ESTIMATION OF THE RECIPROCAL OF THE MEAN OF THE INVERSE GAUSSIAN DISTRIBUTION WITH PRIOR INFORMATION

H.P. Singh, S. Pandit

1. INTRODUCTION

The Inverse Gaussian distribution was first introduced by (Schrodinger, 1915). Stemming from the earlier work by (Tweedie, 1947), the Inverse Gaussian (IG) distribution has received considerable attention in the statistical literature during the last two decades. Over the years it has found applications in different areas ranging from reliability and life testing to meteorology, biology, economics, medicine, market surveys and remote sensing. See (Seshadri, 1999) for an extensive list of applications (Sen and Khattree, 2000). The review paper by (Folks and Chhikara, 1978) has presented various interesting properties of the IG distribution. There are various alternative forms of a IG distribution available in the literature. Out of these we choose to work with the most familiar one having probability density function (p.d.f).

$$\mathrm{f}(x,\mu,\lambda)=\left(\frac{\lambda}{2\pi x^3}\right)^{1/2}\exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\},\; x>0,\lambda>0,\mu>0 \tag{1}$$

Here, the parameters μ and λ are called location and shape parameters respectively. The mean and the variance of the IG distribution corresponding to (1) are respectively given by $\mathrm{E}(X)=\mu$ and $\mathrm{V}(X)=\mu^3/\lambda$. In cases, where the distribution has arisen from the inverse Gaussian process, one may be interested in estimating the reciprocal of the mean $\left(i.e.\frac{1}{\mu}\right)$.

Let $x_1,x_2,...,x_n$ be a random sample of size n from the Inverse Gaussian distribution (1). It is well known that the maximum likelihood estimators for μ and λ are $\hat{\mu}=\bar{x}=\sum_{i=1}^{n}\frac{x_i}{n}$ and $\hat{\lambda}=\frac{n}{V}$, where $V=\sum_{i=1}^{n}\left(\frac{1}{x_i}-\frac{1}{\bar{x}}\right)$.

The sample mean $\bar{x}$ follows Inverse Gaussian distribution with parameters μ and $n\lambda$. The sample mean $\bar{x}$ is unbiased for μ where as $\hat{\lambda}$ is a biased estimator λ. The statistics $(\bar{x}, V)$ are jointly complete sufficient statistics for (μ, λ) if both μ and λ are unknown. When λ is known, the uniformly minimum variance unbiased estimator (UMVUE) of the reciprocal mean $(1/\mu)$ is

$$\hat{\theta}_1 = \left(\frac{1}{\bar{x}} - \frac{1}{n\lambda}\right) \tag{2}$$

with the variance

$$\mathrm{Var}(\hat{\theta}_1) = \frac{1}{n\lambda}\left(\frac{1}{\mu} + \frac{2}{n\lambda}\right) \tag{3}$$

We note that in many practical situations the value of λ is not known, in such a case the maximum likelihood estimator (mle), uniformly minimum variance unbiased estimator (UMVUE) and minimum mean squared error (MMSE) estimators of $(1/\mu)$ are respectively given by

$$\hat{\theta}_{ml} = \frac{1}{\bar{x}}, \qquad \text{(MLE)} \tag{4}$$

$$\hat{\theta}_{UMVUE} = \frac{1}{\bar{x}} - \frac{V}{n(n-1)}, \qquad \text{(UMVUE)} \tag{5}$$

and

$$\hat{\theta}_{MMSE} = \frac{1}{\bar{x}} - \frac{V}{n(n+1)}, \qquad \text{(MMSE)} \tag{6}$$

It is pointed out in (Sen and Khattree, 2000) that the estimators $\hat{\theta}_{ml}$, $\hat{\theta}_{UMVUE}$ and $\hat{\theta}_{MMSE}$ are particular members of the following class of estimators

$$\hat{\theta} = \left\{\frac{1}{\bar{x}} - \alpha V; \alpha \geq 0\right\} \tag{7}$$

The class $\hat{\theta}$ is a convex subspace of the real line. But it does suffer from certain undesirable features. For instance, this class does not ensure the non-negativity of the estimators and unless $\alpha = 0$, with positive probability, any estimator including the UMVUE and MMSE in $\hat{\theta}$ can take negative values and hence be out of the parameter space. Following (Lehmann, 1983, p. 114) one may comment on $\hat{\theta}_{UMVUE}$ as follows "$\hat{\theta}_{UMVUE}$ can take negative values although the estimate is known to be non-negative. Except when *n* is small, the probability of such values is not large, but when they do occur they cause an embarrassment. This difficulty can be avoided by replacing it by zero whenever it leads to a negative value, the resulting estimator of course, will no larger be unbiased". This problem has been further discussed by (Pandey and Malik, 1989).

