STATISTICA, anno LXVIII, nn. 3-4, 2008

RECURRENCE RELATIONS FOR INVERSE AND RATIO MOMENTS OF GENERALIZED ORDER STATISTICS FROM DOUBLY TRUNCATED GENERALIZED EXPONENTIAL DISTRIBUTION

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1. INTRODUCTION

Order statistics arise naturally in many life applications. The moments of order statistics have assumed considerable interest in recent years and have been tabulated quite extensively for several distributions. For an extensive survey, see for example, Joshi and Balakrishnan (1982) have obtained some recurrence relations for product moments of order statistics from the right truncated exponential distribution. Saran and Pushkarana (2004) have obtained some recurrence relations of moments for order statistics from generalized exponential distribution. Haseeb and Hassan (2004) obtained the recurrence relations of single and product moments for generalized order statistics from a general class of distribution (Pareto, Power, Weibull, Burr XII distributions and generalized exponential distribution). Ahmad (2007) obtained the recurrence relations for single product moments of generalized order statistics from doubly truncated Burr XII distribution. In this paper, the results established by Saran and Pushkarna (2000) for order statistics from generalized exponential distribution are extended here for case of generalized order statistics (gos) from doubly truncated generalized exponential distribution.

The pdf of the generalized exponential distribution is given by

$$g(x) = (1-\alpha x)^{(1/\alpha)-1}, \qquad 0 \leq x \leq 1/\alpha, \quad 0 \leq \alpha < 1. \tag{1}$$

The cumulative distribution function of generalized exponential distribution is given by

$$F(x) = 1-(1-\alpha x)^{(1/\alpha)}, \qquad 0 \leq x \leq 1/\alpha, \quad 0 \leq \alpha < 1. \tag{2}$$

From (1) and (2) we have

$$(1-\alpha x) f(x) = 1 - F(x) \tag{3}$$

The probability density function (pdf) of the doubly truncated generalized exponential distribution is given by

$$f_d(x) = \frac{(1-\alpha x)^{(1/\alpha)-1}}{P-Q}, \qquad Q_1 \le x \le P_1, \; 0 \le \alpha < 1. \tag{4}$$

The cumulative distribution function (cdf) of the doubly truncated generalized exponential distribution is given by

$$F_d(x) = Q_2 - \frac{(1-\alpha x)^{(1/\alpha)}}{P-Q}, \qquad Q_1 \le x \le P_1, \; 0 \le \alpha < 1, \tag{5}$$

where Q and 1-P (0 < Q < P < 1) are respectively the proportions of the truncation on the left and the right of the generalized exponential distribution in (1), and

$$Q_1 = \frac{1-(1-Q)^{\alpha}}{\alpha}, \quad \text{and} \quad P_1 = \frac{1-(1-P)^{\alpha}}{\alpha},$$

are, respectively, the points of truncation on the left and the right.
From (4) and (5) we have

$$(1-\alpha x) g(x) = P_2 + 1 - G(x), \tag{6}$$

or equivalently,

$$(1-\alpha x) g(x) = Q_2 - G(x) \quad \text{where}$$

$$Q_2 = \frac{1-Q}{P-Q}, \quad \text{and} \quad P_2 = \frac{1-P}{P-Q}.$$

If $X_{1:n} \le X_{2:n} \le .... \le X_{n:n}$ is a sample of generalized order statistics from generalized exponential distribution given in (1), (n>1, m and k are real numbers and $k \ge 1$) then, the pdf of $X_{(r,n,m,k)}$ $(1 \le r < s \le n)$, is given by Kamps (1995) as follows

$$f_{X(r,n,m,k)}(x) = \frac{C_{r-1}}{(r-1)!}[1-F(x)]^{\gamma_r - 1} f(x) g_m^{r-1}[F(x)], \tag{7}$$

