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THE MULTIDIMENSIONAL MEASUREMENT OF POVERTY: A FUZZY SET APPROACH

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1. INTRODUCTION

The theoretical debate on the measurement of poverty made in the last years substantial improvements, gradually moving from the traditional unidimensional view of poverty to the new multidimensional concept of social exclusion (Hagenaars, 1986; Dagum, 1989; Sen, 1992). The view of poverty as multiple deprivation enriched the explanatory power of this field of research. Moreover, by identifying the dominant dimensions of poverty, it provided the basic information for the design and implementation of structural socioeconomic policies purporting to generate socioeconomic processes to reduce the relative proportion of poor as well as the intensity of poverty.

As frequently happens owing to a great theoretical development, a methodological adjustment is needed, but it is neither immediate nor automatic. That is the case of poverty analyses, where empirical researches still often refer only to income or expenditure.

A multidimensional concept of poverty demands a multidisciplinary analysis and, unlike income or expenditure as the only variable considered in an unidimensional framework, the multidimensional approach introduces and analyzes a vector of variables and attributes retained as indicators of some form of exclusion, deprivation or poverty.

They can be represented by a *m*-order vector of attributes $X=(X_1,X_2,...,X_j,...,X_m)$; the *m*-order vector of attributes considered in a multidimensional approach to the analysis and measurement of poverty includes economic, demographic, social, cultural and political attributes.

A highly efficient and rigorous method to perform a multidimensional analysis of poverty makes use of the fuzzy set theory (Dagum *et al.*, 1992; Cheli and Lemmi, 1995; Bourguignon and Chakravarty, 2003; Dagum and Costa, 2004; Betti *et al.*, 2004; Deutsch and Silber, 2005; Mussard and Pi Alperin, 2008; Kakwani and Silber, 2008*a*, *b*): it purports to arrive at a poverty index as a function of the *m* attributes included in *X*.

The aim of the paper is to construct a set of indicators for the multidimen-

sional analysis of poverty and to apply these indicators to the Italian households, evaluating and comparing the different dimensions of poverty. This paper relies on Costa (2002) and Dagum and Costa (2004) and extends the previous results, related to the period 1993-2000, by analyzing a larger information set which allows to evaluate poverty dynamics over the period 1993-2006.

2. METHODOLOGY

Until the 1970s poverty has been dominantly an economic concept and dealt with personal (individual, family or household) levels of income or expenditure. In this context, poverty is defined as an insufficient command over resources for a person to be able to survive (absolute poverty) or to live according to the standard of living reached in the process of growth and development of a country (relative poverty), or something in between that would be partially but not totally sensitive to the per capita income (quasi absolute or quasi relative).

The absolute poverty approach considers the basic needs a person requires to survive, indeed, to physiological survive. The monetary value of the resources entering into the basic needs determines the poverty line of a population, which plays a central role because it is used to discriminate between poor and non-poor persons. It is a strictly bivalent logic such that the population is partitioned into the poor and non-poor subsets.

A more exhaustive and informative measurement of poverty has to be the outcome of a multidisciplinary and multivariate analytical framework capable of identifying and measuring the main causes that contribute to the observed state of poverty. Hence, it should provide the necessary insights for the design and activation of a structural socioeconomic policy aimed at the steady abatement of the causes of poverty. The analytical framework has to be enriched with the contributions of the European social exclusion school and Sen's capability and entitlement approaches. The measurement of poverty and its policy implications can be strongly enhanced by the application of the fuzzy set theory (see Zadeh, 1965 and Dubois and Prade, 1980 among the others).

It follows a stepwise presentation of a methodological research program for multidimensional analysis and measurement of poverty and its implications for a socioeconomic policy purporting to reduce the extend, intensity and inequality of a poor population.

STEP 1. Identification of the population object of research.

In general, the object of poverty research is a population of households belonging to an economic space (nation or region), or subsets of this population, partitioned with respect to some socioeconomic attribute such as gender, years of schooling, urban-rural and age. It represents the sample space of the analysis:

$$A = (a_1, a_2, ..., a_i, ..., a_n),$$

where n is the cardinality of the set A. In the case of a census, A contains all the households of a population, while, if A is a representative sample of a population,

to each household corresponds a weight n_i equal to the number of households the sample observation a_i represents.

Dagum (1995*b*, 2001) observes how the set A is the factual referent for the analysis and measurement of poverty, and constitutes the essential referent set when dealing with the factual sense and the factual truth in the assessment of the basic assumptions and final propositions of a research on poverty.

