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ESTIMATION OF PARAMETERS OF A TWO-PARAMETER RECTANGULAR DISTRIBUTION AND ITS CHARACTERIZATION BY *k*-th RECORD VALUES

J. Saran, A. Pandey

1. INTRODUCTION

Record values are found in many situations of daily life as well as in many statistical applications. Often we are interested in observing new records and in recording them: e.g. Olympic records or world records in sports. Motivated by extreme weather conditions, Chandler (1952) introduced record values and record value times. Feller (1966) gave some examples of record values with respect to gambling problems. Theory of record values and its distributional properties have been extensively studied in the literature. For more details, see Nevzorov (1987), Arnold, Balakrishnan and Nagaraja (1998), Ahsanullah (1995) and Kamps (1995).

We shall now consider the situations in which the record values (e.g. successive largest insurance claims in non-life insurance, highest water-levels or highest temperatures) themselves are viewed as 'outliers' and hence the second or third largest values are of special interest. Insurance claims in some non-life insurance can be used as an example. Observing successive *k*-th largest values in a sequence, Dziubdziela and Kopociński (1976) proposed the following model of *k*-th record values, where *k* is some positive integer.

Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with cdf $F(x)$ and pdf $f(x)$. Let $X_{j:n}$ denote the *j*-th order statistic of a sample $(X_1, X_2, \ldots, X_n)$. For a fixed $k \geq 1$, we define the sequence $U_1^{(k)}, U_2^{(k)}, \ldots$ of *k*-th upper record times of $X_1, X_2, \ldots$ as follows:

$$U_1^{(k)} = 1$$
$$U_2^{(k)} = \min[j > U_1^{(k)} : X_{j:j+k-1} > X_{U_1^{(k)}:U_1^{(k)}+k-1}],$$
$$U_{n+1}^{(k)} = \min[j > U_n^{(k)} : X_{j:j+k-1} > X_{U_n^{(k)}:U_n^{(k)}+k-1}],$$

$n = 1, 2, \ldots$.

The sequence $\{Y_n^{(k)}, n \geq 1\}$, where $Y_n^{(k)} = X_{U_n^{(k)}}$ is called the sequence of k-th upper record values of the sequence $\{X_n, n \geq 1\}$. For convenience, we define $Y_o^{(k)} = 0$. Note that for $\mathrm{k} = 1$ we have $Y_n^{(1)} = X_{U_n}, n \geq 1$, which are record values of $\{X_n, n \geq 1\}$ (Ahsanullah, 1995).

In this paper, we shall make use of the properties of the k-th upper record values to develop inferential procedures such as point estimation. We shall obtain the best linear unbiased estimates of the parameters of the two-parameter rectangular distribution in terms of k-th upper record values. Ahsanullah (1989) considered the problem of estimation of parameters for the power function distribution based on upper record values ($k = 1$). At the end we give the characterization of the two-parameter rectangular distribution using k-th upper record values.

In order to derive the estimates for the parameters of the two-parameter rectangular distribution and to give its characterization, we need some recurrence relations for single and product moments of k-th upper record values which have been established in the next section.

Applications of random variables having two-parameter rectangular distribution are found in the study of consumption of fuel by an airplane during a flight, the thickness of steel produced by rolling machines of steel plants, development of gambling, lotteries and in the generation of random numbers in simulation experiments.

2. RECURRENCE RELATIONS FOR SINGLE AND PRODUCT MOMENTS

A random variable X is said to have a two-parameter rectangular distribution if its pdf is of the form

$$f(x) = \frac{1}{\theta_2 - \theta_1}, \quad -\infty < \theta_1 < x < \theta_2 < \infty, \tag{1}$$

and the cdf is of the form

$$F(x) = \frac{x - \theta_1}{\theta_2 - \theta_1}, \quad -\infty < \theta_1 < x < \theta_2 < \infty. \tag{2}$$

It can easily be seen that

$$(\theta_2 - x) f(x) = 1 - F(x), \ -\infty < \theta_1 < x < \theta_2 < \infty. \tag{3}$$

The relation in (3) will be employed in this paper to derive recurrence relations for the moments of k-th upper record values from the two-parameter rectangular distribution.