It is to be mentioned that the estimator $\hat{\theta}_1$ in (2) can be used only when the value of the parameter λ is known. The value of λ is not known in most of the practical situations. However, in many practical situations the experimenter has some prior estimate regarding the value of the parameter, either due to his experience or due to his acquaintance with the behavior of the system. Thus the experimenter may have evidence that the value of λ is in the neighborhood of λ_0, a known value. We call λ_0 the experimenter's prior guess see (Pandey and Malik, 1988).

The study of the estimators based on prior point estimate (or guess value) λ_0 revealed that these are better (in terms of mean squared error) than the usual estimators when the guess value is in vicinity of true value. This property necessitated the use of preliminary test of hypothesis to decide whether guess value is in vicinity of true value or not. The intention behind preliminary test was that if guess value is in vicinity of the true value, the estimator based on prior point estimate or guess value λ_0 should be used otherwise usual estimators.

In this paper, we define the estimator of the reciprocal of the mean $(1/\mu)$ by incorporating the additional information λ_0 of λ. The mean square error criterion will be used to judge the merits of the suggested estimator. Numerical illustrations are given in the support of present study.

The relevant references in this connection are (Thompson, 1968), (Singh and Saxena, 2001), (Singh and Shukla, 2000, 2003), (Tweedie, 1945, 56, 57a, 57b), (Travadi and Ratani, 1990), (Jani, 1991), (Kourouklis,1994), (Iwase and Seto, 1983, 85), (Iwase, 1987), (Korwar, 1980), (Srivastava *et al.* 1980), and (Singh and Pandit, 2007, 2008).

2. SUGGESTED ESTIMATOR AND ITS PROPERTIES

If sample size is sufficiently large the maximum likelihood estimators (MLE) are consistent and efficient in terms of mean squared error. If sample size is small, there is relatively little information about the parameter available from the sample and if there is any prior estimate for the parameter, the shrinkage method can be useful, see (Pandey, 1983). Let the prior point estimate λ_0 of the parameter λ be available. Then we define the following estimator for $(1/\mu)$ as

$$\hat{\theta} = \frac{1}{\bar{x}} - \frac{\delta_0}{n}\left\{1 + W\left(\frac{\hat{\delta}}{\delta_0}\right)^p\right\}, \tag{8}$$

where $\delta_0 = 1/\lambda_0, \hat{\delta} = 1/\hat{\lambda} = V/(n-1)$ is the UMVE of $\delta = 1/\lambda$; W is a constant such that MSE of $\hat{\theta}$ is least and p is a non-zero real number.
Taking expectation both sides of (8) we have

$$\mathrm{E}(\hat{\theta}) = \mathrm{E}\left(\frac{1}{\bar{x}}\right) - \left(\frac{1}{n}\right)\left\{\delta_0 + W\delta_0^{1-p}\frac{\mathrm{E}(V^p)}{(n-1)^p}\right\} \tag{9}$$

We know for I.G. distribution (1) that

$$\mathrm{E}\left(\frac{1}{\bar{x}}\right) = \frac{1}{\mu}\left(1 + \frac{\mu}{n\lambda}\right) \tag{10}$$

and

$$\mathrm{E}(V^p) = \frac{2^p}{\lambda^p}\frac{\Gamma\left(\frac{n+2p-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \tag{11}$$

Using (10) and (11) in (9) we have

$$\mathrm{E}(\hat{\theta}) = \frac{1}{\mu} + \frac{1}{n\lambda} - \frac{1}{n}\left\{\delta_0 + W\delta_0^{1-p}\frac{2^p\Gamma\left(\frac{n+2p-1}{2}\right)}{\lambda^p\Gamma\left(\frac{n-1}{2}\right)(n-1)^p}\right\}$$

Thus we get the bias of $\hat{\theta}$ as

$$\mathrm{B}(\hat{\theta}) = \frac{1}{n\lambda} - \frac{1}{n}\left\{\delta_0 + W\delta_0^{1-p}\frac{2^p\Gamma\left(\frac{n+2p-1}{2}\right)}{\lambda^p\Gamma\left(\frac{n-1}{2}\right)}\right\} \tag{12}$$

The mean squared error of $\hat{\theta}$ is given by

$$\mathrm{MSE} = \mathrm{MSE}\left(\frac{1}{\bar{x}}\right) + \left(\frac{\delta^2}{n^2}\right)\left[\eta^2 + W^2\eta^{2(1-p)}\frac{2^{2p}}{(n-1)^{2p}}\cdot\frac{\Gamma\left(\frac{n+4p-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right.$$

$$\left. +2W\eta^{2-p}\frac{2^p}{(n-1)^p}\frac{\Gamma\left(\frac{n+2p-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right]$$

$$-2\left(\frac{\delta^2}{n^2}\right)\left[\eta + W\eta^{1-p}\frac{2^p}{(n-1)^p}\frac{\Gamma\left(\frac{n+2p-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right], \tag{13}$$

where $\delta = \left(\frac{1}{\lambda}\right), \eta = \frac{\delta_0}{\delta} = \frac{\lambda}{\lambda_0}$ and

$$\mathrm{MSE}\left(\frac{1}{\bar{x}}\right) = \left(\frac{1}{n\lambda\mu} + \frac{3}{n^2\lambda^2}\right) = \frac{\delta}{n}\left(\frac{1}{\mu} + \frac{3\delta}{n}\right) \tag{14}$$

