasticity and assessing structural changes in both mean and variance shifts of the correlation coefficients due to external shocks during different phases of the Asian crisis of 1997-98. Our analysis of dynamic conditional correlations of risk premia disequilibria enables us to use an autoregressive model rather than a GARCH model for evaluating the dynamics of conditional correlations\(^\text{18}\). Furthermore, we believe that the Fisher transformation of the dynamic correlation coefficient, which is not adopted by Chiang et al. (2007), leads to more reliable results.

4. MODEL ESTIMATION AND RESULTS

In this Section, we show the results related to the three-step procedure for model estimation and the test for the evaluation of contagion illustrated in Section 3.

In our analysis, we consider five countries, United States (US), United Kingdom (UK), Japan (JP), France (FR), and Italy (IT) using quarterly data series from Q2 1980 to Q3 2009 \((T = 118\) observations). All the data are collected from Thompson Datastream.

4.1. Results for Step 1: Countries disequilibrium risk premia

The first step of the analysis consists in estimating the CCAPM via GMM to obtain, for each country, the values of the two parameters of interest: the intertemporal substitution rate (discount factor) \(\delta\) and the relative risk aversion \(\gamma\).

In the selection of instrumental variables collected in matrix \(Z_t\), i.e. the choice of the information set available at time \(t\) on which investor-consumers base their decisions, we assume that the dependent variable, consumption \(C_t\), is a martingale difference and, thus, is uncorrelated with any macroeconomic variable except for consumption itself\(^\text{19}\). The instruments selection procedure which allows us to identify the variables and their lag length to be included in matrix \(Z_t\) is performed on the basis of two information criteria: MMSC-BIC and MMSC-HQ (Andrews and Lu, 2001). According to these model selection criteria, the result we obtain is that matrix \(Z_t\) is identified by the following variables for each country: \(r_{m,t}\), \(r_{m,t-1}\), \(r_{m,t-2}\), \(r_{m,t-3}\), \(r_{f,t}\), \(r_{f,t-1}\), \(r_{f,t-2}\), \(r_{f,t-3}\), \(\Delta\text{prod}_{t}\), \(\Delta\text{prod}_{t-1}\), \(\Delta\text{prod}_{t-2}\), \(\Delta\text{gdp}_{t}\), \(\Delta\text{gdp}_{t-1}\), \(\Delta\text{gdp}_{t-2}\), \(\Delta\text{gdp}_{t-3}\), \(\text{spread}_{t}\), \(\text{spread}_{t-1}\), \(\text{spread}_{t-2}\), \(\text{spread}_{t-3}\), where

- \(r_m\) is the return of the market portfolio approximated by the Datastream index for the whole stock market;
- \(r_f\) is the return of the risk-free asset defined as the average value of the redemption yield

\(^\text{18}\) See results for the ARCH tests in Tables 5 and 6.

\(^\text{19}\) This approach differs from the one generally followed in the estimation of asset pricing models, where the instrumental variables which should be used are the variables included in the model itself, namely consumption and asset returns. The assumption implied in this approach is that market participants base their consumption/investment decisions only on these variables and hence, for this reason, are assumed as uncorrelated.
\[
    r_{jt} = \frac{\sum_{j} y_{jt} d_{jt} (p_{jt} + a_{jt}) n_{jt}}{\sum_{j} d_{jt} (p_{jt} + a_{jt}) n_{jt}}
\]

where \( y_{jt} \) is the redemption yield to assumed maturity for bond \( j \), \( d_{jt} \) is the duration, \( p_{jt} \) is the price, \( a_{jt} \) is the accrued interest, \( n_{jt} \) is the nominal value, and the summations are over the bonds currently in the Datastream index;

- \( \Delta \text{prod} \) is the \( \Delta \log \) of the industrial production series (seasonal adjusted);
- \( \Delta \text{gdp} \) is the \( \Delta \log \) of the gross domestic product series (seasonal adjusted);
- \( \text{spread} \) is the treasury bill interest rate spread computed as the difference between the long term TBill rate (7-10 years) and the short term TBill rate (1 month).