Let $\{Y_n^{(k)}, n \geq 1\}$ be a sequence of k-th upper record values from (1). Then, the pdf of $Y_n^{(k)}$ and the joint pdf of $Y_m^{(k)}$ and $Y_n^{(k)}$, respectively, are as follows:

$$f_{Y_n^{(k)}}(x)=\frac{k^n}{(n-1)!}[H(x)]^{n-1}[1-F(x)]^{k-1}f(x), \tag{4}$$

for $n=1,2\ldots,$

$$f_{Y_m^{(k)},Y_n^{(k)}}(x,y)=\frac{k^n}{(m-1)!(n-m-1)!}[H(x)]^{m-1}[H(y)-H(x)]^{n-m-1}h(x)[1-F(y)]^{k-1}f(y), x<y, \tag{5}$$

for $1\le m<n,\ n=2,3,\ldots,$
where $H(x)=-\log[1-F(x)]$, log is the natural logarithm and $h(x)=H'(x)$, (Dziubdziela and Kopociński, 1976; Grudzień, 1982).

Theorem 1: Fix a positive integer $k\ge 1$. For $n\ge 1$ and $r=0,1,2,\ldots,$

$$(r+1+k)\ E(Y_n^{(k)})^{r+1}=(r+1)\theta_2\ E(Y_n^{(k)})^r+k\ E(Y_{n-1}^{(k)})^{r+1}. \tag{6}$$

Proof: For $n\ge 1$ and $r=0,1,2,\ldots,$ we have from (4) and (3)

$$\theta_2\ E(Y_n^{(k)})^r-E(Y_n^{(k)})^{r+1}=\frac{k^n}{(n-1)!}\int_{\theta_1}^{\theta_2}x^r[H(x)]^{n-1}[1-F(x)]^k\,dx.$$

Integrating by parts, taking x^r as the part to be integrated and the rest of the integrand for differentiation, we get (6).

Theorem 2: For $1\le m\le n-2,\ r,s=0,1,2,\ldots,$

$$(s+1+k)E[(Y_m^{(k)})^r(Y_n^{(k)})^{s+1}]=\theta_2(s+1)E[(Y_m^{(k)})^r(Y_n^{(k)})^s]+kE[(Y_m^{(k)})^r(Y_{n-1}^{(k)})^{s+1}] \tag{7}$$

and for $m\ge 1,\ r,s=0,1,2,\ldots,$

$$(s+1+k)E[(\mathrm{Y}_{\mathrm{m}}^{(\mathrm{k})})^r(Y_{m+1}^{(k)})^{s+1}]=\theta_2(s+1)E[(Y_m^{(k)})^r(Y_{m+1}^{(k)})^s]+kE(Y_m^{(k)})^{r+s+1}. \tag{8}$$

Proof : From (5), for $1\le m\le n-1$ and $r,s=0,1,2,\ldots,$ we obtain

$$\theta_2E[(Y_m^{(k)})^r(Y_n^{(k)})^s]-E[(Y_m^{(k)})^r(Y_n^{(k)})^{s+1}]=\frac{k^n}{(m-1)!(n-m-1)!}\int_{\theta_1}^{\theta_2}x^r h(x)[H(x)]^{m-1}I(x)dx, \tag{9}$$

where

$$I(x)=\int_x^{\theta_2}y^s[H(y)-H(x)]^{n-m-1}[1-F(y)]^k\,dy,$$

on using the relation in (3). Upon integrating by parts, treating y^s for integration, we get

$$I(x) = -\frac{(n-m-1)}{(s+1)}\int_x^{\theta_2} y^{s+1}[H(y)-H(x)]^{n-m-2}[1-F(y)]^{k-1} f(y)dy$$
$$+\frac{k}{(s+1)}\int_x^{\theta_2} y^{s+1}[H(y)-H(x)]^{n-m-1}[1-F(y)]^{k-1} f(y)dy.$$

Substituting the above expression for $I(x)$ in (9) and simplifying, it leads to (7). Proceeding in a similar manner for the case $n = m + 1$, the recurrence relation given in (8) can easily be established.