The MSE at (13) is minimized for

$$\mathrm{W} = \left(\frac{\delta_0}{\delta}\right)^p\left(\frac{\delta-\delta_0}{\delta}\right)W(p), \tag{15}$$

where

$$\mathrm{W}(p) = \left(\frac{n-1}{2}\right)^p\left[\frac{\Gamma\left(\frac{n+2p-1}{2}\right)}{\Gamma\left(\frac{n+4p-1}{2}\right)}\right]$$

Since δ in (15) is unknown, therefore, replacing δ by its estimate $\hat{\delta}$, we get an estimate of W as

$$\hat{\mathrm{W}} = \left(\frac{\delta_0}{\hat{\delta}}\right)^p\left(\frac{\hat{\delta}-\delta_0}{\delta_0}\right)W(p) \tag{16}$$

Thus replacing W by $\hat{W}$ in (8), we get a class of shrinkage estimators of $(1/\mu)$ as

$$\hat{\theta}_{(p)} = \frac{1}{\bar{x}} - \frac{1}{n}[\delta_0 + W(p)(\hat{\delta} - \delta_0)]$$

$$= \frac{1}{\bar{x}} - \frac{1}{n}\left[\frac{1}{\lambda_0} + W(p)\left\{\widehat{\left(\frac{1}{\lambda}\right)} - \frac{1}{\lambda_0}\right\}\right] \tag{17}$$

As pointed out earlier regarding UMVUE $\hat{\theta}_{UMVUE}$, we note that the resulting family of shrinkage estimators $\hat{\theta}_{(p)}$ can also take negative values for smaller values of n. However, this difficulty can be avoided by replacing it by zero whenever it yields negative values.
The bias of $\hat{\theta}_{(p)}$ is given by (17)

$$B(\hat{\theta}_{(p)}) = \frac{\delta}{n}[\{1 - W(p)\}\{1 - \eta\}] \tag{18}$$

and the MSE of $\hat{\theta}_{(p)}$ is given by

$$MSE(\hat{\theta}_{(p)}) = MSE\left(\frac{1}{\bar{x}}\right) + \frac{\delta^2}{n^2}\left[(\eta - 1)^2(1 - W(p))^2 + \frac{2(W(p))^2}{(n-1)} - 1\right] \tag{19}$$

which is less than $MSE\left(\frac{1}{\bar{x}}\right)$ if

$$1 - \sqrt{G_1} < \eta < 1 + \sqrt{G_1}$$

where

$$G_1 = \frac{1}{(1 - W(p))^2}\left[1 - \frac{2(W(p))^2}{(n-1)}\right]$$

Thus we have the following proposition.

Proposition 2.1 The class of shrinkage estimators $\hat{\theta}_{(p)}$ has smaller relative mean squared error (RMSE) than that of $\hat{\theta}_{ml} = \frac{1}{\bar{x}}$ for $\eta \in (1 - \sqrt{G_1}, 1 + \sqrt{G_1})$.
The MSE expression at (19) can be re-expressed as

$$\mathrm{MSE}(\hat{\theta}_{(p)}) = \mathrm{MSE}(\hat{\theta}_{UMVUE}) + \frac{\delta^2}{n^2}\left[(\eta-1)^2(1-W(p)) - \frac{2(1+W(p))}{(n-1)}\right](1-W(p)), \tag{20}$$

or

$$\mathrm{MSE}(\hat{\theta}_{(p)}) = \mathrm{MSE}(\hat{\theta}_{MMSE}) + \frac{\delta^2}{n^2}\left[(\eta-1)^2(1-W(p))^2 + \frac{2(W(p))^2}{(n-1)} - \frac{2}{(n+1)}\right], \tag{21}$$

where

$$\mathrm{MSE}(\hat{\theta}_{UMVUE}) = \frac{1}{n\mu\lambda} + \frac{2}{n(n-1)\lambda^2} \tag{22}$$

and

$$\mathrm{MSE}(\hat{\theta}_{MMSE}) = \frac{1}{n\mu\lambda} + \frac{2(n+2)}{n^2(n+1)\lambda^2} \tag{23}$$

Now we state the following propositions which can be easily proved from (20) and (21).