Where $\gamma_r = k + (n-r)(m+1) > 0$, for all $1 \le r \le n$.
The joint pdf of $X_{(r,n,m,k)}$ and $X_{(s,n,m,k)}$ $(1 \le i < j \le n)$ is given by Kamps (1995) as follows

$$f_{X(r,n,m,k),X(s,n,m,k)}(x,y)=\frac{C_{s-1}}{(r-1)!(s-r-1)!}[1-F(x)]^{m}g_{m}^{r-1}[F(x)]$$

$$[h_m(F(y))-h_m(F(x))]^{s-r-1}[1-F(y)]^{\gamma_r-1}f(x)f(y), \tag{8}$$

For 0 < x < 1, x < y and r < s, where

$$C_{s-1}=\prod_{i=1}^{s}\gamma_i,\ \ \gamma_i=k+(n-i)(m+1)$$

$$h_m(x)=\begin{cases}\dfrac{-1}{m+1}(1-x)^{m+1}, & m\neq-1\\ -\log(1-x), & m=-1,\end{cases}$$

And $g_m(x)=\int_0^x(1-t)^m\,dt=h_m(x)-h_m(0).$

If m=0 and k=1 we obtain the ordinary order statistics case and if m=-1 we obtain the upper record values case.

The generalized Pareto distribution (GP) introduced by Pickands (1975) has density function

$$f(x)=\begin{cases}k^{-1}(1-\alpha x/k)^{(1/\alpha)-1}, & x>0,\ \alpha\neq0,\ k>0,\\ k^{-1}\exp(-(x/k)), & x>0,\ \alpha=0,\ k>0,\end{cases}$$

where k is the scale parameter and α is the shape parameter, the generalized exponential distribution considered in this paper is a special case of GP when k=1.

We shall denote

$$\mu_{r,s,n,m,k}^{-t}=E\left(\frac{1}{X_{r,n,m,k}^{t}}\right)$$

$$\mu_{r,s,n,m,k}^{t,-w}=E\left(\frac{X_{r,n,m,k}^{t}}{X_{s,n,m,k}^{w}}\right)$$

2. RECURRENCE RELATIONS BASED ON THE NON-TRUNCATED CASE

2.1. *Recurrence relations of inverse moments*

Theorem 1.

Fix a positive integer k, for $n\in N$, $m\in R$, $1\leq r\leq n$, $t=1,2,...$

$$\mu_{r,n,m,k}^{-t} = \frac{1}{\gamma_r - \alpha t}[\gamma_r \mu_{r-1,n,m,k}^{-t} + t\mu_{r,n,m,k}^{-t-1}]. \quad (9)$$

Proof.
Using (7) we have

$$\mu_{r,n,m,k}^{-t-1} - \alpha\mu_{r,n,m,k}^{-t} = \frac{C_{r-1}}{(r-1)!}\int_0^{1/\alpha} x^{-t-1}(1-\alpha x)f(x)[1-F(x)]^{\gamma_r-1} g_m^{r-1}[F(x)]dx$$

Using (3) we have

$$\mu_{r,n,m,k}^{-t-1} - \alpha\mu_{r,n,m,k}^{-t} = \frac{C_{r-1}}{(r-1)!}\int_0^{1/\alpha} x^{-t-1}[1-F(x)]^{\gamma_r} g_m^{r-1}[F(x)]dx \quad (10)$$

Integrating (10) by parts, treating x^{-t-1} for integration and the rest of integrand for differentiation, we obtain

$$\mu_{r,n,m,k}^{-t-1} - \alpha\mu_{r,n,m,k}^{-t} = \frac{\gamma_r C_{r-1}}{t(r-1)!}\int_0^{1/\alpha} x^{-t} f(x)[1-F(x)]^{\gamma_r-1} g_m^{r-1}[F(x)]dx$$

$$+\frac{\gamma_r C_{r-1}}{t(r-2)!}\int_0^{1/\alpha} x^{-t} f(x)[1-F(x)]^{\gamma_r} g_m^{r-2}[F(x)]dx$$

Simplifying the resulting equation, we derive the recurrence relation given in (9).