3. ESTIMATION OF THE PARAMETERS θ_1 AND θ_2

It can be shown, on using (1), (2) and (4), that

$$E(Y_n^{(k)}) = \frac{k^n}{(n-1)!}\int_{\theta_1}^{\theta_2} x\left[-\log\left(\frac{\theta_2-x}{\theta_2-\theta_1}\right)\right]^{n-1}\left(\frac{\theta_2-x}{\theta_2-\theta_1}\right)^{k-1}\frac{1}{\theta_2-\theta_1}dx,$$

which simplifies to

$$E(Y_n^{(k)}) = \theta_2 - (\theta_2-\theta_1)\left(\frac{k}{k+1}\right)^n. \tag{10}$$

Also

$$E(Y_n^{(k)})^2 = \theta_2^2 + (\theta_2-\theta_1)^2\left(\frac{k}{k+2}\right)^n - 2\theta_2(\theta_2-\theta_1)\left(\frac{k}{k+1}\right)^n.$$

Hence, one can easily obtain

$$Var(Y_n^{(k)}) = (\theta_2-\theta_1)^2\left[\left(\frac{k}{k+2}\right)^n - \left(\frac{k}{k+1}\right)^{2n}\right]. \tag{11}$$

Further, on using the recurrence relations given in equations (6) and (7) in the relation

$$Cov(Y_m^{(k)}, Y_n^{(k)}) = E(Y_m^{(k)}Y_n^{(k)}) - E(Y_m^{(k)})E(Y_n^{(k)}),$$

we obtain

$$Cov(Y_m^{(k)}, Y_n^{(k)}) = \left(\frac{k}{k+1}\right)Cov(Y_m^{(k)}, Y_{n-1}^{(k)}), \quad n > m.$$

Applying it recursively, it can easily be verified that

$$Cov(Y_m^{(k)}, Y_n^{(k)}) = \left(\frac{k}{k+1}\right)^{n-m} \mathrm{var}(Y_m^{(k)}), \qquad n > m. \tag{12}$$

Let us consider the following transformation

$$Z_1^{(k)} = Y_1^{(k)},$$

$$Z_i^{(k)} = \left(\frac{k+2}{k}\right)^{\frac{i-1}{2}} \left(Y_i^{(k)} - \frac{k}{k+1} Y_{i-1}^{(k)}\right), \qquad i = 2, 3, \ldots, n\,.$$

Then on using (10), we obtain

$$E(Z_1^{(k)}) = \left(\frac{k}{k+1}\right)\theta_1 + \frac{\theta_2}{k+1}, \tag{13}$$

$$E(Z_i^{(k)}) = \left(\frac{k+2}{k}\right)^{\frac{i-1}{2}} \frac{\theta_2}{k+1}, \qquad i = 2, 3, \ldots, n. \tag{14}$$

Similarly, on using (11), we obtain

$$Var(Z_i^{(k)}) = \frac{k(\theta_2 - \theta_1)^2}{(k+2)(k+1)^2}, \quad i = 1, 2, \ldots, n. \tag{15}$$

Also, it can be shown that

$$Cov(Z_i^{(k)}, Z_j^{(k)}) = 0, \qquad i \neq j,\ 1 \le i < j \le n. \tag{16}$$

Let $\boldsymbol{Z}' = (Z_1^{(k)}, Z_2^{(k)}, \ldots, Z_n^{(k)})$. Then

$$E(\boldsymbol{Z}) = \boldsymbol{A\theta}, \tag{17}$$

where

$$\boldsymbol{A} = \begin{pmatrix} \dfrac{k}{k+1} & \dfrac{1}{k+1} \\ 0 & \left(\dfrac{k+2}{k}\right)^{\frac{1}{2}} \dfrac{1}{k+1} \\ \vdots & \vdots \\ 0 & \left(\dfrac{k+2}{k}\right)^{\frac{n-1}{2}} \dfrac{1}{k+1} \end{pmatrix}, \quad \boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}.$$

The best linear unbiased estimates $\hat{\theta}_1, \hat{\theta}_2$ of θ_1 and θ_2, respectively, based on $Y_1^{(k)}, Y_2^{(k)}, \ldots, Y_n^{(k)}$ are given by

$$\hat{\boldsymbol{\theta}} = \begin{pmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{pmatrix} = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{Z}. \tag{18}$$