Proposition 2.2 The class of shrinkage estimators $\hat{\theta}_{(p)}$ has smaller relative mean squared error (RMSE) than that of UMVUE

$$\hat{\theta}_{UMVUE} = \left(\frac{1}{\bar{x}} - \frac{V}{n(n-1)}\right), \text{ for } \eta \in (1-\sqrt{G_2}, 1+\sqrt{G_2}).$$

where

$$G_2 = \left\{\frac{2(1-(W(p))^2)}{(n-1)(1-W(p))^2}\right\}^{1/2}.$$

Proposition 2.3 The class of shrinkage estimators $\hat{\theta}_{(p)}$ has smaller RMSE than that of MMSE estimator

$$\hat{\theta}_{MMSE} = \left(\frac{1}{\bar{x}} - \frac{V}{n(n+1)}\right) \text{ for } \eta \in (1-\sqrt{G_3}, 1+\sqrt{G_3}),$$

where

$$G_3 = \left[\frac{2}{(1-W(p))^2} \left\{ \frac{1}{(n+1)} - \frac{(W(p))^2}{(n-1)} \right\} \right]^{1/2} .$$

3. SPECIAL CASE

In this section we discuss the properties of an estimator $\hat{\theta}_{(1)}$ (say), which is a particular member of the proposed class of shrinkage estimators $\hat{\theta}_{(p)}$ defined at (17).

For p=1, the estimator $\hat{\theta}_{(p)}$ reduces to the estimator $\hat{\theta}_{(1)}$, thus

$$\hat{\theta}_{(1)} = \frac{1}{\bar{x}} - \frac{1}{n} \left\{ \delta_0 + \left(\frac{n-1}{n+1} \right) (\hat{\delta} - \delta_0) \right\}$$

$$= \hat{\theta}_{MMSE} - \frac{2\delta_0}{n(n+1)} \tag{24}$$

Putting p=1 in (18) and (19) we get the bias and MSE of $\hat{\theta}_{(1)}$ respectively as

$$\mathrm{B}(\hat{\theta}_{(1)}) = \frac{2(1-\eta)\delta}{n(n+1)} \tag{25}$$

and

$$\mathrm{MSE}(\hat{\theta}_{(1)}) = \mathrm{MSE}(\hat{\theta}_{MMSE}) + \frac{4\delta^2}{n^2(n+1)^2} \eta(\eta - 2), \tag{26}$$

where $\mathrm{MSE}(\hat{\theta}_{MMSE})$ is given by (23).

In order to compare the bias of $\hat{\theta}_{MMSE}$ with $B(\hat{\theta}_{(1)})$, we write the bias of $\hat{\theta}_{MMSE}$ as

$$\mathrm{B}(\hat{\theta}_{MMSE}) = \frac{2\delta}{n(n+1)} \tag{27}$$

It follows from (25) and (27) that

$$\left| B(\hat{\theta}_{(1)}) \right| < \left| B(\hat{\theta}_{MMSE}) \right| \text{ if}$$

$$0 < \eta < 2 \tag{28}$$

Further from (26) we note that

$$\text{MSE}(\hat{\theta}_{(1)}) < \text{MSE}(\hat{\theta}_{MMSE}) \text{ if}$$

$$0 < \eta < 2 \tag{29}$$

Thus we state the following theorem.

Theorem 3.1 The estimator $\hat{\theta}_{(1)}$ is less biased as well more efficient than MMSE estimator $\hat{\theta}_{MMSE}$ iff $0 < \eta < 2$.

Remark 3.1 It is to be noted that the estimator $\hat{\theta}_{MMSE}$ is the minimum mean squared error estimator so it has less MSE than that of usual estimator $\hat{\theta}_{ml} = 1/\bar{x}$ and UMVUE $\hat{\theta}_{UMVUE}$. Thus it is interesting to note from (29) that the estimator $\hat{\theta}_{(1)}$ is more efficient than the estimators $\hat{\theta}_{ml}, \hat{\theta}_{UMVUE}$ and $\hat{\theta}_{MMSE}$ under the condition $0 < \eta < 2$.

4. NUMERICAL ILLUSTRATION AND CONCLUSIONS

To get tangible idea about the performance of the proposed class of shrinkage estimators $\hat{\theta}_{(p)}$ over usual estimator $\hat{\theta}_{ml} = \frac{1}{\bar{x}}$, UMVUE $\hat{\theta}_{UMVUE} = \left\{\frac{1}{\bar{x}} - \frac{V}{n(n-1)}\right\}$ and MMSE estimator $\hat{\theta}_{MMSE} = \left\{\frac{1}{\bar{x}} - \frac{V}{n(n+1)}\right\}$, we have computed the percent relative efficiencies (PRE's) of $\hat{\theta}_{(p)}$ with respect to $\hat{\theta}_{ml}, \hat{\theta}_{UMVUE}$ and $\hat{\theta}_{MMSE}$ respectively using the following formulae:

$$\text{PRE}(\hat{\theta}_{(p)}, \hat{\theta}_{ml}) = \frac{A}{(A + A_1)} \times 100 \tag{30}$$

$$\text{PRE}(\hat{\theta}_{(p)}, \hat{\theta}_{UMVUE}) = \frac{n A_2}{(A + A_1)} \times 100 \tag{31}$$

$$\text{PRE}(\hat{\theta}_{(p)}, \hat{\theta}_{MMSE}) = \frac{n A_3}{(A + A_1)} \times 100 \tag{32}$$

where $A = \left(3 + \frac{n}{C^2}\right)$, (33)

$$A_1 = \left[(\eta-1)^2(1-W(p))^2 + \frac{2(W(p))^2}{(n-1)} - 1 \right], \quad (34)$$

$$A_2 = \left(\frac{1}{C^2} + \frac{2}{n-1} \right), \quad (35)$$

$$A_3 = \left(\frac{1}{C^2} + \frac{2(n+2)}{n(n+1)} \right), \quad (36)$$

$C = \left(\frac{\mu}{\lambda} \right)^{1/2}$, is the coefficient of variation.

The PRE's have been computed for different values of $p = \pm 1, \pm 2$. $n = 5, 10, 15$; $\eta = 0.25(0.25)1.75$; and C = 1,5. The computed values are displayed in tables 4.1(a). 4.2(a) and 4.3(a).

We have also computed the range of η for different values of p, n, η and C; and presented in tables 4.1(b), 4.2(b) and 4.3(b). We note that

$$\text{W}(-2) = \frac{(n-7)(n-9)}{(n-1)^2}$$

$$\text{W}(-1) = \frac{(n-5)}{(n-1)}$$

$$\text{W}(1) = \frac{(n-1)}{(n+1)}$$

$$\text{W}(2) = \frac{(n-1)^2}{(n+5)(n+3)}$$

It is observed from tables 4.1(a), 4.2(a) and 4.3(a) that the percent relative efficiencies of $\hat{\theta}_{(p)}$, $p = \pm 2, \pm 1$; with respect to $\hat{\theta}_{ml}$, $\hat{\theta}_{UMVUE}$ and $\hat{\theta}_{MMSE}$ respectively ($i.e. PRE(\hat{\theta}_{(p)}, \hat{\theta}_{ml}), PRE(\hat{\theta}_{(p)}, \hat{\theta}_{UMVUE})$ *and* $PRE(\hat{\theta}_{(p)}, \hat{\theta}_{MMSE})$):

(i) attain their maximum at $\eta = 1$ (i.e. when λ coincide with λ_0),
(ii) decrease as *n* increases,
(iii) increase as the value of coefficient of variation (*C*) increases,
(iv) is more than 100% for the largest range of dominance of η when *p*=1, (*i.e.* $\hat{\theta}_{(1)}$), but the gain in efficiency is smaller compared to other estimators $\hat{\theta}_{(-2)}$,

$\hat{\theta}_{(-1)}$ and $\hat{\theta}_{(2)}$. However, the estimator $\hat{\theta}_{(2)}$ seems to be the good intermediate choice between the estimators $\hat{\theta}_{(-2)}$, $\hat{\theta}_{(-1)}$ and $\hat{\theta}_{(1)}$.

Comparing the results of tables 4.1(a), 4.2(a) and 4.3(a) it is seen that the gain in efficiency by using $\hat{\theta}_{(p)}$ over $\hat{\theta}_{ml}$ is the largest followed by $\hat{\theta}_{UMVUE}$ and then $\hat{\theta}_{MMSE}$. Also the proposed family of shrinkage estimators $\hat{\theta}_{(p)}$ is more efficient than $\hat{\theta}_{ml}$ for widest range of η followed by $\hat{\theta}_{UMVUE}$ and $\hat{\theta}_{MMSE}$ see tables 4.1(b), 4.2(b) and 4.3(b). It is expected too.

Thus we conclude that the suggested family of shrinkage estimators $\hat{\theta}_{(p)}$ is to be recommended for its use in practice when

(i) the guessed value λ_0 moves in the vicinity of the true value λ.
(ii) sample size *n* is small (*i.e.* in the situations where the sampling is costly) and
(iii) the population is heterogeneous.