2.2. *Recurrence relations of ratio moments*

Theorem 2.
For integers $1 < r < s \leq n$ and real m, $k \geq 1$

$$\mu_{r,n,m,k}^{t,-w} = \frac{1}{\gamma_s - \alpha w}[\gamma_s\, \mu_{r,s-1,n,m,k}^{t,-w} - w\mu_{r,s,n,m,k}^{t,-w-1}]. \quad (11)$$

Proof.
Using (8) we have

$$E(X_{r,n,m,k}^{t} X_{s,n,m,k}^{-w-1} - \alpha X_{r,n,m,k}^{t} X_{s,n,m,k}^{-w}) = \frac{C_{r-1}}{(r-1)!(s-r-1)!}\int_{x=0}^{1/\alpha}\int_{y=x}^{1/\alpha} x^{t}\, y^{-w-1}(1-\alpha x)$$

$$f(x)f(y)[1-F(x)]^{m} g_m^{r-1}[F(x)][h_m[F(y)]-h_m[F(x)]]^{s-r-1}[1-F(y)]^{\gamma_s-1} dydx \quad (12)$$

Using (3), we have

$$\mu_{r,s,n,m,k}^{t,-w-1} - \alpha\, \mu_{r,s,n,m,k}^{t,-w} = \frac{C_{r-1}}{(r-1)!(s-r-1)!} \int_0^{1/\alpha} x^t f(x)[1-F(x)]^m g_m^{r-1}[F(x)] I(x) dx \tag{13}$$

Where

$$I(x) = \int_{y-x}^{1/\alpha} y^{-w-1}[h_m[F(y)] - h_m[F(x)]]^{s-r-1}[1-F(y)]^{\gamma_s} dy$$

Integrating I(x) by parts treating y^{-w-1} for integration and the rest of integrand for differentiation, we obtain

$$I(x) = -\frac{\gamma_r}{w} \int_{y=x}^{1/\alpha} y^{-w} f(y)[h_m[F(y)] - h_m[F(x)]]^{s-r-1}[1-F(y)]^{\gamma_s - 1} dy$$

$$+ \frac{s-r-1}{w} \int_{y=x}^{1/\alpha} y^{-w} f(y)[h_m[F(y)] - h_m[F(x)]]^{s-r-2}[1-F(y)]^{\gamma_s} dy \tag{14}$$

Substituting from (14) into (13) we have

$$\mu_{r,s,n,m,k}^{t,-w-1} - \alpha\, \mu_{r,s,n,m,k}^{t,-w} = -\frac{\gamma_s}{w} \mu_{r,s,n,m,k}^{t,-w} + \frac{\gamma_s}{w} \mu_{r,s-1,n,m,k}^{t,-w}.$$

Simplifying the above equation, we obtain the recurrence relation given in (11).

3. RECURRENCE RELATIONS BASED ON DOUBLY TRUNCATED CASE

3.1. *Recurrence relations of inverse moments*

Theorem 3.

For integers $r \geq 1,\ t \geq 0,\quad Q_1 \leq x \leq P_1,\ l_1 = \frac{C_{r-1}}{C_{r-2}^*},\ C_{r-1}^* = \prod_{i=1}^{r} \nu_i$,

$\nu_i = (k+m) + (n-1-i)(m+1),\ m \neq -1$

$$\begin{aligned} \mu_{r,n,m,k}^{-t} &= \frac{1}{\gamma_r - \alpha t}[\gamma_r \mu_{r-1,n,m,k}^{-t} - t\, \mu_{r,n,m,k}^{-t-1}] \\ &+ \frac{P_2 l_1}{\gamma_r - \alpha t}[\mu_{r-1,n-1,m,m+k}^{-t} - \mu_{r,n-1,m,m+k}^{-t}] \end{aligned} \tag{15}$$

Proof.
Using (7), we have

$$\mu_{r,n,m,k}^{-t-1} - \alpha\, \mu_{r,n,m,k}^{-t} = \frac{C_{r-1}}{(r-1)!}\int_{Q_1}^{P_1} x^{-t-1}(1-\alpha x) f_d(x)[1-F_d(x)]^{\gamma_r-1} g_m^{r-1}[F_d(x)]dx$$