STEP 2. The multidimensional analysis concept of poverty.

A multivariate concept of poverty demands a multidisciplinary analysis. A main socioeconomic conceptual development was introduced by Sen (1985) and further developed in several other contributions of this author. In his analysis of poverty, Sen deals with the concepts of functioning, capabilities and entitlement.

Unlike income or expenditure as the only variable considered in the unidimensional measurement of poverty, the social exclusion approach introduces and analyzes a vector of variables and attributes retained as indicators of some form of deprivation or poverty.

Research on social exclusion identifies a long list of economic and social phenomena (Silver, 1995). They are represented by the *m*-order vector of attributes $X = (X_1, X_2, ..., X_j, ..., X_m)$. The *m*-order vector of attributes considered in a multivariate approach to the analysis and measurement of poverty includes economic, social, cultural, family and political attributes represented by continuous and discrete quantitative, and dichotomic and politomic qualitative variables.

STEP 3. Choice of the set of socioeconomic attributes related to the state of poverty.

Based on the information available, *e.g.*, a sample survey or a census, we select the socioeconomic attributes whose lack of, or partial (insufficient) possession of any of those attributes, contributes to the state of a household poverty. The set of poverty indicators used in this paper is illustrated in Section 3.

STEP 4. Equivalence scale.

In the multidimensional analysis framework, the equivalence scale is needed to transform the income variable, entering as an element of the *m*-order vector *X*, into equivalent levels of income corresponding to the assumption that all households are of size two.

In the following, we refer to the equivalence scale built by Carbonaro (1985, 2002) for the "Commissione di indagine sulla povertà" created by the Presidency of the Counsel of Ministers of the Italian Government. Table 1 presents the output of the Carbonaro's equivalence scale for the case of an income of 1000 euro.

TABLE 1

Carbonaro's equivalence scale

Household size	1	2	3	4	5	6	7+
Income y_i	1000	1000	1000	1000	1000	1000	1000
Equivalent income y_i^e	1667	1000	752	613	526	463	417

STEP 5. Specification of a poverty line.

This is not a main step in the multidimensional approach; it is a derived proposition. Once we estimate the multidimensional poverty index μ_B, we make

$$\mu_B = H = F(Z) \Rightarrow Z = F^{-1}(\mu_B) = F^{-1}(H)$$

where Z is an imputed poverty line in the multidimensional approach. The symbol H stands for the head-count ratio, *i.e.*, the percentage of households that are poorer than the average μ_B, and F stands for the distribution of equivalent income.

STEP 6. Measuring poverty: the fuzzy set approach.

A highly efficient and rigorous method to perform a multivariate analysis of poverty, including social exclusion and Sen's capability approaches, makes use of the fuzzy set theory. Cerioli and Zani (1990) applied fuzzy set theory to estimate the poverty in the Province of Parma (Italy). Dagum *et al.* (1992), Cheli *et al.* (1994), Martinetti (1994), Cheli and Lemmi (1995), Betti and Verma (1999), Betti *et al.* (2004), Lemmi and Betti (2006) among the others made further contributions and applications.

The fuzzy set theory allows to:

(a) measure each household relative level of poverty or deprivation;

(b) estimate the average poverty index of the population of households; and

(c) measure the relative deprivation and poverty corresponding to each component or attribute included in X.

The latter index is of a paramount importance for its policy implications. It identifies the most important variables or dimensions of poverty that need to be addressed to achieve a structural reduction of poverty, *i.e.*, to implement a structural socioeconomic policy purporting to target institutional, behavioral, technological and social structural changes with the scope of generating dynamic economic processes of growth and development with less social exclusion, decreasing absolute and relative levels of poverty, and more equity.

In the framework of the fuzzy set approach we need to define some fundamental tools, such as:

(i) the set B of poor households;

(ii) the degree of membership to the set B of the i-th household;

(iii) the poverty ratio of the i-th household; and

(iv) the poverty ratio of the population.

The subset of poor households B includes any household a_i which presents some degree of poverty in at least one of the m attributes of X.

The degree of membership to the fuzzy set B of the i-th household ($i=1, ..., n$) with respect to the j-th attribute ($j=1, ... , m$) is defined as

$$\mu_B(X_j(a_i)) = x_{ij}, \qquad 0 \le x_{ij} \le 1$$

In particular:

(i) $x_{ij} = 1$ iff the *i*-th household does not possess the *j*-th attribute;

(ii) $x_{ij} = 0$ iff the *i*-th household possesses the *j*-th attribute; and

(iii) $0 < x_{ij} < 1$ iff the *i*-th household possesses the *j*-th attribute with an intensity belonging to the open interval (0,1).