Table 1 shows the estimates of coefficients \( \hat{\delta} \) and \( \hat{\gamma} \) for each country. From the estimate results and the corresponding standard errors reported in Table 1, it can be easily noted that all coefficients are significant. According to the J-tests, the over-identifying restrictions implied by the model are not rejected for all countries. The estimation of the risk aversion rate allows the measurement of the equilibrium risk premium, \( ERP_{i,j} = \hat{\gamma}_i w_{i,j} \), for each country \( i \).

### TABLE 1

<table>
<thead>
<tr>
<th>Country</th>
<th>( \hat{\delta} )</th>
<th>( \hat{\gamma} )</th>
<th>J-Test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.9802</td>
<td>0.0832</td>
<td>20.73</td>
</tr>
<tr>
<td></td>
<td>(0.0065)</td>
<td>(0.0187)</td>
<td>(0.9949)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.9783</td>
<td>0.3391</td>
<td>22.38</td>
</tr>
<tr>
<td></td>
<td>(0.0069)</td>
<td>(0.0294)</td>
<td>(0.9890)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.9921</td>
<td>0.3209</td>
<td>19.48</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0321)</td>
<td>(0.9974)</td>
</tr>
<tr>
<td>France</td>
<td>0.9771</td>
<td>0.1165</td>
<td>23.23</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0820)</td>
<td>(0.9933)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.9828</td>
<td>0.6349</td>
<td>17.57</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0375)</td>
<td>(0.9992)</td>
</tr>
</tbody>
</table>

### TABLE 2

Descriptive statistics for variables \( X_{i,t} \)

<table>
<thead>
<tr>
<th></th>
<th>( X_{i,t} )</th>
<th>( X_{i,t} )</th>
<th>( X_{i,t} )</th>
<th>( X_{i,t} )</th>
<th>( X_{i,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.068</td>
<td>-0.330</td>
<td>-0.313</td>
<td>-0.100</td>
<td>-0.626</td>
</tr>
<tr>
<td>Median</td>
<td>-0.061</td>
<td>-0.321</td>
<td>-0.230</td>
<td>-0.087</td>
<td>-0.636</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.171</td>
<td>-0.170</td>
<td>-0.058</td>
<td>0.170</td>
<td>-0.061</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.332</td>
<td>-0.656</td>
<td>-0.702</td>
<td>-0.446</td>
<td>-0.924</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.082</td>
<td>0.082</td>
<td>0.110</td>
<td>0.115</td>
<td>0.145</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.403</td>
<td>-0.774</td>
<td>-0.552</td>
<td>-0.364</td>
<td>1.028</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.873</td>
<td>4.532</td>
<td>3.945</td>
<td>4.178</td>
<td>5.544</td>
</tr>
<tr>
<td>Jarque-Bera Test</td>
<td>7.011</td>
<td>23.511</td>
<td>10.472</td>
<td>9.514</td>
<td>52.589</td>
</tr>
<tr>
<td>Prob. (JB Test)</td>
<td>0.030</td>
<td>&lt; 0.001</td>
<td>0.005</td>
<td>0.009</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

Using Equation (3), we obtain the time series of risk premium disequilibria for each country, denoted as \( X_{i,t} \), measured as the distance between equilibrium and actual observed risk premia (descriptive statistics are reported in Table 2).
4.2. Results for Step 2: Measurement of disequilibria amplification

The second step of the analysis starts with the identification of financial crises in each country on the basis of the standardized values of variables $X_{i,t}$. In particular, we detect a crisis in country $i$ at time $t$ when the standardized value of the series is lower than $-2$ (which obviously corresponds to $-2\sigma_{X_{i}}$ for the non-standardized variables). This procedure allows us to address the limitation of the arbitrary choice of the crisis periods (Billio and Pelizzon, 2003). For each country, we report the quarters which are detected as turmoil periods in Table 3, whereas Figure 1 shows the dynamics of the standardized series $X_{i,t}$.