Using (6), we have

$$\mu_{r,n,m,k}^{-t-1} - \alpha\, \mu_{r,n,m,k}^{-t} = \frac{C_{r-1}}{(r-1)!}\int_{Q_1}^{P_1} x^{-t-1}[1-F_d(x)]^{\gamma_r} g_m^{r-1}[F_d(x)]dx$$

$$+\frac{P_2 C_{r-1}}{(r-1)!}\int_{Q_1}^{P_1} x^{-t-1}[1-F_d(x)]^{\gamma_r-1} g_m^{r-1}[F_d(x)]dx$$

Integrating by parts, treating x^{-t-1} for integration and the rest of integrand for differentiation, we obtain

$$\mu_{r,n,m,k}^{-t-1} - \alpha\, \mu_{r,n,m,k}^{-t} = \frac{C_{r-1}}{t(r-2)!}\int_{Q_1}^{P_1} x^{-t} f_d(x)[1-F_d(x)]^{\gamma_r} g_m^{r-2}[F_d(x)]dx$$

$$-\frac{C_{r-1}\gamma_r}{t(r-1)!}\int_{Q_1}^{P_1} x^{-t} f_d(x)[1-F_d(x)]^{\gamma_{r-1}-1} g_m^{r-1}[F_d(x)]dx$$

$$+\frac{P_2 C_{r-1}}{t(r-2)!}\int_{Q_1}^{P_1} x^{-t} f_d(x)[1-F_d(x)]^{\gamma_{r-1}-1} g_m^{r-2}[F_d(x)]dx$$

$$-\frac{P_2 C_{r-1}\gamma_r}{t(r-1)!}\int_{Q_1}^{P_1} x^{-t} f_d(x)[1-F_d(x)]^{\gamma_{\gamma_r}-2} g_m^{r-1}[F_d(x)]dx$$

$$\mu_{r,n,m,k}^{-t-1} - \alpha\, \mu_{r,n,m,k}^{-t} = \frac{\gamma_r}{t}\mu_{r-1,n,m,k}^{-t} - \frac{\gamma_r}{t}\mu_{r,n,m,k}^{-t} + \frac{P_2 l_1}{t}[\mu_{r-1,n-1,m,m+k}^{-t} - \mu_{r,n-1,m,m+k}^{-t}]$$

Simplifying the last equation, we derive the recurrence relation given in (15).

3.2. *Recurrence relations of ratio moments*

For integers $r, s \geq 1,\ t, w \geq 0,\ Q_1 \leq x < y \leq P_1,\ l_2 = \dfrac{C_{r-1}}{C_{r-1}^*}, C_{r-1}^* = \prod_{i=1}^{r} \nu_i$,

$$\mu_{r,s,n,m,k}^{t,-w} = \frac{1}{\gamma_s - \alpha w}[\gamma_s\, \mu_{r,s-1,n,m,k}^{t,-w} - w\mu_{r,s-1,n,m,k}^{t,-w-1}]$$

$$+\frac{P_2 l_2}{\gamma_s - \alpha w}[\mu^{t,-w}_{r,s,n-1,m,m+k} - \mu^{t,-w}_{r,s-1,n-1,m,m+k}]. \tag{16}$$

Proof.
Using (8), we have

$$\mu^{t,-w-1}_{r,n,m,k} - \alpha\, \mu^{t,-w}_{r,n,m,k} = \frac{C_{r-1}}{(r-1)!(s-r-1)!}\int_{x=Q_1}^{P_1}\int_{y=x}^{P_1} x^t y^{-w-1}(1-\alpha\, y) f_d(x) f_d(y)$$

$$[1-F_d(x)]^m g_m^{r-1}[F_d(x)][h_m[F_d(y)]-h_m[F_d(x)]]^{s-r-1}[1-F_d(y)]^{\gamma_s-1} dydx \tag{17}$$

Using (6), we have

$$\mu^{t,-w-1}_{r,n,m,k} - \alpha\, \mu^{t,-w}_{r,n,m,k} =$$

$$\frac{C_{r-1}}{(r-1)!(s-r-1)!}\int_{x=Q_1}^{P_1} x^t f_d(x)[1-F_d(x)]^m g_m^{r-1}[F_d(x)] I_1 dx$$

$$+\frac{P_2 C_{r-1}}{(r-1)!(s-r-1)!}\int_{x=Q_1}^{P_1} x^t f_d(x)[1-F_d(x)]^m g_m^{r-1}[F_d(x)] I_2 dx \tag{18}$$

where

$$I_1 = \int_{y=x}^{P_1} y^{-w-1}[h_m[F_d(y)]-h_m[F_d(x)]^{s-r-1}[1-F_d(y)]^{\gamma_s} dy$$