The poverty ratio of the *i*-th household $\mu_B(a_i)$, *i.e.*, the degree of membership of the *i*-th household to the fuzzy set *B* is defined as the weighted average of x_{ij},

$$\mu_B(a_i) = \sum_{j=1}^{m} x_{ij} w_j \Big/ \sum_{j=1}^{m} w_j ,$$

where w_j is the weight attached to the *j*-th attribute.

The poverty ratio $\mu_B(a_i)$ measures the degree of poverty of the *i*-th household as a weighting function of the *m* attributes. Hence, it measures the relative deprivation, degree of social exclusion, and insufficient capability of the *i*-th household to reach a living standard of the society to which it belongs.

The weight w_j attached to the *j*-th attribute stands for the intensity of deprivation of X_j. It is an inverse function of the degree of deprivation of this attribute by the population of households. The smaller the number of households and the amount of their deprivation of X_j, the greater the weight w_j.

A weight that fulfils the above property is proposed by Cerioli and Zani (1990)[1] and can be represented with the following expression:

$$w_j = \log\left[n \Big/ \sum_{i=1}^{n} x_{ij} n_i \right] \geq 0 ,$$

with $\sum_{i=1}^{n} x_{ij} n_i > 0$ and where n_i is the weight attached to the *i*-th sample observation when the data are extracted from a sample survey.

The requirement that $\sum_{i=1}^{n} x_{ij} n_i > 0$ means that it is not considered an attribute X_j such that $x_{ij} = 0$ for all *i*. This would be an irrelevant attribute and should be excluded because there is not any deprivation in X_j.

The weight w_j is zero when $\sum_{i=1}^{n} x_{ij} n_i = n$, *i.e.*, when the *j*-th attribute is not possessed by any of the *n* households, hence, $x_{ij} = 1$, *i*=1, ..., *n*.

[1] Further interesting proposals on the weighting structure are developed, among the others, by Cheli and Lemmi (1995) and Betti and Verma (1999).

Finally, the poverty ratio of the population μ_B is simply obtained as a weighted average of the poverty ratio of the *i*-th household $\mu_B(a_i)$

$$\mu_B = \sum_{i=1}^{n} \mu_B(a_i) n_i \Big/ \sum_{i=1}^{n} n_i \, .$$

In addition to the multidimensional poverty ratio of the *i*-th household $\mu_B(a_i)$ and of the population μ_B, the fuzzy set framework also allows to simply obtain an unidimensional poverty ratio for each of the *j* attributes considered.

While the multidimensional poverty ratio for the *i*-th household $\mu_B(a_i)$ is the weighted average of x_{ij}, with weight w_j, the unidimensional poverty ratio for the *j*-th indicator is the weighted average of x_{ij}, with weight n_i:

$$\mu_B(X_j) = \sum_{i=1}^{n} x_{ij} n_i \Big/ \sum_{i=1}^{n} n_i \, .$$

In this way it also possible to obtain the multidimensional poverty ratio of the population μ_B as the weighted average of $\mu_B(X_j)$, with weight w_j:

$$\mu_B = \sum_{i=1}^{n} \mu_B(a_i) n_i \Big/ \sum_{i=1}^{n} n_i = \sum_{j=1}^{m} \mu_B(X_j) w_j \Big/ \sum_{j=1}^{m} w_j \, .$$

3. DATA

The data used in this study are from the Survey on Households Income and Wealth (SHIW), a multidimensional survey on Italian households performed every two years by the Bank of Italy. While the main focus of the SHIW is on income and wealth, it includes also relevant information about demographic characteristics, housing, health, education and training. For more detailed information on the SHIW see the Bank of Italy documentation, as some more specialistic paper (Brandolini and Cannari, 1994).

In the following are analysed the data related to survey year 1993, 1995, 1998, 2000, 2002, 2004 and 2006. Their corresponding sample sizes are 8089, 8135, 7147, 8001, 8011, 8012 and 7768 households, respectively.

The information provided by the SHIW allow to construct the following set of composite indicators on the basis of both household and individual data:

1. Household equivalent disposable income, *i.e.*, total household income minus taxes and social contributions divided by the corresponding value of the equivalence scale;
2. Gender, age and job status of the household head;
3. Educational achievement of the household head and his/her father;

4. Educational achievement of the household spouse and her/his father;
5. Professional occupation of the household head;
6. Household size, number of senior members and job status of the household head and the other household members;
7. Typology and heating services of the household residence;
8. Occupancy title and location of the household residence;
9. Household size and dimension (in square meters) of the household residence;
10. Household size and number of bathrooms in the household residence;
11. Ratio between the number of the household members with income and the household size.