TABLE 4.1(a)

Percent relative efficiency of $\hat{\theta}_{(p)}$ *over* $\hat{\theta}_{ml}$

C	**P →**	**– 1**			**– 2**		
	$\eta\downarrow$ n →	**5**	**10**	**15**	**5**	**10**	**15**
	0.25	110.11	103.82	103.87	105.79	106.74	105.15
	0.50	111.30	106.28	104.95	110.35	107.28	105.30
1	**0.75**	112.04	107.81	105.61	113.27	107.61	105.40
	1.00	112.28	108.33	105.83	114.29	107.72	105.43
	1.25	112.04	107.81	105.61	113.27	107.61	105.40
	1.50	111.30	106.28	104.95	110.35	107.28	105.30
	1.75	110.11	103.82	103.87	105.79	106.74	105.15
	0.25	129.78	116.36	122.90	115.84	131.80	132.41
	0.50	134.03	129.17	130.86	130.61	135.03	133.67
	0.75	136.72	138.31	136.15	141.44	137.05	134.43
5	**1.00**	137.63	141.65	138.01	145.45	137.73	134.69
	1.25	136.72	138.31	136.15	141.44	137.05	134.43
	1.50	134.03	129.17	130.86	130.61	135.03	133.67
	1.75	129.79	116.36	122.90	115.84	131.80	132.41
		1			**2**		
	0.25	109.82	106.84	105.15	108.40	106.29	104.90
	0.50	110.35	106.93	105.18	111.42	107.23	105.30
	0.75	110.66	106.99	105.20	113.31	107.80	105.54
1	**1.00**	110.77	107.01	105.21	113.96	107.99	105.62
	1.25	110.64	106.99	105.20	113.31	107.80	105.54
	1.50	110.35	106.93	105.18	111.42	107.23	105.30
	1.75	109.82	106.84	105.15	108.40	106.29	104.90
	0.25	128.79	132.43	132.44	124.03	129.25	130.48
	0.50	130.61	132.97	132.68	134.45	134.72	133.62
	0.75	131.73	133.29	132.82	141.59	138.23	135.57
5	**1.00**	132.11	133.40	132.87	144.14	139.44	136.24
	1.25	131.73	133.29	132.82	141.59	138.23	135.58
	1.50	130.61	132.97	132.68	134.45	134.72	133.62
	1.75	128.79	132.43	132.44	124.03	129.25	130.48

TABLE 4.2(a)

Percent relative efficiency of $\hat{\theta}_{(p)}$ *over* $\hat{\theta}_{UMVUE}$

C	P →	– 1			– 2		
	$\eta\downarrow$ n →	**5**	**10**	**15**	**5**	**10**	**15**
1	**0.25**	103.23	97.61	98.92	99.17	100.35	100.10
	0.50	104.35	99.10	99.95	103.40	100.86	100.30
	0.75	105.03	101.36	100.60	106.20	101.17	100.40
	1.00	105.26	101.85	100.80	107.10	101.27	100.40
	1.25	105.03	101.36	100.60	106.20	101.17	100.40
	1.50	104.35	99.10	99.95	103.40	100.86	100.30
	1.75	103.23	97.61	98.92	99.17	100.35	100.10
5	**0.25**	109.51	89.74	93.64	97.74	101.65	100.90
	0.50	113.09	99.62	99.61	110.20	104.14	101.80
	0.75	115.35	106.67	103.70	119.30	105.70	102.40
	1.00	116.13	109.25	105.10	122.70	106.22	102.60
	1.25	115.35	106.67	103.70	119.30	105.70	102.40
	1.50	113.09	99.62	99.69	110.20	104.14	101.80
	1.75	109.51	89.74	93.64	97.74	101.65	100.90
		1			**2**		
1	**0.25**	102.96	100.45	100.10	101.60	99.93	99.91
	0.50	103.45	100.54	100.20	104.50	100.81	100.30
	0.75	103.75	100.59	100.20	106.20	101.35	100.50
	1.00	103.85	100.61	100.20	106.80	101.53	100.60
	1.25	103.75	100.59	100.20	106.20	101.35	100.50
	1.50	103.45	100.54	100.20	104.50	100.81	100.30
	1.75	102.96	100.45	100.10	101.60	99.93	99.91
5	**0.25**	108.66	102.14	100.90	104.70	99.68	99.41
	0.50	110.20	102.55	101.10	113.40	103.90	101.80
	0.75	111.15	102.80	101.20	119.50	106.61	103.30
	1.00	111.47	102.88	101.20	121.60	107.54	103.80
	1.25	111.15	102.80	101.20	119.50	106.61	103.30
	1.50	110.20	102.55	101.10	113.40	103.90	101.80
	1.75	108.66	102.14	100.90	104.70	99.68	99.41

TABLE 4.3(a)