Integrating I_1 by parts treating y^{-w-1} for integration and the rest of the integrand for differentiation, we obtain

$$I_1 = \frac{-\gamma_s}{w}\int_{y=x}^{P_1} y^{-w} f_d(y)[h_m[F_d(y)]-h_m[F_d(x)]]^{s-r-1}[1-F_d(y)]^{\gamma_s-1} dy$$

$$+\frac{s-r-1}{w}\int_{y=x}^{P_1} y^{-w} f_d(y)[h_m[F_d(y)]-h_m[F_d(x)]]^{s-r-2}[1-F_d(y)]^{\gamma_{s-1}-1} dy \tag{19}$$

and

$$I_2 = \int_{y=x}^{P_1} y^{-w-1}[h_m[F_d(y)]-h_m[F_d(x)]]^{s-r-1}[1-F_d(y)]^{\gamma_s-1} dy$$

Integrating I_2 by parts treating y^{-w-1} for integration and the rest of the integrand for differentiation, we obtain

$$I_2 = \frac{-(\gamma_s - 1)}{w} \int_{y=x}^{P_1} y^{-w} f_d(y)[h_m[F_d(y)] - h_m[F_d(x)]]^{s-r-1}[1 - F_d(y)]^{\gamma_s - 2} dy$$

$$+ \frac{s-r-1}{w} \int_{y=x}^{P_1} y^{-w} f_d(y)[h_m[F_d(y)] - h_m[F_d(x)]]^{s-r-2}[1 - F_d(y)]^{\gamma_{s-1} - 2} dy \tag{20}$$

Subsitituting from (19) and (20) into (18) we have,

$$\mu_{r,n,m,k}^{t,-w-1} - \alpha\, \mu_{r,n,m,k}^{t,-w} = \frac{-\gamma_s C_{r-1}}{w(r-1)!(s-r-1)!} \int_{x=Q_1}^{P_1} \int_{y=x}^{P_1} x^t y^{-w} f_d(x) f_d(y)$$

$$[1 - F_d(x)]^m [h_m[F_d(y)] - h_m[F_d(x)]]^{s-r-1} g_m^{r-1}[F_d(x)][1 - F_d(y)]^{\gamma_s - 1} dydx$$

$$+ \frac{C_{r-1}}{w(r-1)!(s-r-2)!} \int_{x=Q_1}^{P_1} \int_{y=x}^{P_1} x^t y^{-w} f_d(x) f_d(y)[1 - F_d(x)]^m [h_m[F_d(y)]$$

$$- h_m[F_d(x)]]^{s-r-2} g_m^{r-1}[F_d(x)][1 - F_d(y)]^{\gamma_{s-1} - 1} dydx$$

$$- \frac{(\gamma_s - 1) P_2 C_{r-1}}{w(r-1)!(s-r-1)!} \int_{x=Q_1}^{P_1} \int_{y=x}^{P_1} x^t y^{-w} f_d(x) f_d(y)[1 - F_d(x)]^m g_m^{r-1}[F_d(x)]$$

$$[h_m[F_d(y)] - [F_d(x)]]^{s-r-1}[1 - F_d(y)]^{\gamma_s - 2} dydx$$

$$+ \frac{P_2 C_{-1}}{w(r-1)!(s-r-2)!} \int_{x=Q_1}^{P_1} \int_{y=x}^{P_1} x^t y^{-w} f_d(x) f_d(y)[1 - F_d(x)]^m g_m^{r-1}[F_d(x)]$$

$$[h_m[F_d(y)] - h_m[F_d(x)]]^{s-r-2}[1 - F_d(y)]^{\gamma_{s-1} - 2} dydx \tag{21}$$