All indicators but the 5th are composite, the first and the last five are defined on the basis of household data, the 2nd and the 6th by mixing household and individual data, while the 3rd, the 4th and the 5th refer only to individual data.

In order to define the degree of membership x_{ij} to the set *B* of the *i*-th household, $i = 1, 2, ..., n$, with respect to the *j*-th indicator, $j = 1, 2, ..., m$ it is possible to follow a three steps procedure.

First, for each indicator X_j it is necessary to build a table containing the possible simple or composite outcomes of X_j.

Second, to each outcome it is associated a value, in the closed unit interval [0,1], which represents, for the *j*-th indicator, the degree of membership to *B* corresponding to the given outcome.

Third, for the *i*-th household, $i = 1, 2, ..., n$, is observed the outcome with respect to the *j*-th indicator and it is assigned the corresponding degree of membership to *B*, which for the *i*-th household is x_{ij}.

Tables 2-12 in the Appendix report the degrees of membership assigned to the outcomes of the $m = 11$ indicators considered.

Only for Table 2 it is necessary to add some explanation about the definition of the degrees of membership. First, total net household income, *y*, is transformed into total net equivalent household income, y^e, by using Carbonaro's equivalence scale. Second, are calculated the 5th and the 25th percentile of y^e, respectively $y^e_{0.05}$ and $y^e_{0.25}$. Third, the possible outcomes of X_1 are classified as:

(i) $y^e < y^e_{0.05}$,

(ii) for $y^e_{0.05} \leq y^e \leq y^e_{0.25}$ and

(iii) $y^e > y^e_{0.25}$.

Fourth, to the case

(i) $y^e < y^e_{0.05}$ is assigned degree of membership to *B* equal to 1, to the case

(iii) $y^e > y^e_{0.25}$ is assigned degree of membership to *B* equal to 0, to the case

(ii) $y^e_{0.05} \leq y^e \leq y^e_{0.25}$ is assigned degree of membership to *B* between 0 and 1,

assuming a linearly decreasing path $a + by_i^e$ from $y_{0.05}^e$ to $y_{0.25}^e$ with $a + by_{0.05}^e = 1$, $a + by_{0.25}^e = 0$: therefore for $y_{0.05}^e \leq y_i^e \leq y_{0.25}^e$ the degree of membership to B is $(y_{0.25}^e - y_i^e)/(y_{0.25}^e - y_{0.05}^e)$.

The immediate and natural criticism to the degrees of membership outlined in Tables 2-12 is to consider the choices carried out as arbitrary and subjective. It is certainly a valid criticism, but is also important to observe how the unidimensional framework implies $x_{ij} = 1$ for i = 1, ... , n and j = 2, ..., m, that is clearly an unlikely proposal: the only correct alternative to Tables 2-12 is to suggest a different assignation of the x_{ij}, varying their values, but without setting to one all x_{ij}.

In the next paragraph are illustrated and discussed the results related to the fuzzy set poverty ratios obtained by means of the 11 indicators considered.

4. RESULTS

The main results of the analysis consist in the construction of unidimensional poverty ratios by attribute, which allow to obtain multidimensional poverty measures. Table 13 reports these results by year and Figure 1 illustrates the row mean of Table 13, that is the mean of the poverty ratios by attribute over the 7 surveys analyzed.

TABLE 13

Unidimensional poverty ratios $\mu_B(X_j)$ by attribute and by year

	1993	1995	1997	2000	2002	2004	2006
$\mu_B(X_1)$	0.131	0.132	0.135	0.135	0.139	0.140	0.141
$\mu_B(X_2)$	0.061	0.071	0.072	0.068	0.068	0.061	0.066
$\mu_B(X_3)$	0.519	0.490	0.457	0.466	0.480	0.461	0.432
$\mu_B(X_4)$	0.381	0.353	0.326	0.331	0.326	0.309	0.283
$\mu_B(X_5)$	0.262	0.281	0.275	0.257	0.265	0.251	0.263
$\mu_B(X_6)$	0.045	0.051	0.055	0.053	0.056	0.048	0.053
$\mu_B(X_7)$	0.134	0.134	0.115	0.108	0.106	0.097	0.083
$\mu_B(X_8)$	0.125	0.127	0.106	0.103	0.111	0.107	0.105
$\mu_B(X_9)$	0.404	0.394	0.364	0.361	0.360	0.349	0.334
$\mu_B(X_{10})$	0.141	0.184	0.169	0.157	0.152	0.136	0.126
$\mu_B(X_{11})$	0.059	0.05	0.045	0.045	0.045	0.038	0.038

Among the poverty ratios by attribute, $\mu_B(X_3)$ presents the maximum values: the third indicator, X_3, that is the educational achievement of the household head, emerges as the most important cause of poverty. It is followed by X_9, *i.e.*, the household size and dimension of the residence. In the third place comes X_4, *i.e.*, the educational achievement of the household spouse. The attribute X_5, *i.e.*, the professional occupation of the household head, is in the fourth place.