The analysis continues by modelling the endogenous process of risk premia disequilibria through the estimation of the DCC multivariate GARCH model using (standardized) $X_{i,t}$ achieved in Step 1 of our analysis as dependent variables. In the first stage of the model estimation procedure, one univariate GARCH model is specified for each country. According to BIC information criterion, we identify GARCH(1,1) specification for modelling standardized series $X_{i,t}$ of all countries. The results of the univariate GARCH models summarized in Table 4 show a high level of volatility persistence: $\alpha_{i,1} + \beta_{i,1}$ is very close to 1 for all countries. Moreover, the highly significance of $\hat{\beta}_{i,1}$ coefficients highlights the fact that the GARCH specification is particularly suitable for analyzing risk premium disequilibria.

The parameter estimation for the DCC(1,1) component are reported in the last rows of Table 4. Estimate of coefficient $b$ is highly significant, whereas, for parameter $a$, we do not reject the null hypothesis $H_0: a = 0$ for a level of significance of 10%. Despite the non-significant estimate of parameter $a$, the model is able to detect a common time-series pattern which is shown by the dynamics of the conditional correlations between the United States and the other four countries depicted in Figure 2. Conditional correlation coefficients are also characterized by a high level of persistence: the sum of DCC parameters is close to 1.

---

20 From Table 3 and Figure 1, we notice crisis periods which are common to all the countries: Q4 1998 and, except for Japan, Q1 2001. The former period is associated to the well-known Russian crisis started in August 1998 which was probably triggered by the crisis that affected many Asian markets in late 1997, while the latter to the dot-com bubble burst (2000-01) and the subsequent trough of business cycle detected in the U.S. by the National Bureau of Economic Research in the period between March and November 2001 (see http://www.nber.org/cycles/cyclesmain.html). Table 3 also shows an additional turmoil period which is common to the three European countries examined in our analysis: Q4 2002. An interesting and quite surprising feature which arises from the results in Table 3 is that the crisis started in 2007-2008 is not, de facto, detected as financial turmoil period which caused disequilibrium in risk premia. The only exceptions are Q4 2008 for Japan and Q1 2009 for the United States.
Table 3

Identification of the quarters of financial turmoil (standardized $X_{ij} < -2$)

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>U.K.</th>
<th>Japan</th>
<th>France</th>
<th>Italy*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3 1981</td>
<td>✔</td>
<td></td>
<td></td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q3 1982</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q1 1988</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q2 1990</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q4 1990</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q2 1995</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q4 1998</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q2 2001</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q4 2001</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q3 2002</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q4 2002</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q4 2008</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Q1 2009</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>

* For Italy, we consider values lower than -1.5 (the only observation lower than -2 is Q3 1982).

In the third step of our analysis, we analyze the exogenous shifts in the dynamics of correlation coefficients estimated by the DCC MV-GARCH model, focusing on the relationships between the United States and the other countries. This choice is based on the a priori assumption that U.S. market is the originator of both the 2000-01 turmoil period and the crisis started in 2007-08 and that U.S. financial-economic situation has a strong influence for the financial mood in other countries. This assumption could be easily relaxed by investigating all the pair-wise correlation coefficients between countries.
### TABLE 4
Results of coefficient estimates for DCC MV-GARCH model

