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SOME DYNAMIC GENERALIZED INFORMATION MEASURES IN THE CONTEXT OF WEIGHTED MODELS

S.S. Maya, S.M. Sunoj

1. INTRODUCTION

Length of time during a study period has been considered as prime variable of interest in many fields such as reliability, survival analysis, economics, business etc. In particular, consider an item under study, then information about the remaining (past) lifetime is also an important component in many applications. Kullback and Leibler information and entropy of order α developed by Renyi (1961) are some of the important dynamic measures that have been applied by several authors to study the effect of age (time) in these situations. For more details and recent works, we refer to Ebrahimi and Kirmani (1996a, 1996b), Di Crescenzo and Longobardi (2002, 2004), Asadi *et al.* (2004, 2005a, 2005b) and the references therein.

Let X and Y be two absolutely continuous non negative random variables that describe the lifetimes of two items. Denote $f(t)$, $F(t)$ and $\bar{F}(t)=1-F(t)$ the probability density function (pdf), distribution function (df) and survival function (sf) of X respectively and $g(t)$, $G(t)$ and $\bar{G}(t)=1-G(t)$ be the corresponding functions of Y. Moreover, let $\lambda_X(t)=\frac{f(t)}{F(t)}$ and $\lambda_Y(t)=\frac{g(t)}{G(t)}$ be the reversed hazard rates of X and Y respectively. Kullback and Leibler (1951) introduced a directed divergence (also known as information divergence, information gain, or relative entropy), a distance measure of the difference between two probability distributions: from a "true" probability distribution $f(t)$ to an arbitrary (reference) probability distribution $g(t)$ is given by

$$I(X,Y)=\int_0^\infty f(x)\log\frac{f(x)}{g(x)}dx\,. \tag{1}$$

Clearly, equation (1) is a ruler to measure the similarity (closeness) between two distributions $f(t)$ and $g(t)$. In view of the wide applicability of (1), Ebrahimi and Kirmani (1996b) proposed a measure of discrimination between two residual life distributions based on (1) given by

$$I_{X,Y}(t) = \int_t^\infty \frac{f(x)}{\overline{F}(t)} \log\left(\frac{f(x)/\overline{F}(t)}{g(x)/\overline{G}(t)}\right) dx\,; \qquad t > 0\,. \tag{2}$$

$I_{X,Y}(t)$ measures the relative entropy of $(X-t|X>t)$ and $(Y-t|Y>t)$. It is useful to compare the residual life times of two items, which have both survived up to time t (for more details see Ebrahimi (1998, 2001), Asadi *et al.* (2005a, 2005b)). Along the similar lines of the measure (2), Di Crescenzo and Longobardi (2004) defined a information distance between the past lives $(X|X \le t)$ and $(Y|Y \le t)$ as

$$\overline{I}_{X,Y}(t) = \int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(x)/F(t)}{g(x)/G(t)}\right) dx\,; \qquad t > 0\,. \tag{3}$$

Given that at time t, two items have been found to be failing, $\overline{I}_{X,Y}(t)$ measures the discrepancy between their past lives.

Similar to the discrimination information measure (2), closeness between two residual distributions is also measured by the Renyi information divergence of order α (see Asadi *et al.* (2005a, 2005b)), given by

$$I_{X,Y}(\alpha,t) = \frac{1}{(\alpha-1)} \log \int_t^\infty \frac{f^\alpha(x) g^{(1-\alpha)}(x)}{\overline{F}^\alpha(t)\overline{G}^{(1-\alpha)}(t)} dx\,. \tag{4}$$

Also, Asadi *et al.* (2005a) defined Renyi discrimination implied by F and G as

$$\overline{I}_{X,Y}(\alpha,t) = \frac{1}{(\alpha-1)} \log \int_0^t \frac{f^\alpha(x) g^{(1-\alpha)}(x)}{F^\alpha(t)G^{(1-\alpha)}(t)} dx\,. \tag{5}$$

Although a wide variety of research has been carried out for studying these dynamic information measures (2) to (5), however, in the present paper we further examine it by measuring a distance (similarity) between a true distribution and an observed (weighted) distribution and obtain relationships between these distributions. Further, some bounds and inequalities related to these measures are also proved. Finally, the relationship between weighted models and their unweighted counter part using the dynamic generalized information measures for residual lifetime models are also examined and proved certain characterization results arising out of it.