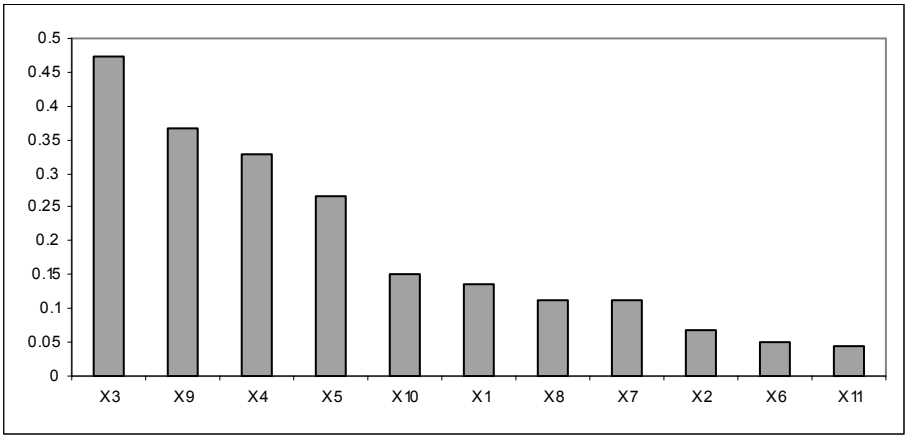

Figure 1 – Poverty ratios by attribute, mean over the 7 surveys analyzed.

It is remarkable that, for the seven surveys, the identified four main causes of poverty that make the highest contribution to the levels of the multidimensional poverty ratio are ranked in the same order.

The poverty ratios $\mu_B(X_1)$, related to the first income-based indicator, are quite stable across time, ranging from 0.131 of the 1993 to 0.141 of 2006 and are characterized by an increasing trend. The indicator X_1 is in the sixth place from 1993 to 2002 and in the fifth in 2004 and 2006.

Unidimensional poverty ratios concur to the multidimensional measure together with the weights w_j which are reported in Table 14.

TABLE 14

Weight w_j attached to the j-th attribute by year

	1993	1995	1998	2000	2002	2004	2006
w_1	0.88	0.88	0.87	0.87	0.86	0.86	0.85
w_2	1.22	1.15	1.14	1.17	1.17	1.22	1.18
w_3	0.28	0.31	0.34	0.33	0.32	0.34	0.36
w_4	0.42	0.45	0.49	0.48	0.49	0.51	0.55
w_5	0.58	0.55	0.56	0.59	0.58	0.60	0.58
w_6	1.34	1.29	1.26	1.27	1.26	1.32	1.28
w_7	0.87	0.87	0.94	0.97	0.97	1.01	1.08
w_8	0.90	0.90	0.97	0.99	0.95	0.97	0.98
w_9	0.39	0.40	0.44	0.44	0.44	0.46	0.48
w_{10}	0.85	0.73	0.77	0.80	0.82	0.87	0.90
w_{11}	1.23	1.30	1.34	1.35	1.35	1.42	1.42

In the 7 analysed surveys, the highest w_j is, on average, w_{11}, indicating how household size and number of income receivers strongly influences the poverty status of an household. In all the years but 1993, w_{11} is the highest weight in the

multidimensional poverty ratio. From the other side, the lowest weight is, on average, w_3, signalling how educational level of the household head is not possessed by many of the households and how the degrees of membership x_{i3}, $i = 1, ..., n$, are generally high. Furthermore, it is interesting to observe how the weights attached to the income-based indicator X_1 are quite stable overtime, ranging from 0.85 of 2006 to 0.88 of the 1993 and 1995.

