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MODIFIED INTERVENED POISSON DISTRIBUTION

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1. INTRODUCTION

Cohen (1960) introduced positive Poisson distribution to describe a chance mechanism whose observational apparatus becomes active only when atleast one event occurs. (Singh, 1978) obtained a numerical example to illustrate the statistical application of the positive Poisson distribution in such situations. Later a modified version of positive Poisson distribution is introduced by (Shanmugam, 1985) which is termed as the intervened Poisson distribution (IPD). An advantage of the IPD is that it provides information on how effective various preventive actions taken by health service agents, where positive Poisson fails. The IPD is applicable in several areas such as reliability analysis, queuing problems, epidemiological studies, etc. For example, see (Shanmugam, 1985, 1992) and (Huang and Fung, 1989). During the observational period, the failed units are either replaced by new units or rebuilt. This kind of replacement changes the reliability of a system as only some of its components have longer life. (Scollnik, 2006) introduced a generalized version of the IPD namely intervened generalized Poisson distribution ($IGPD$).

In this paper, we propose a modified version of intervened Poisson distribution which extends the IPD and an advantage of this distribution over the IPD is that it stretches the probability in all directions so that clustering of probabilities at initial values of operating mechanism is overlooked.

In section 2, we present the derivation of the IPD and the $IGPD$ and in section 3, we present the derivation of a new modified version of the IPD, which we call the modified intervened Poisson distribution (or in short $MIPD$) and study some of its important properties. Further estimation of parameters of $MIPD$ by method of factorial moments, method of mixed moments and method of maximum likelihood are discussed in section 4 and illustrated using certain real life data sets. It is to be noted that neither uniformly best estimators for unknown parameters nor uniformly best statistics exist for the $MIPD$. Further results concerning the statistical inference in case of the $MIPD$ will be published as a sequel.

2. THE *IPD* AND THE *IGPD*

In this section, we present the derivation of the *IPD* and the *IGPD* .

Let U_1 be the number of cholera cases in a household. The event $U_1 = 0$ is not observable since observational apparatus (That is, diagnosis) is activated only when $U_1 > 0$. The random variable U_1 being of a rare event, it is appropriate to consider a zero truncated Poisson distribution for the positive integer valued random variable with probability mass function (pmf)

$$h(u_1) = \frac{\lambda^{u_1}}{u_1!}(e^{\lambda} - 1)^{-1} \tag{1}$$

with $\lambda > 0$ for those values of u_1 on the positive integers, and zero elsewhere.

In this example of cholera incidence, health service agencies and others resort to various preventive measures. These have the effect of changing λ from one incidence to another. We may assume that such effect results in changing λ to $\rho\lambda$. Let U_2 be the number of cases that occurred after preventive treatments were applied. The random variable U_2 is Poisson with mean $\rho\lambda$ and it is statistically independent of U_1 . Assume that our observational apparatus has a record of only the random variable $U = U_1 + U_2$, the total number of rare events occurred al-together is an IPD with parameters λ and ρ , which we denoted here after as $IPD(\lambda, \rho)$ .The pmf of U is given

$$g(u) = \frac{[(1+\rho)^u - \rho^u]}{e^{\lambda\rho}(e^{\lambda} - 1)} \frac{\lambda^u}{u!} \tag{2}$$

with $\lambda > 0$ and $\rho \geq 0$, for those values of u on the positive integers and zero elsewhere. The mean and variance of the $IPD(\lambda, \rho)$ are

$$E(U) = \mu = \lambda \left[(1+\rho) + \frac{1}{(e^{\lambda} - 1)} \right] \tag{3}$$

and

$$Var(U) = \mu - \left[\frac{\lambda}{e^{\lambda} - 1} \right]^2 e^{\lambda} \tag{4}$$

Additional properties of the *IPD* are given in (Shanmugam, 1985).

A random variable W is said to follow the generalized Poisson distribution (*GPD*) of Consul (1989) with parameters θ and δ , if its pmf is given by

$$h_2(w) = \frac{\theta(\theta + w\delta)e^{-(\theta+w\delta)}}{w!} \tag{5}$$

with $\theta > 0$ and $0 \le \delta < 1$, for those values of w on the non-negative integers, and zero elsewhere.