With the use of $\gamma_s - 2 \equiv \nu - 1 = [(n-1) - s](m+1) + (m+k) - 1$ and $\gamma_{s-1} - 2 \equiv \nu_{s-1} - 1 = [(n-1) - (s-1)(m+1) + (m+k) - 1$, we obtain

$$\mu_{r,s,n,m,k}^{t,-w-1} - \alpha\, \mu_{r,s,n,m,k}^{t,-w} = -\frac{\gamma_s}{w} \mu_{r,s,n,m,k}^{t,-w} + \frac{\gamma_s}{w} \mu_{r,s-1,n,m,k}^{t,-w}$$
$$- \frac{P_2 l_2}{w} \mu_{r,s,n-1,m,m+k}^{t,-w} + \frac{P_2 l_2}{w} \mu_{r,s-1,n-1,m,m+k}^{t,-w}$$

Simplifying the last equation, we derive the recurrence relation given in (16).

4. REMARKS AND CONCLUSION

a) When t=-u, (u any positive value), (9) is the recurrence relation for single moments of (gos) from the general form of the distribution $1-F(x)=[ah(x)+b]^c$ obtained by Haseeb and Hassan (2004) with $h(x)$=x, $a=\alpha$ and $c=1/\alpha$.

b) When m=0, k=1 and t=-u in (9), we obtain the recurrence relation of order statistics

$$\mu_{r:n}^{u}=\frac{1}{n+\alpha u}[u\,\mu_{r:n}^{u-1}+n\,\mu_{r-1:n}^{u}]$$

Which is given by Saran and Pushkarna (2000).

c) When w=-z in (11) is the recurrence relation for product moments of gos from general form of the distribution $1-F(x)=[ah(x)+b]^c$ obtained by Haseeb and Hassan (2004) with $h(x)$=x, b=1, $a=\alpha$ and $c=1/\alpha$.

d) By setting m=0, k=1 and w=-z in (11) we obtain the recurrence relation of order statistics

$$\mu_{r,s:n}^{t,z}=\frac{1}{n-s+\alpha z+1}[(n-s+1)\mu_{r,s-1:n}^{t,z}+z\,\mu_{r,s:n}^{t,z-1}],$$

Which is given by Saran and Pushkarna (2000).

e) When $P_2=0$ in (15), we obtain the recurrence relation (9).

f) When $P_2=0$ in (16), we obtain the recurrence relation (11).

Using these equations one can obtain the inverse and ratio moments of generalized order statistics numerically for any size of observations and can also derive the relations for variance and covariance for that distribution, from (11) we can obtain the variance of $X_{r,n,m,k}$ as

$$Var\left(\frac{X_{r,n,m,k}}{X_{s,n,m,k}}\right)=E\left(\frac{X_{r,n,m,k}^{2}}{X_{s,n,m,k}^{2}}\right)-\left[E\left(\frac{X_{r,n,m,k}}{X_{s,n,m,k}}\right)\right]^{2},$$

and we can obtain the covariance of $X_{r,n,m,k}$ and $X_{s,n,m,k}$ from (11) as

$$Cov(X_{r,n,m,k},X_{s,n,m,k})=\mu_{r,s,n,m,k}-\mu_{r,n,m,k}*\mu_{s,n,m,k},$$

These moments may be used to find best linear unbiased estimate of the parameter of the generalized exponential distribution.

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ACKNOWLEDGMENT

The author would like to thank the referees for helpful comments and suggestions, also I thank Dr. M. Atallah, Math. Dept. Faculty of Science, Tanta Univ. Egypt. For his advice which led to improve this paper.

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SUMMARY

Recurrence relations for inverse and ratio moments of generalized order statistics from doubly truncated generalized exponential distribution

In this article, some recurrence relations of inverse and ratio moments for generalized order statistics from doubly truncated and non-truncated generalized exponential distribution are derived. From our results, we deduce the recurrence relations for single and product moments of generalized order statistics from general class distribution obtained by Haseeb and Hassan (2004), also we deduce the recurrence relations for single and product moments of order statistics from generalized exponential distribution obtained by Saran and Pushkarna (2000).