Percent relative efficiency of $\hat{\theta}_{(p)}$ *over* $\hat{\theta}_{MMSE}$

C	P →	– 1			– 2		
	$\eta\downarrow$ n →	**5**	**10**	**15**	**5**	**10**	**15**
1	**0.25**	100.93	97.28	98.82	96.98	100.02	100.04
	0.50	102.03	99.59	99.85	101.15	100.53	100.18
	0.75	102.70	101.02	100.47	103.83	100.83	100.27
	1.00	102.92	101.51	100.68	104.76	100.94	100.30
	1.25	102.70	101.02	100.47	103.83	100.83	100.27
	1.50	102.03	99.59	99.85	101.15	100.53	100.18
	1.75	100.93	97.28	98.82	96.97	100.02	100.04
5	**0.25**	102.75	88.36	93.02	91.70	100.08	100.23
	0.50	106.11	98.09	99.05	103.40	102.54	101.18
	0.75	108.23	105.03	103.06	111.97	104.07	101.75
	1.00	108.96	107.56	104.46	115.15	104.59	101.95
	1.25	108.23	105.03	103.06	111.97	104.07	101.75
	1.50	106.12	98.09	99.05	103.40	102.54	101.18
	1.75	102.75	88.36	93.03	91.70	100.08	100.23
		1			**2**		
1	**0.25**	100.67	100.11	100.04	99.37	99.60	99.80
	0.50	101.15	100.20	100.07	102.13	100.48	100.18
	0.75	101.44	100.25	100.09	103.87	101.01	100.41
	1.00	101.54	100.27	100.09	104.46	101.19	100.48
	1.25	101.44	100.25	100.09	103.87	101.01	100.41
	1.50	101.15	100.20	100.07	102.16	100.48	100.18
	1.75	100.67	100.12	100.04	99.37	99.60	99.80
5	**0.25**	101.96	100.56	100.25	98.19	98.15	98.76
	0.50	103.40	100.97	100.43	106.44	102.30	101.14
	0.75	104.29	101.21	100.54	112.09	104.96	102.62
	1.00	104.59	101.30	100.58	114.11	105.88	103.13
	1.25	104.29	101.21	100.54	112.09	104.96	102.62
	1.50	103.40	100.96	100.43	106.44	102.30	101.14
	1.75	101.96	100.56	100.25	98.19	98.15	98.76

TABLE 4.1(b)

Range of η for different values of n

n → Estimator ↓	5	10	15
$\hat{\theta}_{(-2)}$	(0.0000, 2.8708)	(0.0000,2.0383)	(0.0000, 2.3186)
$\hat{\theta}_{(-1)}$	(0.0000, 2.0000)	(0.0000, 3.1715)	(0.0000, 4.3700)
$\hat{\theta}_{(1)}$	(0.0000, 3.6458)	(0.0000, 6.0745)	(0.0000, 8.5498)
$\hat{\theta}_{(2)}$	(0.0000, 2.2374)	(0.0000, 2.6774)	(0.0000, 3.1481)

TABLE 4.2(b)

Range of η for different values of n

n → Estimator ↓	5	10	15
$\hat{\theta}_{(-2)}$	(0.0000, 2.1067)	(0.3006, 1.6994)	(0.3033, 1.6966)
$\hat{\theta}_{(-1)}$	(0.1591, 1.8409)	(0.0609, 1.9391)	(0.3780, 1.9622)
$\hat{\theta}_{(1)}$	(0.0000, 2.2574)	(0.0000, 2.2209)	(0.0000, 2.2099)
$\hat{\theta}_{(2)}$	(0.0693, 1.9306)	(0.1436, 1.8564)	(0.1658, 1.8342)

TABLE 4.3(b)

Range of η for different values of n

n → Estimator ↓	5	10	15
$\hat{\theta}_{(-2)}$	(0.0446,1.9554)	(0.3348,1.6651)	(0.3278,1.6722)
$\hat{\theta}_{(-1)}$	(0.2402,1.7598)	(0.1299,1.8701)	(0.1061,1.8939)
$\hat{\theta}_{(1)}$	(0.0000,2.0000)	(0.0000,2.0000)	(0.0000,2.0000)
$\hat{\theta}_{(2)}$	(0.1635,1.8365)	(0.1951,1.8049)	(0.2056,1.7944)

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School of Studies in Statistics HOUSILA P. SINGH
Vikram University, Ujjain, India SHEETAL PANDIT

REFERENCES

J.L. FOLKS, R.S. CHHIKARA, (1978), *The Inverse Gaussian distribution and its application*, a review, "Journal of Royal Statistical Society", B, 40, pp. 263-289.

R.V. HOGG, (1974), *Adoptive robust procedures, A partial review and some suggestions for future applications and theory*, "Journal of the American Statistical Association", 69, pp. 909-923.

KOSEI IWASE, (1987), *UMVU estimation for the Inverse Gaussian distribution I* $(\mu, c\mu^2)$ *with known C*, "Communication Statistics", 16, 5, pp. 1315-1320.

KOSEI IWASE, NORAKI SETO, (1983), *UMVUE's for the Inverse Gaussian distribution*, "Journal of the American Statistical Association", 78, pp. 660-663.

KOSEI IWASE, NORAKI SETO, (1985), *UMVU estimators of the mode and limits of an interval for the Inverse Gaussian distribution*, "Communication in Statistics, Theory and Methods", 14, pp. 1151-1161.