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. GARCH(1,1)</td>
<td>( \omega_{US} )</td>
<td>0.1001</td>
<td>0.0667</td>
<td>1.5003</td>
<td>0.1335</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{US} )</td>
<td>0.3695</td>
<td>0.1567</td>
<td>2.3581</td>
<td>0.0184</td>
</tr>
<tr>
<td></td>
<td>( \beta_{US} )</td>
<td>0.5970</td>
<td>0.0826</td>
<td>7.227</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>U.K. GARCH(1,1)</td>
<td>( \omega_{UK} )</td>
<td>0.1063</td>
<td>0.0714</td>
<td>1.4881</td>
<td>0.1367</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{UK} )</td>
<td>0.2218</td>
<td>0.1633</td>
<td>1.3583</td>
<td>0.1744</td>
</tr>
<tr>
<td></td>
<td>( \beta_{UK} )</td>
<td>0.7040</td>
<td>0.0827</td>
<td>8.5112</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Japan GARCH(1,1)</td>
<td>( \omega_{JP} )</td>
<td>0.4164</td>
<td>0.1714</td>
<td>2.4300</td>
<td>0.0151</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{JP} )</td>
<td>0.0658</td>
<td>0.0976</td>
<td>0.6738</td>
<td>0.5005</td>
</tr>
<tr>
<td></td>
<td>( \beta_{JP} )</td>
<td>0.5279</td>
<td>0.1575</td>
<td>3.3525</td>
<td>0.0008</td>
</tr>
<tr>
<td>France GARCH(1,1)</td>
<td>( \omega_{FR} )</td>
<td>0.0076</td>
<td>0.0320</td>
<td>0.2391</td>
<td>0.8111</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{FR} )</td>
<td>0.0000</td>
<td>0.0038</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>( \beta_{FR} )</td>
<td>0.9921</td>
<td>0.0329</td>
<td>30.136</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Italy GARCH(1,1)</td>
<td>( \omega_{IT} )</td>
<td>0.1943</td>
<td>0.1850</td>
<td>1.0504</td>
<td>0.2935</td>
</tr>
<tr>
<td></td>
<td>( \alpha_{IT} )</td>
<td>0.3905</td>
<td>0.4269</td>
<td>0.9148</td>
<td>0.3603</td>
</tr>
<tr>
<td></td>
<td>( \beta_{IT} )</td>
<td>0.5013</td>
<td>0.1310</td>
<td>3.8261</td>
<td>0.0001</td>
</tr>
<tr>
<td>DCC(1,1)</td>
<td>( \alpha )</td>
<td>0.0478</td>
<td>0.0306</td>
<td>1.5624</td>
<td>0.1182</td>
</tr>
<tr>
<td></td>
<td>( \beta )</td>
<td>0.9467</td>
<td>0.0159</td>
<td>59.446</td>
<td>&lt; 0.0001</td>
</tr>
</tbody>
</table>

#### 4.3. Results for Step 3: Test for contagion between countries

From Figure 2 we can observe that, since mid 1998, correlations between the U.S. and the other countries have strongly increased. In order to evaluate if this shift is exogenously determined and due to contagion phenomena, we analyze the dynamic conditional correlations through the autoregressive model specified in Equation (4), where the dummy variables correspond to the detected crisis periods reported in Table 3. In particular, we evaluate four recent financial turmoil periods: \( DM_{1,t} \) represents the impulse dummy variable for Q4 1998, \( DM_{2,t} \) defines Q4 2001, \( DM_{3,t} \) identifies Q4 2002, and \( DM_{4,t} \) is the impulse dummy for Q1 2009\(^{21}\). The results of the autoregressive models illustrated in Table 5 show that, expect for the autoregressive component coefficient which is highly significant for all estimated models, the only significant dummy variable is \( DM_{4,t} \) in the model which evaluates the correlation between the United States and Japan. The estimates of variables \( DM \) are non-significant in all the other analyzed cases, thus highlighting absence of contagion between countries. Last two rows of Table 5 shows the p-values related to the LM Breusch-Godfrey test for assessing residual autocorrelation up to four lags and the ARCH test which investigates the pres-

\(^{21}\) In addition to the three dummy variables referred to the crisis periods endogenously detected by analyzing series \( X_{ij} \) and reported in Table 3, with the purpose of evaluating the possible contagion effects during the financial crisis started in 2007-08, we also include in our analysis a dummy variable which corresponds to the period 2008-09.
ence of conditional heteroskedasticity. All the p-values reported in Table 5 are highly significant with respect to the significance level usually employed, stressing the fact that the AR(1) specification used is suitable, as well as the AR(2) model for $\rho_{UK-FR,t}$, and we do not need to specify an ARCH model for the stochastic component\textsuperscript{22}.