(Scollnik, 2006) obtained $IGPD$ as in the following. Let V_1 follows zero-truncated GPD with parameters α, β and V_2 follows GPD with parameters $\gamma\alpha$ and β. Assume that V_1 and V_2 are statistically independent. Then the random variable $V = V_1 + V_2$ is an IGPD with parameters α, β and γ and its pmf is

$$g_2(v) = \frac{\alpha([1+\gamma][(1+\gamma)\alpha + v\beta]^{v-1} - \gamma[\gamma\alpha + v\beta]^{v-1})}{e^{\gamma\alpha+v\beta}(e^{\theta}-1)v!} \tag{6}$$

with $\alpha > 0$, $\gamma \ge 0$ and $0 \le \beta < 1$.

3. A MODIFIED VERSION OF IPD AND ITS PROPERTIES

In this section we define a modified version of intervened Poisson distribution ($MIPD$) and derive some of its important properties.

Let Y be a positive integer valued random variable following the IPD with parameters λ and ρ_1, and let Z be a non-negative integer valued random variable having Poisson distribution with mean $\lambda\rho_2$, in which $\lambda > 0$ and $\rho_j \ge 0$ for each $j = 1,2$. Assume that Y and Z are statistically independent. Then the distribution of $X = Y + 2Z$ is called modified intervened Poisson distribution with parameters λ, ρ_1 and ρ_2 which we written as $MIPD(\lambda, \rho_1, \rho_2)$. Clearly $MIPD(\lambda, \rho, 0)$ is $IPD(\lambda, \rho)$. Thus the $MIPD(\lambda, \rho_1, \rho_2)$ is an extended class of discrete distributions which include both positive Poisson distribution and the $IPD(\lambda, \rho)$ as its special case. Also, this type of an extension opens up the possibility of a second intervention. Now we have the following results.

Result 3.1 The probability mass function $f_x = P(X = x)$ of $MIPD(\lambda, \rho_1, \rho_2)$ is the following for $x = 1,2,...$ in which $\lambda > 0, \rho_j \ge 0$ for each $j = 1,2$.

$$f_x = k \sum_{r=0}^{\left[\frac{x-1}{2}\right]} \frac{\eta_{x-2r}(\lambda\rho_2)^r}{(x-2r)!r!} \tag{7}$$

where

$$\eta_x = \lambda^x (1+\rho_1)^x - \rho_1{}^x] \tag{8}$$

and

$$k = \frac{1}{(e^{\lambda} - 1)e^{\lambda(\rho_1+\rho_2)}} \tag{9}$$

Proof: The Probability mass function f_x of X is given by

$$f_x = P(X = x)$$

$$= \sum_{r=0}^{\left[\frac{x-1}{2}\right]} P(Y = x-2r)P(Z = r / Y = x-2r)$$

$$= \sum_{r=0}^{\left[\frac{x-1}{2}\right]} \frac{[(1+\rho_1)^{x-2r} - \rho_1{}^{x-2r}]}{e^{\lambda\rho_1}(e^{\lambda} - 1)} \frac{\lambda^{x-2r}}{x-2r!} \frac{(\lambda\rho_2)^r e^{-\lambda\rho_2}}{r!}$$

$$= k \sum_{r=0}^{\left[\frac{x-1}{2}\right]} \frac{\eta_{x-2r}}{(x-2r)!} \frac{(\lambda\rho_2)^r}{r!}$$

where η_x and k are as given in (8) and (9).

Result 3.2 Probability generating function (pgf) of X following $MIPD(\lambda, \rho_1, \rho_2)$ with pmf (7) is the following.

$$Q(s) = \frac{(e^{\lambda s} - 1)e^{\lambda s(\rho_1+\rho_2 s)}}{(e^{\lambda} - 1)e^{\lambda(\rho_1+\rho_2)}} \tag{10}$$

Proof: The pgf of X with pmf (7) is

$$Q(s) = E(s^X)$$

$$= k \sum_{x=1}^{\infty} s^x \sum_{r=0}^{\left[\frac{x-1}{2}\right]} \frac{\eta_{x-2r}}{(x-2r)!} \frac{(\lambda\rho_2)^r}{r!}$$

$$= k \sum_{x=1}^{\infty} \sum_{r=0}^{\left[\frac{x-1}{2}\right]} s^x \frac{(\lambda\rho_2)^r}{(x-2r)!r!} [[\lambda(1+\rho_1)]^{x-2r} - [\lambda\rho_1]^{x-2r}] \tag{11}$$

Simplifying (11) by applying the definition of exponential series, we get (10).