We have

$$(\boldsymbol{A}'\boldsymbol{A}) = \begin{pmatrix} \left(\dfrac{k}{k+1}\right)^2 & \dfrac{k}{(k+1)^2} \\ \dfrac{k}{(k+1)^2} & T \end{pmatrix},$$

where

$$T = \left(\frac{1}{k+1}\right)^2 + \left(\frac{k+2}{k}\right)\left(\frac{1}{k+1}\right)^2 + \left(\frac{k+2}{k}\right)^2\left(\frac{1}{k+1}\right)^2 + \ldots + \left(\frac{k+2}{k}\right)^{n-1}\left(\frac{1}{k+1}\right)^2$$

$$= \frac{k}{2(k+1)^2}\left\{\left(\frac{k+2}{k}\right)^n - 1\right\}.$$

Now, one can easily obtain

$$(\boldsymbol{A}'\boldsymbol{A})^{-1} = \frac{(k+1)^4}{\left[\dfrac{(k+2)}{2}\left\{\left(\dfrac{k+2}{k}\right)^{n-1} - 1\right\}\right]k^2} \begin{pmatrix} T & -\dfrac{k}{(k+1)^2} \\ -\dfrac{k}{(k+1)^2} & \left(\dfrac{k}{k+1}\right)^2 \end{pmatrix}.$$

Substituting for $(\boldsymbol{A}'\boldsymbol{A})^{-1}$ in (18) and simplifying the resulting expression, we obtain

$$\hat{\theta}_1 = \frac{2(k+1)^4}{k^2(k+2)\left[\left(\dfrac{k+2}{k}\right)^{n-1} - 1\right]}$$

$$\times \left[\left(T\frac{k}{k+1} - \frac{k}{(k+1)^3}\right)Z_1^{(k)} - \frac{k}{(k+1)^3}\left\{\left(\frac{k+2}{k}\right)^{\frac{1}{2}} Z_2^{(k)} + \ldots + \left(\frac{k+2}{k}\right)^{\frac{n-1}{2}} Z_n^{(k)}\right\}\right]$$

and

$$\hat{\theta}_2 = \frac{2(k+1)^4}{k^2(k+2)\left[\left(\frac{k+2}{k}\right)^{n-1} - 1\right]}\left[\frac{k^2}{(k+1)^3}\left\{\left(\frac{k+2}{k}\right)^{\frac{1}{2}} Z_2^{(k)} + \ldots + \left(\frac{k+2}{k}\right)^{\frac{n-1}{2}} Z_n^{(k)}\right\}\right].$$

Hence, on using (15) and (16), we obtain

$$Var(\hat{\theta}_1) = \frac{2(\theta_2 - \theta_1)^2 (k+1)^2}{k(k+2)^2} \frac{T}{\left[\left(\frac{k+2}{k}\right)^{n-1} - 1\right]},$$

$$Var(\hat{\theta}_2) = \frac{2k(\theta_2 - \theta_1)^2}{(k+2)^2\left[\left(\frac{k+2}{k}\right)^{n-1} - 1\right]}$$

and

$$Cov(\hat{\theta}_1, \hat{\theta}_2) = -\frac{2(\theta_2 - \theta_1)^2}{(k+2)^2\left[\left(\frac{k+2}{k}\right)^{n-1} - 1\right]}.$$

The generalized variance $\hat{\Sigma}$ of $\hat{\theta}_1$ and $\hat{\theta}_2 (\hat{\Sigma} = Var(\hat{\theta}_1) Var(\hat{\theta}_2) - (Cov(\hat{\theta}_1, \hat{\theta}_2))^2)$ is

$$\frac{2(\theta_2 - \theta_1)^4}{(k+2)^3\left[\left(\frac{k+2}{k}\right)^{n-1} - 1\right]}.$$

On considering the two k-th upper record values $Y_s^{(k)}$ and $Y_r^{(k)}$ (s > r) it follows from (10) and (12) that the best linear unbiased estimates of θ_1 and θ_2 based on these two k-th record values are as follows:

$$\theta_1^* = \left(\frac{k+1}{k}\right)^r Y_r^{(k)} - \left(\left(\frac{k+1}{k}\right)^r - 1\right)\theta_2^*,$$

$$\theta_2^* = \frac{Y_s^{(k)} - \left(\frac{k+1}{k}\right)^{r-s} Y_r^{(k)}}{1 - \left(\frac{k+1}{k}\right)^{r-s}}.$$

The variances and covariances of θ_1^* and θ_2^* are

$$Var(\theta_1^*) = \frac{[(k+1)^{2r} - k^r(k+2)^r]}{k^r(k+2)^r}(\theta_2-\theta_1)^2 + \left(\left(\frac{k+1}{k}\right)^r - 1\right)^2 Var(\theta_2^*),$$

$$Var(\theta_2^*) = \frac{\left[\left(\frac{k+2}{k}\right)^{r-s} - \left(\frac{k+1}{k}\right)^{2(r-s)}\right]}{\left(\frac{k+2}{k}\right)^r\left[1-\left(\frac{k+1}{k}\right)^{(r-s)}\right]^2}(\theta_2-\theta_1)^2$$

and

$$Cov(\theta_1^*, \theta_2^*) = -\left(\left(\frac{k+1}{k}\right)^r - 1\right)Var(\theta_2^*).$$