P.N. JANI, (1991), *A class of shrinkage estimators for the scale parameter of the exponential distribution*, "IEEE Transaction on Reliability", 40, pp. 60-70.

R.M. KORWAR, (1980), *On the UMVUE's of the variation and its reciprocal of an Inverse Gaussian distribution*, "Journal of the American Statistical Association", 75, pp. 734-735.

S. KOUROUKLIS, (1994), *Estimation in the two-parameter exponential distribution with prior information*, "IEEE Transaction on Reliability", 43, 3, pp. 446-450.

E.L. LEHMANN, (1983), *Theory of point estimation*, Wiley, New York, pp. 114.

B.N. PANDEY, (1983), *Shrinkage estimation of the exponential scale parameter*, "IEEE Transaction on Reliability", 32, 2, pp. 203-205.

B.N. PANDEY, H.J. MALIK, (1988), *Some improved estimators for a measure of dispersion of an Inverse Gaussian distribution*, "Communication in Statistics, Theory and Methods", 17, 11, pp. 3935-3949.

B.N. PANDEY, H.J. MALIK, (1989), *Estimation of the mean and the reciprocal of the mean of the Inverse Gaussian distribution*, "Communication Statistics, Simulation", 18(3), pp. 1187-1201.

E. SCHRONDINGER, (1915), *Zur theories der fall-und Steigvursuche on Tailchenmit Brownsher Bewegung*, "Physikaliche Zeitschrift", 16, pp. 289-295.

SEN, R. KHATTREE, (2000), *Revisiting the problem of estimating the Inverse Gaussian parameters*, "IAPQR Transactions", 25, 2, pp. 63-79.

R. SESHADRI, (1999), *The Inverse Gaussian distribution, Statistical Theory and Application*, Springer Verlag, New York.

H.P. SINGH, S.K. SHUKLA, (2000), *Estimation in the two-parameter Weibull distribution with prior information*, "IAPQR. Transactions", 25, 2, pp. 107-117.

H.P. SINGH, S. SAXENA, (2001), *Improved estimation in one-parameter Exponential distribution with prior information*, "Gujarat Statistical Review", 28, 1-2, pp. 25-35.

H.P. SINGH, S.K. SHUKLA, (2003), *A family of shrinkage estimators of the square of mean in normal distribution*, "Statistical Papers", Verlag, 44, pp. 433-442.

H.P. SINGH, S. PANDIT, (2007), *Improved estimation if Inverse Gaussian shape parameter and measure of dispersion with prior information*, "Journal of statistical Theory and Practice", 1, 2, pp. 205-226.

H.P. SINGH, S. PANDIT, (2008), *Estimation of shape parameter and measure of dispersion of Inverse Gaussian distribution using prior information*, "Bulletin of statistics and economics", 2, S08, pp. 81-100.

V.K. SRIVASTAVA, T.D. DWIVEDI, S. BHATNAGAR, (1980), *Estimation of the square of mean in normal population*, "Statistica", XL, 4, pp. 455-466.

J.R. THOMPSON, (1968), *Some shrinkage technique for estimating the mean*, "Journal of the American Statistical Association", 63, pp. 113-123.

R.J. TRAVEDI, R.T. RATANI, (1990), *On estimation of reliability function for Inverse Gaussian distribution with known coefficient of variation*, "IAPQR, Transactions", 5, 2, pp. 29-37.
M.C.K. TWEEDIE, (1945), *Inverse Statistical Variate*, Nature, pp. 155-453.
M.C.K. TWEEDIE, (1947), *Functions of statistical variate with given means, with special inference to laplacian distributions*, "Proceedings of the Cambridge Philosophical Society", 43, pp. 41-49.
M.C.K. TWEEDIE, (1956), *Some statistical properties of Inverse Gaussian distribution.* "Virginia Journal Science", 7, pp. 160-165.
M.C.K. TWEEDIE, (1957 a), *Statistical properties of Inverse Gaussian distribution*, I, "Annals of Mathematical Statistics", 28, pp. 362-377.
M.C.K. TWEEDIE, (1957 b), *Statistical properties of Inverse Gaussian distribution*, II, "Annals of Mathematical Statistics", 28, pp. 696-705.

SUMMARY

Estimation of the reciprocal of the mean of the Inverse Gaussian distribution with prior information

This paper considers the problem of estimating the reciprocal of the mean $(1/\mu)$ of the Inverse Gaussian distribution when a prior estimate or guessed value λ_0 of the shape parameter λ is available. We have proposed a class of estimators $\hat{\theta}_{(p)}$, say, for $(1/\mu)$ with its mean squared error formula. Realistic conditions are obtained in which the estimator $\hat{\theta}_{(p)}$ is better than usual estimator, uniformly minimum variance unbiased estimator (UMVUE) and the minimum mean squared error estimator (MMSE). Numerical illustrations are given in support of the present study.