2. WEIGHTED DISTRIBUTIONS

The weighted distributions arise naturally as a result of observations generated from a stochastic process and recorded with some weight function. The concept of weighted distributions was introduced by Rao (1965) in connection with modeling statistical data and in situations where the usual practice of employing standard distributions for the purpose was not found appropriate. Various fields of applications of weighted distributions include analysis of family size, study of albinism, human heredity, aerial survey and visibility bias, line transcend sampling, renewal theory, cell cycle analysis and pulse labeling, efficacy of early screening for disease, etiological studies, statistical ecology and reliability modeling. A survey of research in this area is available in Patil and Rao (1977), Jones (1990), Gupta and Kirmani (1990), Navarro *et al.* (2001), Nair and Sunoj (2003), Di Crescenzo and Longobardi (2006) and Oluyede and Terbeche (2007).

Let X be a random variable (rv) having an absolutely continuous df $F(t) = P(X \le t)$ with support (a,b), a subset of the real line where $a = \inf\{t : F(t) > 0\}$ and $b = \sup\{t : F(t) < 1\}$. In a weighted distribution problem, a realization t of X enters into an investigators record with probability proportional to $w(t)$. Obviously the recorded t is not an observation on X but rather an observation on a weighted rv X_w. If the pdf of X is $f(t)$, reversed hazard rate of X is $\lambda(t)$ and $w(.)$ a non-negative function satisfying $\mu = E(w(X)) < \infty$, then the rv X_w with pdf $f_w(t)$, df $F_w(t)$ and reversed hazard rate $\lambda_w(t)$ corresponding to the weighted rv are given by

$$f_w(t) = \frac{w(t) f(t)}{\mu_w} \tag{6}$$

$$F_w(t) = \frac{E(w(X) \mid X \le t)}{\mu_w} F(t) \tag{7}$$

and

$$\lambda_w(t) = \frac{w(t)}{E(w(X) \mid X \le t)} \lambda(t), \tag{8}$$

where $\mu_w = E(w(X))$, is a normalizing constant. For more results and properties, we refer to Sunoj and Maya (2006). When $w(t) = t^\beta$, the corresponding distribution is a size-biased model and its pdf is given by

$$f_s(t) = \frac{t^\beta f(t)}{\mu_\beta} \tag{9}$$

where $0 < \mu_\beta(t) = \int_0^\infty x^\beta f(x)dx$. The size-biased df and reversed hazard rate are given by

$$F_s(t) = \frac{\overline{m}_\beta(t)F(t)}{\mu_\beta} \tag{10}$$

and

$$\lambda_s(t) = \frac{t^\beta}{\overline{m}_\beta(t)}\lambda(t) \tag{11}$$

where $\overline{m}_\beta(t) = E(X^\beta \mid X \le t)$ is the conditional moment function.

Remark 1. When $\beta = 1$, the size-biased model reduces to length biased one and the pdf, df and reversed hazard rate are obtained by putting $\beta = 1$ in the equations (9), (10) and (11).