From the unidimensional poverty ratios $\mu_B(X_j)$ and from the weights w_j, it is possible to obtain the multidimensional poverty ratio: Table 15 reports the quantities $\mu_B(X_j)w_j / \mu_B$, $j = 1, ..., 11$, *i.e.*, the contribution to the multidimensional poverty ratio of the 11 indicators used in the analysis. Furthermore, Figure 2 illustrates the row mean of Table 15, that is the average contribution to the multidimensional poverty ratio of the 11 indicators over the 7 surveys.

TABLE 15

Contribution to the multidimensional poverty ratio by attribute and by year (per cent values)

	1993	1995	1997	2000	2002	2004	2006
X_1	8.94	8.79	9.03	9.17	9.25	9.52	9.59
X_2	5.72	6.17	6.35	6.17	6.18	5.89	6.22
X_3	11.44	11.50	11.96	12.06	11.89	12.36	12.61
X_4	12.36	12.09	12.21	12.40	12.33	12.56	12.42
X_5	11.80	11.73	11.86	11.83	11.87	12.02	12.22
X_6	4.72	4.98	5.32	5.28	5.42	5.07	5.40
X_7	9.04	8.87	8.32	8.15	8.04	7.85	7.17
X_8	8.74	8.62	7.96	7.93	8.24	8.29	8.23
X_9	12.31	12.07	12.29	12.46	12.41	12.72	12.74
X_{10}	9.29	10.26	10.04	9.85	9.68	9.40	9.09
X_{11}	5.64	4.92	4.68	4.71	4.69	4.34	4.31

The greatest contribution to multidimensional poverty ratio is given, on average, by the 9th indicator, related to the dimension of the household residence, but it is possible to distinguish a wider set of indicators, represented by X_9, X_4, X_3 and X_5, which are the main factors of poverty with a contribution to μ_B of about 49%. In the analyzed period the structure of poverty is therefore made by education and activity of the household head and by the dimension of the household residence. The income-based indicator X_1 gives, on average, a contribution of about 9% to the overall measure, while the influence of X_{10} (household size and number of bathrooms is around 10%. The indicator which less contributes to μ_B is X_{11}, the ratio income earners / household size.

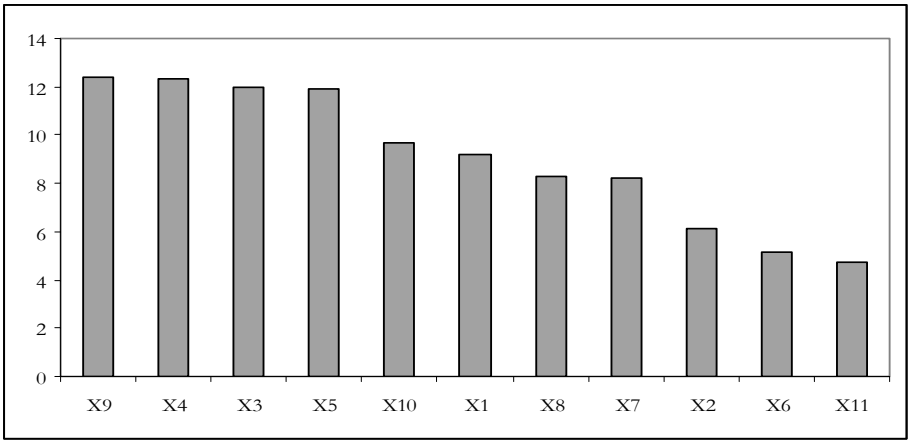

Figure 2 – Average contribution to the multidimensional poverty ratio of the 11 indicators.

Finally, Table 16 reports the multidimensional poverty ratio μ_B.

TABLE 16

Multidimensional fuzzy set poverty ratio μ_B by year

	1993	1995	1997	2000	2002	2004	2006
μ_B	0.144	0.149	0.142	0.138	0.140	0.131	0.129

The lowest diffusion of poverty occurs in 2006, followed by 2004 and 2000, while the maximum of poverty refers to 1995. The multidimensional fuzzy set poverty ratio μ_B presents a decreasing trend, contrasting (Figure 3) with the trend of the univariate fuzzy set poverty ratio $\mu_B(X_1)$. The levels of μ_B are higher than the values of $\mu_B(X_1)$ from 1993 to 1998, in 2000 and 2002 the two poverty indicators show similar results, while in 2004 and 2006 μ_B is about one percentage point below $\mu_B(X_1)$. By taking into account only the equivalent income, that on the basis of $\mu_B(X_1)$, 2006 and 2004 are the poorest years, while the lowest diffusion of poverty occurs in 1993 and 1995. The decreasing trend of μ_B can be explained by the relevant reduction of $\mu_B(X_3)$ (from 0.52 in 1993 to 0.43 in 2006), $\mu_B(X_4)$ (from 0.38 in 1993 to 0.28 in 2006) and $\mu_B(X_9)$ (from 0.40 in 1993 to 0.33 in 2006).