It can easily be shown that the generalized variance $\Sigma^* = Var(\theta_1^*)Var(\theta_2^*) - (Cov(\theta_1^*, \theta_2^*))^2$ is minimum when s = n and r = 1. Hence the best linear unbiased estimates of θ_1 and θ_2 based on two selected k-th record values are

$$\tilde{\theta}_1 = \frac{k+1}{k}Y_1^{(k)} - \frac{\tilde{\theta}_2}{k},$$

$$\tilde{\theta}_2 = \frac{Y_n^{(k)} - \left(\frac{k+1}{k}\right)^{1-n} Y_1^{(k)}}{1-\left(\frac{k+1}{k}\right)^{1-n}}.$$

Also, on using (11) and (12), one can obtain

$$Var(\tilde{\theta}_1) = \frac{(\theta_2-\theta_1)^2}{k(k+2)} + \frac{Var(\tilde{\theta}_2)}{k^2},$$

$$Var(\tilde{\theta}_2) = \frac{\{(k+1)^{2(n-1)} - k^{n-1}(k+2)^{n-1}\}}{\left(\left(\frac{k+1}{k}\right)^{n-1} - 1\right)^2 k^{n-2}(k+2)^n}(\theta_2-\theta_1)^2$$

and

$$Cov(\tilde{\theta}_1, \tilde{\theta}_2) = -\frac{Var(\tilde{\theta}_2)}{k}.$$

Let

$$e_1 = \frac{Var(\hat{\theta}_1)}{Var(\tilde{\theta}_1)}, \quad e_2 = \frac{Var(\hat{\theta}_2)}{Var(\tilde{\theta}_2)} \text{ and } e_{12} = \frac{Cov(\hat{\theta}_1, \hat{\theta}_2)}{Cov(\tilde{\theta}_1, \tilde{\theta}_2)}.$$

The generalized variance $\tilde{\Sigma}$ of $\tilde{\theta}_1, \tilde{\theta}_2$ is

$$\frac{\{(k+1)^{2(n-1)} - k^{n-1}(k+2)^{n-1}\}}{k^{n-1}(k+2)^{n+1}\left(\left(\dfrac{k+1}{k}\right)^{n-1} - 1\right)^2}(\theta_2 - \theta_1)^4.$$

Further, it can be seen that $e_{12} = e_2$.

In Tables 1 and 2, we tabulate the values of e_1 and e_2, for $k = 2$ and $k = 3$, respectively and for $n = 2, 4, 5, 10, 15, 20$ and 30. It can be seen from the tables that efficiency of the best linear unbiased estimate of θ_1 based on two k-th record values are very high compared to corresponding estimate based on a complete set of k-th record values.

TABLE 1

Values of e_1 *and* e_2 *for* $k = 2$

n	e_1	e_2
2	1.0000	1.0000
4	0.9965	0.9506
5	0.9969	0.9141
10	0.9996	0.7272
15	1.0000	0.6148
20	1.0000	0.5592
30	1.0000	0.5170

TABLE 2

Values of e_1 *and* e_2 *for* $k = 3$

n	e_1	e_2
2	1.0000	1.0000
4	0.9968	0.9688
5	0.9970	0.9148
10	0.9989	0.7846
15	0.9998	0.6492
20	1.0000	0.5613
30	1.0000	0.4725

Remark : Although θ_2 can be accurately estimated by taking large n, the estimate of θ_1 does not improve with increasing n (asymptotically for $k = 2$, $Var(\hat{\theta}_1) = 1/8$ and for k=3, $Var(\hat{\theta}_1) = 1/15$).

4. CHARACTERIZATION

This section contains characterization of the two-parameter rectangular distribution. Characterization is a condition involving certain properties of a random variable X, which identifies the associated cdf $F(x)$. The property that uniquely determines $F(x)$ may be based on function of random variables whose joint distribution is related to that of X. A characterization can be used in the construction of goodness of fit tests and in examining the consequences of modeling assumptions made by an applied scientist.

In this paper we shall use the record moment sequence to determine $F(x)$ uniquely. We shall use the following result of Lin (1986).