Result 3.3 The mean and variance of the $MIPD(\lambda,\rho_1,\rho_2)$ are the following.

$$E(X)=\lambda\left[(1+\rho_1+2\rho_2)+\frac{1}{(e^{\lambda}-1)}\right] \tag{12}$$

and

$$Var(X)=E(X)-\left[\left(\frac{\lambda}{(e^{\lambda}-1}\right)^2 e^{\lambda}+2\lambda\rho_2\right] \tag{13}$$

Proof is simple and hence omitted.

Result 3.4 The r-th factorial moment $\mu_{[r]}$ of $MIPD(\lambda,\rho_1,\rho_2)$ with pgf (10) is the following, for $r=1,2,...$

$$\mu_{[r]}=\frac{r!}{(e^{\lambda}-1)}[e^{\lambda}\sum_{j=0}^{\left[\frac{r}{2}\right]}\frac{(\lambda\rho_2)^j}{j!(r-2j)!}[[\lambda(1+\rho_1+2\rho_2)]^{r-2j}-[\lambda(\rho_1+2\rho_2)]^{r-2j}] \tag{14}$$

Proof. The factorial moment generating function (fgmf) $F(t)$ of $MIPD(\lambda,\rho_1,\rho_2)$ with pgf (3.4) is

$$F(t)=\sum_{r=0}^{\infty}\mu_{[r]}\frac{t^r}{r!} \tag{15}$$

$$=Q(1+t)$$

$$=k(e^{\lambda(1+t)}-1)e^{\lambda\rho_1(1+t)+\lambda\rho_2(1+t)^2}$$

$$=\frac{[e^{\lambda}e^{\lambda[t(1+\rho_1+2\rho_2)+\rho_2 t^2]}-e^{\lambda[t(\rho_1+2\rho_2)+\rho_2 t^2]}]}{(e^{\lambda}-1)}$$

$$=\frac{1}{(e^{\lambda}-1)}\left[e^{\lambda}\sum_{r=0}^{\infty}\sum_{j=0}^{\left[\frac{r}{2}\right]}\frac{(\lambda\rho_2)^j}{j!(r-2j)!}[[\lambda(1+\rho_1+2\rho_2]^{r-2j}-[\lambda(\rho_1+2\rho_2]^{r-2j}]\right] \tag{16}$$

On equating coefficient of $\frac{t^r}{r!}$ on right side expressions of (15) and (16), we get (14).

Remark 3.1 When $\rho_2 = 0$, $\rho_1 = \rho$ then $\mu_{[r]} = \frac{\lambda^r}{(e^{\lambda} - 1)}[e^{\lambda}(1+\rho)^r - \rho^r]$, which is the r th factorial moment of $IPD(\lambda, \rho)$.

Result 3.5 A simple recurrence relation for the probabilities of $MIPD(\lambda, \rho_1, \rho_2)$ is the following for $x = 1, 2, ..$ with $f_0 = 0$.

$$(x+1) f_{x+1} = \lambda(1+\rho_1) f_x + 2\lambda\rho_2 f_{x-1} + k\lambda \sum_{j=0}^{\left[\frac{x}{2}\right]} \frac{(\lambda\rho_1)^{x-2j}}{(x-2j)!} \frac{(\lambda\rho_2)^j}{j!} \tag{17}$$

Proof. From (10), we have the following *pgf* of $MIPD(\lambda, \rho_1, \rho_2)$

$$Q_X(s) = \sum_{x=1}^{\infty} f_x s^x$$

$$= \frac{(e^{\lambda s} - 1) e^{\lambda s(\rho_1 + \rho_2 s)}}{(e^{\lambda} - 1) e^{\lambda(\rho_1 + \rho_2)}} \tag{18}$$

On differentiating (18) with respect to s, we have

$$\sum_{k=1}^{\infty} x f_x s^{x-1} = c(e^{\lambda s} - 1) e^{\lambda s(\rho_1 + \rho_2 s)} [\lambda(\rho_1 + 2\rho_2 s) + \lambda] + c\lambda e^{\lambda s(\rho_1 + \rho_2 s)}$$

$$= \lambda(1+\rho_1) Q(s) + 2\lambda\rho_2 Q(s) + k\lambda e^{\lambda s(\rho_1 + \rho_2 s)} \tag{19}$$

On equating coefficient of s^x on both sides of (19) we get (17).