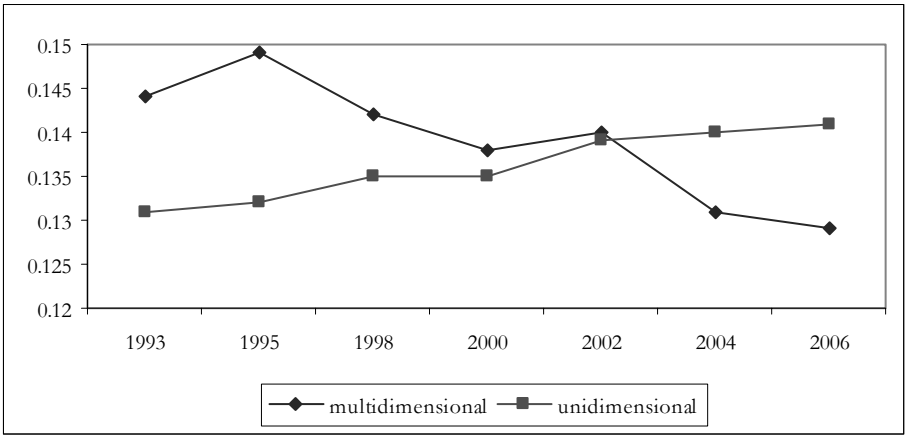

Figure 3 – Multidimensional fuzzy poverty ratio μ_B and unidimensional fuzzy poverty ratio $\mu_B(X_1)$.

5. CONCLUSIONS

The information provided by the SHIW allows to obtain a set of 11 composite indicators for Italian households. Among these indicators the main factors of poverty are identified in the education and the activity of the household head and in the dimension of the household residence. It is quite interesting to observe the great stability of poverty structure from 1993 to 2006: the 7 surveys analyzed share the same problems in the field of social exclusion.

The multidimensional approach offers fuzzy set poverty ratios for: (i) each household; (ii) the population of households; and (iii) the population of households by attribute. These ratios accurately represent the state of poverty, social exclusion and deprivation of the poor, and clearly identify the causes of poverty by order of importance.

By identifying the poverty structure, the multidimensional approach can be extremely useful in order to implement socio-economic actions to reduce poverty diffusion: on the basis of the previous results, these actions should be addressed to reform educational system and labour market and to improve housing conditions.

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APPENDIX

TABLE 2

Household equivalent disposable income

	Degree of membership
$y_i < y^e_{0.05}$	1
$y^e_{0.05} \leq y_i \leq y^e_{0.25}$	$by_i + a$
$y_i > y^e_{0.25}$	0

TABLE 3

Gender, age and occupational status of the household head

	Degree of membership		
	< 25 years	25 - 65 years	> 65 years
Male employed* head of the household	0	0	0
Male unemployed head of household, employed spouse	0.4	0.2	0.3
Male unemployed head of household, unemployed spouse	1	1	1
Male unemployed head of household, no spouse	1	1	1
Female employed head of the household, employed spouse	0	0	0
Female employed head of the household, unemployed spouse	0.2	0.5	0.2
Female employed head of the household, no spouse	0	0	0
Female employed head of the household with children and no spouse	0.5	0.2	0.5
Female unemployed head of the household, employed spouse	0.5	0.3	0.4
Female unemployed head of the household, unemployed spouse	1	1	1
Female unemployed head of the household, no spouse	1	1	1
Female unemployed head of the household with children and no spouse	1	1	1

* employed comprehends also retired workers

TABLE 4

Educational achievement of the household head and his/her father

	Head of the household				
Father	None	Elementary school	Junior high school	Senior high school	University degree
None	1	0.6	0.2	0	0
Elementary school	1	1	0.5	0	0
Junior high school	1	1	1	0	0
Senior high school	1	1	1	0.4	0
University degree	1	1	1	0.5	0

TABLE 5

Educational achievement of the household spouse and her/his father

	Spouse				
Father	None	Elementary school	Junior high school	Senior high school	University degree
None	1	0.6	0.2	0	0
Elementary school	1	1	0.5	0	0
Junior high school	1	1	1	0	0
Senior high school	1	1	1	0.4	0
University degree	1	1	1	0.5	0

TABLE 6

Professional occupation of the household head

	Degree of membership
Teacher	0
Manager	0
Self employed	0
Office worker	0.2
Blue-collar worker	0.3
Unemployed	1
Job pensioner	0.2
Non-job pensioner	1