Furthermore, we develop an additional analysis in which we consider alternative dummy variables with respect to the ones reported in Table 5. In particular, to provide a more broad analysis of contagion effects between the analyzed countries, we evaluate a larger window length for the crisis period than the single quarter observation illustrated in Table 3. We examine three well-known turmoil periods associated to three step dummy variables: $DM^{*}_{1,t}$ identifies the Asian and Russian crises (Q4 1997 – Q2 1998), $DM^{*}_{2,t}$ refers to the dot-com bubble burst (Q1 2000 – Q4 2002), and $DM^{*}_{3,t}$ indicates the 2007-08 stock market crash (Q3 2008 – end of sample). It is easy to note that the three new dummies $DM^{*}_{l,t}$ include the four quarters which are identified by dummies $DM_{l,t}$ in the autoregressive model of Table 5\textsuperscript{23}. These step dummy variables enables us to detect potential exogenous structural breaks in the series of dynamic correlation coefficients for more extended time windows than dummies $DM_{l,t}$. The results related to the autoregressive models with dummies $DM^{*}_{l,t}$ illustrated in Table 6 show that the dummy variable $DM^{*}_{3,t}$ is significant for each correlation coefficient $\rho_{ij,t}$ between the U.S. and all the other countries. This result can be considered as evidence of contagion: the financial crisis that started in 2007-08 in the United States, which is commonly assumed to be the originator country, infected all the countries we considered, causing a significant (and persistent, as illustrated in Figure 2) shift in the correlations after mid 2008\textsuperscript{24}.

Therefore, the results of our analysis are twofold: defining crisis periods relying on the statistical criterion of $2\sigma$ is no contagion, expect for U.S.–Japan; vice versa, defining crises over wider time periods we find evidence of contagion between the United States and all the other countries for the latest financial crisis related to the 2007-08 stock market crash.

\textsuperscript{22} The approach for testing contagion developed by Chiang et al. (2007) refers to a GARCH(1,1) specification for analyzing the dynamic conditional correlations because of the presence of conditional heteroskedasticity in financial market returns.

\textsuperscript{23} It must be noted that dummies $DM^{*}_{l,t}$ may suffer from the limitation described in Billio and Pelizzon (2003) regarding the arbitrary choice of the time window length for crisis periods. However, these periods are identified using dummies $DM_{l,t}$ which are endogenously detected using the $-2\sigma_{X_t}$ condition and are not affected from that limitation.

\textsuperscript{24} Results in Table 6 also show a quite curious outcome: the dummy variable $DM^{*}_{1,t}$ is significant for the correlation coefficient between Italy and other 3 countries, namely U.S., U.K. and France. This result is not easily interpretable and will require a more in depth investigation.
Figure 2 – Dynamics of conditional correlation coefficients estimated by the DCC MV-GARCH between the U.S. and the other countries.

Table 5

Results for the autoregressive model estimates with impulse dummy variables referred to the detected crisis periods:

\[ DM_{ij} = \text{Q4 1998}, \quad DM_{ij} = \text{Q4 2001}, \quad DM_{ij} = \text{Q4 2002}, \quad DM_{ij} = \text{Q1 2009} \]

\[ \rho_{ij} \]

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P-value Test: 0.67 0.27 0.63 0.24 0.56 0.96* 0.70 0.94 0.94 0.68

BG(4) P-value Test: 0.88 0.94 0.93 0.42 0.54 0.36 0.93 0.77 0.93 0.59

(Standard error in parenthesis)