Result 3.6 Recurrence relation for the factorial moments $\mu_{[r]}$ of $MIPD(\lambda, \rho_1, \rho_2)$ is the following for $r \geq 1$ with $\mu_{[0]} = 0$

$$\mu_{[r+1]} = \lambda[(1+\rho_1) + 2\rho_2]\mu_{[r]} + 2\lambda\rho_2 r \mu_{[r-1]} + r! c\lambda \sum_{j=0}^{[r/2]} \frac{(\lambda(\rho_1 + 2\rho_2))^{r-2j} (\lambda\rho_2)^j}{(r-2j)!} \frac{1}{j!} \tag{20}$$

Proof: From the expression (15), the fmgf $F(t)$ of the $MIPD(\lambda,\rho_1,\rho_2)$ is

$$F(t)=\sum_{r=0}^{\infty}\frac{t^r\mu_{[r]}}{r!}$$

$$=Q(1+t)$$

$$=k(e^{\lambda(1+t)}-1)e^{\lambda[\rho_1(1+t)+\rho_2(1+t)^2]} \tag{21}$$

On differentiating (21) with respect to t, we have

$$\sum_{r=1}^{\infty}\frac{t^{r-1}\mu_{[r]}}{(r-1)!}=k(e^{\lambda(1+t)}-1)e^{\lambda(1+t)(\rho_1+\rho_2)}\lambda[(\rho_1+\rho_2)+\rho_2(1+2t)]$$
$$+k\lambda e^{\lambda(1+t)(\rho_1+\rho_2+\rho_2 t)}e^{\lambda(1+t)}$$

$$=\lambda(\rho_1+2\rho_2+1)\sum_{r=0}^{\infty}\frac{t^r\mu_{[r]}}{r!}+2\lambda\rho_2\sum_{r=0}^{\infty}\frac{t^{r+1}\mu_{[r]}}{r!}+c\lambda e^{\lambda(1+t)(\rho_1+\rho_2(1+t))} \tag{22}$$

Equating coefficient of $\frac{t^r}{r!}$ on both sides of (22), we obtain (20).

4. ESTIMATION

In this section we discuss the estimation of the parameters λ, ρ_1 and ρ_2 of the $MIPD(\lambda,\rho_1,\rho_2)$ by method of factorial moments, method of mixed moments and method of maximum likelihood. The parameters λ, ρ_1 and ρ_2 of the $MIPD(\lambda,\rho_1,\rho_2)$ have been estimated by the method of factorial moments as in the following. The first three factorial moments $\mu_{[1]}$ $\mu_{[2]}$ and $\mu_{[3]}$ of the $MIPD$ are equated to the corresponding sample factorial moments $m_{[1]}$, $m_{[2]}$ and $m_{[3]}$. Thus, we have the following system of equations:

$$\lambda\left[(1+\rho_1+2\rho_2+\frac{1}{(e^{\lambda}-1)}\right]=m_{[1]} \tag{23}$$

$$\lambda\left[(1+\rho_1+2\rho_2)\mu_{[1]}+\frac{\lambda^2e^{\lambda}(1+\rho_1+2\rho_2)}{(e^{\lambda}-1)}+2\lambda\rho_2\right]=m_{[2]} \tag{24}$$

$$\lambda(1+\rho_1+2\rho_2)\mu_{[2]}+4\lambda\rho_2\mu_{[1]} + \frac{2e^{\lambda}}{(e^{\lambda}-1)}\left[\lambda^3\frac{(1+\rho_1)^2}{2}+2\lambda^3\rho_2(1+\rho_1)+\lambda\rho_2(1+2\lambda\rho_2)\right]=m_{[3]} \quad (25)$$

Now factorial moment estimates of the parameters λ, ρ_1 and ρ_2 are obtained by solving the non-linear system of equations (23), (24) and (25) numerically by using mathematical softwares such as *MATHCAD, MATHEMATICA, MATHLAB* etc.

In method of mixed moments, the parameters λ, ρ_1 and ρ_2 of the $MIPD(\lambda,\rho_1,\rho_2)$ are estimated by using the first two sample factorial moments and the first observed frequency of the distribution. Thus the estimates are obtained by solving the equations (23), (24) along with the following equation

$$k\lambda=\frac{p_1}{N} \quad (26)$$

where p_1 is the observed frequency corresponding to the first observed value, N, the observed total frequency and k is as given in (9).