Now we define a generalized information measure of discrimination between the past lives $(X \mid X \le t)$ and $(X_w \mid X_w \le t)$ corresponding to (3) as

$$\overline{I}_{X,X_w}(t) = \int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(x)/F(t)}{f_w(x)/F_w(t)}\right)dx. \tag{12}$$

Equation (12) is directly related to the past entropy $\overline{H}(t)$ (see Di Crescenzo and Longobardi (2002)) as

$$\overline{I}_{X,X_w}(t) = -\int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f_w(x)}{F_w(t)}\right)dx - \overline{H}(t) \tag{13}$$

where $\overline{H}(t) = -\int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(x)}{F(t)}\right)dx$. Equation (13) measures the discrepancy between the past lives of original rv X and weighted rv X_w. More importantly, $\overline{I}_{X,X_w}(t)$ may be a useful qualitative tool for measuring how far the true density is away from a weighted density. On the other hand, when the original and weighted density functions are equal then $\overline{I}_{X,X_w}(t) = 0$ *a.e.*

Remark 2. Equation (12) may be useful in the determination of a weight function and therefore for the selection of a suitable weight function in an observed mechanism, we can choose a weight function for which the distance measure in

(12) is small. Moreover, the generalized information measures are all asymmetric in $f(t)$ and $f_w(t)$, therefore, for reversing the roles of $f(t)$ and $f_w(t)$ in (12), say $\overline{I}_{X_W,X}(t)$ and equate with (12) for a symmetric measure implies the weight function is unity, *i.e.*, $f_w(t) \equiv f(t)$.

Now, substituting (6) and (7) in (12), we get

$$\overline{I}_{X,X_w}(t) = \log[E(w(X)|X \le t)] - E(\log w(X)|X \le t). \tag{14}$$

For the size biased model, (14) becomes

$$\overline{I}_{X,X_s}(t) = \log \overline{m}_\beta(t) - \beta(\log \overline{G}(t)) \tag{15}$$

where $\log \overline{G}(t) = E(\log X|X \le t)$ is the geometric vitality function for the right truncated distribution. When $\beta = 1$, equation (15) reduces to the discrimination measure between the original and a length-biased model.

Theorem 1. $\overline{I}_{X,X_w}(t)$ is independent of t if and only if the weight function takes the form $w(t) = (F(t))^{\theta-1}; \theta > 0$.

Proof. Suppose that $\overline{I}_{X,X_w}(t)$ is independent of t, that is

$$\overline{I}_{X,X_w}(t) = k. \tag{16}$$

Comparing (14) and (16) and differentiating with respect to t, we get

$$\frac{d}{dt}\log[E(w(X)|X \le t)] - \frac{d}{dt}E(\log w(X)|X \le t) = 0. \tag{17}$$

Now using the relationship

$$E(w(X)|X \le t) = \frac{1}{F(t)}\int_0^t w(x)f(x)dx \tag{18}$$

Differentiating (18) with respect to t and using (8), we have

$$\frac{d}{dt}\log[E(w(X)|X \le t)] = \lambda_w(t) - \lambda(t). \tag{19}$$

Again from the definition of

$$E(\log w(X)|X \le t)F(t) = \int_0^t \log w(x)f(x)dx, \tag{20}$$

and differentiating (20) with respect to t, we obtain

$$\frac{d}{dt}E(\log w(X)\big|X \le t) = (\log w(t) - E(\log w(X)\big|X \le t))\lambda(t)\,. \tag{21}$$

Taking logarithm of equation (8), we have

$$\log w(t) = \log\left(\frac{\lambda_w(t)}{\lambda(t)}\right) + \log[E(w(X)\big|X \le t)] \tag{22}$$

Substituting (22) in (21) and using (16), we get

$$\frac{d}{dt}E(\log w(X)\big|X \le t) = \left(\log\left(\frac{\lambda_w(t)}{\lambda(t)}\right) - k\right)\lambda(t) \tag{23}$$

Now substituting (19) and (23) in equation (17) and simplifying, we obtain

$$\left[\log\frac{\lambda(t)}{\lambda_w(t)} - k\right] - \frac{\lambda_w(t)}{\lambda(t)} + 1 = 0\,. \tag{24}$$

Putting $u(t) = \dfrac{\lambda(t)}{\lambda_w(t)}$ and on differentiating (24) we get

$$\frac{u^{'}(t)}{u(t)}\left(1 - \frac{1}{u(t)}\right) = 0$$

which implies that either $u^{'}(t) = 0$ or $u(t) = 1$. But as X and X_w are not equal $u(t) \ne 1$. So $u'(t) = 0$. Hence we have $u'(t) = 0$, which implies that there exists a non-negative constant θ such that $\lambda^w(t) = \theta\lambda(t)$. Now using (8) and (9) we get $w(t) = (F(t))^{\theta-1}; \theta > 0$.