* = significant at 10%
** = significant at 5%
*** = significant at 1%
TABLE 6
Results for the autoregressive model estimates with impulse dummy variables referred to the detected crisis periods:
\[ DM_{1,t}^* = \text{Asian and Russian crises: Q4 1997 – Q4 1998}, \]
\[ DM_{2,t}^* = \text{dot-com bubble: Q1 2000 – Q4 2002}, \]
\[ DM_{3,t}^* = \text{2007-08 stock market crash: Q3 2008 – end} \]

\[ \rho_{ij,t} - \sum_{t=1}^{k} \beta_i \rho_{ij,t-k} \]

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(Standard error in parenthesis)
* = significant at 10%
** = significant at 5%
*** = significant at 1%

5. CONCLUSIONS

Present-day dynamics of financial markets show various synchronous amplification processes which can be easily confused with contagion. In this paper, we propose a new procedure for testing the existence of contagion effects between countries. More specifically, we develop a methodology that discriminates between amplification processes endogenously generated and the exogenous transmission of shocks which can be attributed to financial contagion phenomena. We suggest to model time-varying conditional correlations in risk premia disequilibria including deterministic variables representing crisis periods. In this way, we introduce a new test of contagion which is able to detect correlation shifts derived from idiosyncratic shocks originated in another country, ruling out the endogenous amplifications. Moreover, this approach allows us to test the existence of contagion in forms which are rigorously consistent with the characteristics of modern financial markets in which the behaviours of both professionals and less-experienced market participants play a fundamental role in the amplification processes of volatility. Therefore, the test procedure we propose discriminates between the different causes of amplification and detects the cases in which contagion occurs.

Within our procedure, we identify disequilibria measuring the equilibrium risk premia and computing the distance between the equilibrium level and the risk premia actually observed for five countries, namely United States, United Kingdom, Japan, France, and Italy, using quarterly data from 1990 to 2010. Focusing on the analysis of contagion phenomena between countries from mid 1998 onwards and examining quarters representing crisis periods, we find evidence of
significant contagion effects from the U.S. to the other countries for the 2007-08 financial crisis.

Comparing our results with the existing literature, the transmission of idiosyncratic shocks across countries turns out to be less frequent than some results achieved in the statistical literature, but it is clearly detected during the latest financial crisis. This result is not surprisingly since the so-called “subprime crisis” has its own origin exactly in the U.S. financial system which, as we pointed out, is the generator of the amplification processes of volatility and, hence, has easily affected worldwide financial markets.

REFERENCES

F.H. KNIGHT, (1921), Risk, uncertainty, and profit, Houghton Mifflin, Boston.
A statistical procedure for testing financial contagion

The aim of the paper is to provide an analysis of contagion through the measurement of the risk premia disequilibria dynamics. In order to discriminate among several disequilibrium situations we propose to test contagion on the basis of a two-step procedure: in the first step we estimate the preference parameters of the consumption-based asset pricing model (CCAPM) to control for fundamentals and to measure the equilibrium risk premia in different countries; in the second step we measure the differences among empirical risk premia and equilibrium risk premia in order to test cross-country disequilibrium situations due to contagion. Disequilibrium risk premium measures are modelled by the multivariate DCC-GARCH model including a deterministic crisis variable. The model describes simultaneously the risk premia dynamics due to endogenous amplifications of volatility and to exogenous idiosyncratic shocks (contagion), having controlled for fundamentals effects in the first step. Our approach allows us to achieve two goals: (i) to identify the disequilibria generated by irrational behaviours of the agents, which cause increasing in volatility that is not explained by the economic fundamentals but is endogenous to financial markets, and (ii) to assess the existence of contagion effect defined by exogenous shift in cross-country return correlations during crisis periods. Our results show evidence of contagion from the United States to United Kingdom, Japan, France, and Italy during the financial crisis which started in 2007-08.