Theorem 3 (Lin, 1986): Let n_0 be any fixed non-negative integer, $-\infty \le a < b \le \infty$, and $g(x) \ge 0$ be an absoluetly continuous function with $g'(x) \ne 0$ a.e. on (a, b). Then the sequence of functions $\{(g(x))^n e^{-g(x)}, n \ge n_o\}$ is complete in $L(a, b)$ iff $g(x)$ is strictly monotone on (a, b).

Theorem 4: A necessary and sufficient condition for a random variable X to be distributed with pdf given by (1) is that

$$(r+1+k)\ E(Y_n^{(k)})^{r+1} = (r+1)\theta_2\ E(Y_n^{(k)})^r + k\ E(Y_{n-1}^{(k)})^{r+1}, \tag{19}$$

for $n = 1, 2, \ldots$ and $r = 0, 1, 2, \ldots$, where $k \ge 1$ is any fixed positive integer.

Proof: The necessary part follows from (6). On the other hand if the recurrence relation (19) is satisfied, we get

$$\frac{(r+1+k)k^n}{(n-1)!}\int_{\theta_1}^{\theta_2} x^{r+1}[H(x)]^{n-1}[1-F(x)]^{k-1} f(x)dx$$

$$= \frac{(r+1)\theta_2 k^n}{(n-1)!}\int_{\theta_1}^{\theta_2} x^r [H(x)]^{n-1}[1-F(x)]^{k-1} f(x)dx$$

$$+ \frac{k^n}{(n-2)!}\int_{\theta_1}^{\theta_2} x^{r+1}[H(x)]^{n-2}[1-F(x)]^{k-1} f(x)dx.$$

Integrating the last integral on the right-hand side of the above equation by parts, we get

$$\frac{(r+1+k)k^n}{(n-1)!}\int_{\theta_1}^{\theta_2} x^{r+1}[H(x)]^{n-1}[1-F(x)]^{k-1} f(x)dx$$
$$=\frac{(r+1)\theta_2 k^n}{(n-1)!}\int_{\theta_1}^{\theta_2} x^{r}[H(x)]^{n-1}[1-F(x)]^{k-1} f(x)dx$$
$$+\frac{k^{n+1}}{(n-1)!}\int_{\theta_1}^{\theta_2} x^{r+1}[H(x)]^{n-1}[1-F(x)]^{k-1} f(x)dx$$
$$-\frac{k^n(r+1)}{(n-1)!}\int_{\theta_1}^{\theta_2} x^{r}[H(x)]^{n-1}[1-F(x)]^{k} dx,$$

which on simplification reduces to

$$\int_{\theta_1}^{\theta_2} x^r[H(x)]^{n-1}[1-F(x)]^{k-1}[(r+1+k)xf(x)+(r+1)(1-F(x))-(r+1)\theta_2 f(x)-kxf(x)]dx=0.$$

Now on using *Theorem 3* with $g(x)=-\log[1-F(x)]=H(x),$ it follows that

$$(\theta_2-x)f(x)=[1-F(x)],$$

which proves that $f(x)$ has the form (1).

5. CONCLUDING REMARKS

In this paper the best linear unbiased estimates of the parameters of a two-parameter rectangular distribution have been obtained using some properties of the k-th record values. The efficiency of these estimates based on two k-th record values has been compared with that of the corresponding estimates based on a complete set of k-th record values and has been presented in the tabular form. It has been seen that efficiency of the best linear unbiased estimate of θ_1 based on two k-th record values are very high compared to corresponding estimate based on a complete set of k-th record values.

Some recurrence relations for single and product moments of k-th upper record values from a two-parameter rectangular distribution have also been established. These recurrence relations have been used to characterize a two-parameter rectangular distribution.

Department of Statistics JAGDISH SARAN
University of Delhi, India

Department of Statistics APARNA PANDEY
P.G.D.A.V. College,
New Delhi, India

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SUMMARY

Estimation of parameters of a two-parameter rectangular distribution and its characterization by k-th record values

In this paper, we shall make use of the properties of the *k*-th upper record values to develop the inferential procedures such as point estimation. We shall obtain the best linear unbiased estimates of parameters of a two-parameter rectangular distribution based on *k*-th record values. The efficiency of the best linear unbiased estimates of the parameters based on two *k*-th record values has been compared with that of the corresponding estimates based on a complete set of *k*-th record values. At the end we give the characterization of the two-parameter rectangular distribution using *k*-th upper record values.