TABLE 7

Household size, number of senior member and job-status of the household head and the other components of the household

Household size	N. of senior members	Household head	Other components	Degree of membership
1	1	Employed or job pensioner		0
1	1	Unemployed or non-job pensioner		1
2	1	Unemployed or non-job pensioner	Employed	0
2	1	Employed or job pensioner	Unemployed	0
3	1	Employed or job pensioner	1 Unemployed	0
3	1	Employed or job pensioner	2 Unemployed	0.8
3	1	Unemployed or non-job pensioner	2 Employed	0
3	1	Unemployed or non-job pensioner	1 Employed	0.5
3	1	Unemployed or non-job pensioner	2 Unemployed	1
3	2	Employed or job pensioner	1 Employed or job pensioner	0
3	2	Employed or job pensioner	2 Unemployed or non-job pensioner	1
3	2	Unemployed or non-job pensioner	2 Employed or job pensioner	0
3	2	Unemployed or non-job pensioner	1 Employed or job pensioner	1
3	3	Employed or job pensioner	1 Employed or job pensioner	0
3	3	Employed or job pensioner	2 Unemployed or non-job pensioner	1
3	3	Unemployed or non-job pensioner	2 Employed or job pensioner	0
3	3	Unemployed or non-job pensioner	1 Employed or job pensioner	1
> 3	≥ 1	Employed or job pensioner	2 Employed or job pensioner	0
> 3	≥ 1	Employed or job pensioner	1 Employed or job pensioner	0.4
> 3	≥ 1	Employed or job pensioner	0 Employed or job pensioner	1
> 3	≥ 1	Unemployed or non-job pensioner	3 Employed or job pensioner	0
> 3	≥ 1	Unemployed or non-job pensioner	2 Employed or job pensioner	0.5
> 3	≥ 1	Unemployed or non-job pensioner	1 Employed or job pensioner	1

TABLE 8

Typology and heating service of the household residence

Typology	Heating service	
	YES	NO
Luxury	0	0
Mid-range	0	0.25
Modest	0	0.50
Low-income	0.40	0.75
Very low-income	0.80	1

TABLE 9

Occupancy title and location of the household residence

Occupancy title	Location		
	Highly residential	Middle class neighbourhood	Run-down neighbourhood
Owned	0	0	0.5
Rented or sublet	0	0.3	1
Occupied under redemption agreement	0	0.2	0.5
Occupied in usufruct	0	0.2	0.5
Occupied free of charge	0	0.3	1

TABLE 10

Household size and number of bathrooms of the residence

Household size	Number of bathrooms	Degree of membership
Whatever	≥ 2	0
1	1	0
2	1	0
3	1	0.3
4	1	0.6
≥ 5	1	1

TABLE 11

Household size and dimension (in square meters) of the residence

Household size	Square meters	Degree of membership
1	< 50	1
1	50 – 65	0.5
1	65 – 80	0.25
1	> 80	0
2	< 60	1
2	60 – 75	0.5
2	75 – 90	0.25
2	> 90	0
3	< 70	1
3	70 – 85	0.5
3	85 -100	0.25
3	> 100	0
4	< 80	1
4	80 – 95	0.5
4	95 -110	0.25
4	> 110	0
≥ 5	< 100	1
≥ 5	100 – 120	0.5
≥ 5	120 – 140	0.25
≥ 5	> 140	0

TABLE 12

Ratio between the number of the household members perceiving an income and the household size

Household size	Value of the ratio	Degree of membership
1	0	1
1	1	0
2	0	1
2	≥ 0.5	0
3	0	1
3	≥ 0.33	0
4	0	1
4	0.25	0.4
4	≥ 0.5	0
5	0	1
5	0.2	0.5
5	≥ 0.4	0
6	0	1
6	1/6	0.75
6	2/6	0.25
6	≥ 0.5	0
≥ 7	0	1
≥ 7	0.14 – 0.29	0.75
≥ 7	0.30 – 0.58	0.25
≥ 7	> 0.58	0

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SUMMARY

The multidimensional measurement of poverty: a fuzzy set approach

By using fuzzy set theory a multidimensional analysis of poverty of Italian households is performed on the basis of SHIW data. A set of composite indicators is constructed in order to analyze different dimensions of poverty. For each indicator is calculated an unidimensional poverty ratio, thus allowing a comparison among indicators on the dimensions of poverty. Finally, a multidimensional poverty ratio is obtained.