In method of maximum likelihood, the parameters λ, ρ_1 and ρ_2 of the $MIPD(\lambda,\rho_1,\rho_2)$ are estimated by maximizing the following log likelihood function with respect to the parameters.

$$\log L=\sum_{x=1}^{z} p_x \log f_x \quad (27)$$

where p_x is the observed frequency of x events and z is the highest value of x observed. Thus the maximum likelihood estimates of the parameters λ, ρ_1 and ρ_2 are obtained by solving the following system of normal equations.

$$k\sum_{x=1}^{z}\frac{p_x}{q_x}\sum_{r=0}^{\left[\frac{x-1}{2}\right]}\eta_{x-2r}\frac{(x-r)(\lambda\rho_2)^r}{(x-2r)!r!}=\lambda dN \quad (28)$$

$$k\sum_{x=1}^{z}\frac{p_x}{f_x}\sum_{r=0}^{\left[\frac{x-1}{2}\right]}\eta_{x-2r-1}\frac{(\lambda\rho_2)^r}{(x-2r-1)!r!}=\frac{dN}{\lambda} \quad (29)$$

$$\frac{k}{\rho_2}\sum_{x=1}^{z}\frac{p_x}{f_x}\sum_{r=0}^{\left[\frac{x-1}{2}\right]}\eta_{x-2r}\frac{(\lambda\rho_2)^r}{(x-2r)!(r-1)!}=\lambda N, \quad (30)$$

where

$$d = \frac{k\left[e^{\lambda}(1+\rho_1+\rho_2)-(\rho_1+\rho_2)\right]}{(e^{\lambda}-1)}, \tag{31}$$

η_x and k are as given respectively in (6) and (7).

We present the fitting of positive Poisson distribution (PPD), intervened Poisson distribution (IPD), intervened generalized Poisson distribution ($IGPD$) and the modified intervened Poisson distribution ($MIPD$) to the following two data sets by the method of factorial moments, the method of mixed moments and the method of maximum likelihood in Tables 1 and 2.

TABLE 1

Comparison of fit of MIPD using various methods of estimation for the first data set

		Expected frequency by method of											
		Factorial moments				Mixed moments				Maximum likelihood			
x	Obsrvd. freqency	PPD	IPD	IGPD	MIPD	PPD	IPD	IGPD	MIPD	PPD	IPD	IGPD	MIPD
1	213	210	200	204	228	213	213	213	213	210	191	206	217
2	128	122	134	137	115	120	125	120	127	122	140	134	120
3	37	52	50	43	38	51	48	48	41	52	48	43	43
4	18	12	12	12	15	12	11	15	13	12	17	13	16
5	3	3	3	3	3	3	3	3	4	3	3	3	3
6	1	1	1	1	1	1	1	2	2	1	1	1	1
7	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	400	400	400	400	400	400	400	400	400	400	400	400	400
Estimates of parameters		$\hat{\lambda}=1.16$	$\hat{\lambda}=0.56$ $\hat{\rho}=0.47$	$\hat{\alpha}=1.16$ $\hat{\gamma}=0.068$ $\hat{\beta}=1.01$	$\hat{\lambda}=1.16$ $\hat{\rho}_1=0.36$ $\hat{\rho}_2=0.10$	$\hat{\lambda}=1.15$	$\hat{\lambda}=0.68$ $\hat{\rho}=0.39$	$\hat{\alpha}=0.43$ $\hat{\gamma}=0.068$ $\hat{\beta}=1$	$\hat{\lambda}=0.61$ $\hat{\rho}_1=0.48$ $\hat{\rho}_2=0.02$	$\hat{\lambda}=1.16$	$\hat{\lambda}=0.69$ $\hat{\rho}=0.47$	$\hat{\alpha}=0.40$ $\hat{\gamma}=0.08$ $\hat{\beta}=1.1$	$\hat{\lambda}=0.53$ $\hat{\rho}_1=0.48$ $\hat{\rho}_2=0.02$
Chi-square value		7.66	7.49	4.83	3.08	7.37	7.03	3.65	2.98	7.66	6.14	3.27	1.69
P value		0.1	0.112	0.306	0.544	0.129	0.129	0.455	0.56	0.105	0.189	0.51	0.79

TABLE 2

Comparison of fit of MIPD using various methods of estimation for the second data set

		Expected frequency by method of											
		Factorial moments				Mixed moments				Maximum likelihood			
x	Obsrvd. freqency	PPD	IPD	IGPD	MIPD	PPD	IPD	IGPD	MIPD	PPD	IPD	IGPD	MIPD
1	1062	1029	1033	1038	1060	1062	1062	1062	1062	1029	1083	1034	1062
2	263	293	284	276	272	282	240	242	266	288	240	286	258
3	120	138	143	135	126	106	142	129	114	143	107	128	118
4	50	40	40	42	44	58	38	50	43	40	62	43	42
5	22	20	20	29	15	16	35	30	25	20	30	28	30
6	7	12	12	12	10	10	17	21	20	12	10	11	21
7	6	1	1	1	5	0	0	0	3	1	1	2	2
8	2	1	1	1	2	0	0	0	1	1	1	2	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0
10+	1	0	0	0	0	0	0	0	0	0	0	0	0
Total	1534	1534	1534	1534	1534	1534	1534	1534	1534	1534	1534	1534	1534
Estimates of parameters		$\hat{\lambda}=1.02$	$\hat{\lambda}=0.28$ $\hat{\rho}=0.85$	$\hat{\alpha}=0.14$ $\hat{\gamma}=0.28$ $\hat{\beta}=0.003$	$\hat{\lambda}=0.61$ $\hat{\rho}_1=0.48$ $\hat{\rho}_2=0.02$	$\hat{\lambda}=0.65$	$\hat{\lambda}=0.60$ $\hat{\rho}=0.38$	$\hat{\alpha}=0.12$ $\hat{\gamma}=0.30$ $\hat{\beta}=0.005$	$\hat{\lambda}=0.301$ $\hat{\rho}_1=0.42$ $\hat{\rho}_2=0.40$	$\hat{\lambda}=1.02$	$\hat{\lambda}=0.33$ $\hat{\rho}=0.15$	$\hat{\alpha}=0.15$ $\hat{\gamma}=0.32$ $\hat{\beta}=0.005$	$\hat{\lambda}=0.28$ $\hat{\rho}_1=0.40$ $\hat{\rho}_2=0.42$
Chi-square value		9.82	9.41	6.69	4.67	19.17	16.37	5.35	3.89	9.8	9.68	5.8	3.63
P value		0.08	0.09	0.25	0.46	0.005	0.006	0.375	0.565	0.08	0.085	0.326	0.69

The first data set given in Table 1 indicates the distribution of number of articles on theoretical Statistics and Probability for years1940-49 and initial letter N-R of the author's name. For reference, see (Kendall, 1961). The second data set given in Table 2 represents the distribution of 1534 biologists according to the number of research papers to their credit in the review of applied entomology, volume 24, 1936**.** For details see (Williams, 1944).

Based on chi-square values and *P* values in the tables, it can be concluded that $MIPD(\lambda, \rho_1, \rho_2)$ gives the best fit compared to the existing models such as positive Poisson distribution, intervened Poisson distribution and intervened generalized Poisson distribution.

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REFERENCES

A.C. COHEN (1960), *Estimating parameters in a conditional Poisson distribution*, "Biometrics", 16, pp. 203-211.

P.C. CONSUL (1989), *Generalized Poisson distribution: properties and applications*, Marcel Dekker, New York.

M. HUANG, K.Y. FUNG (1989), *Intervened truncated Poisson distribution*, "Sankhya" series 51, pp. 302-310.

M.G. KENDAL (1961), *Natural law in science*, "Journal of Royal Statistical Society", series A 124, pp. 1-18.

D.P.M. SCOLLNIK (1995), *Bayesian analysis of an intervened Poisson distribution*, "Commu-nications in Statistics-Theory & Methods", 24, pp. 735-754.

D.P.M. SCOLLNIK (1998), *On the analysis of the truncated generalized Poisson distribution using a Bayesian method*, "Australian Bulletin", 28, pp. 135-152.

D.P.M. SCOLLNIK (2006), *On the intervened generalized Poisson distribution*, "Communication in Statistics-Theory & Methods", 35, pp. 953-963.

R. SHANMUGAM (1985), An *intervened Poisson distribution and its medical application*, "Biometrics", 41, pp. 1025-1029.

R. SHANMUGAM (1992), *An inferential procedure for the Poisson intervention parameter,* "Biometrics", 48, pp. 559-565.

J. SINGH (1978), *A characterization of positive Poisson distribution and its application*, "SIAM Journal of Applied Mathematics", 34, pp. 545-48.

C.B. WILLIAMS (1944), *Number of publications written by biologists*, "Annals of Eugenics", 12, pp. 143-146.

SUMMARY

Modified intervened Poisson distribution

In this paper, we develop modified intervened Poisson distribution (MIPD) and consider some of its properties. Some real life data sets are given here to illustrate MIPD is the best fit among intervened generalized Poisson distribution (IGPD), intervened Poisson distribution (IPD) and Positive Poisson distribution (PPD).