Conversely assuming $w(t) = (F(t))^{\theta-1}$ and using (7), we obtain

$$F^w(t) = (F(t))^{\theta}\,. \tag{25}$$

From (25) and (12) we get that for $t > 0$, $\overline{I}_{X,X^w}(t) = \theta - 1 - \log\theta$, which is independent of t.

Remark 3. From equation (25) and (12), it is clear that for $t > 0$, $\overline{I}_{X,X^w}(t) = \theta - 1 - \log\theta$, a constant, is true for any df $F(t)$. Thus the distance between a true probability distribution $f(t)$ and a weighted distribution $f_w(t)$ is always a constant if and only if the weight function is of the form $w(t) = (F(t))^{\theta-1}; \theta > 0$.

Corollary 1. $\overline{I}_{X,X_w}(t)$ is independent of t if and only if the distribution functions of X and X_w satisfy the proportional reversed hazard model (see Di Crescenzo (2000) and Di Crescenzo and Longobardi (2002)).

Corollary 2. When X_w is a length biased model (*i.e.*, when $w(t) = t$), then $\overline{I}_{X,X_w}(t) = k$ characterizes power distribution with df

$$F(t) = \left(\frac{t}{b}\right)^c, \ 0 < t < b, \ b, c > 0. \tag{26}$$

Proof. It can be proved easily from the relationship

$$F(t) = \exp\left(-\int_t^b \frac{(\overline{\beta}'(x)w(x) + \overline{\beta}(x)w'(x))}{(1-\overline{\beta}(x))w(x)}dx\right), \tag{27}$$

where $\overline{\beta}(x) = \frac{\lambda(x)}{\lambda_w(x)}$ (see (Sunoj and Maya, 2006)). Now from Theorem 1 $\frac{\lambda(x)}{\lambda_w(x)} = \frac{1}{\theta}$, and using (27), the form of the model (26) is direct.

We next consider the Renyi discrimination function between two distributions for the past lifetime $(X|X \le t)$ and $(X_w|X_w \le t)$ implied by $F(t)$ and $F_w(t)$. Using equation (4), it is given by

$$\overline{I}_{X,X_w}(\alpha,t) = \frac{1}{(\alpha-1)}\log\int_0^t \frac{f^\alpha(x) f_w^{(1-\alpha)}(x)}{F^\alpha(t) F_w^{(1-\alpha)}(t)}dx. \tag{28}$$

Equation (28) using (6) and (7), becomes

$$\overline{I}_{X,X_w}(\alpha,t) = \log[E(w(X)|X \le t)] + \frac{1}{(\alpha-1)}\log[E(w^{(1-\alpha)}(X)|X \le t)]. \tag{29}$$

For the size-biased model is

$$\overline{I}_{X,X_s}(\alpha,t) = \log \overline{m}_\beta(t) + \frac{1}{(\alpha-1)}\log \overline{m}_{\beta(1-\alpha)}(t).$$

Remark 4. When $\alpha = 0$, then (28) reduces to (12).

Theorem 2. The Renyi divergence measure for the past life $\bar{I}_{X,X_w}(\alpha,t)$ is independent of t if and only if the weight function is $w(t) = (F(t))^{\theta-1}$; $\theta > 0$.

Proof. The proof is similar to that of Theorem 1.

Corollary 3. When $w(t) = t$, then Theorem 2 characterizes power distribution with df (19).

3. INEQUALITIES FOR DYNAMIC GENERALIZED INFORMATION MEASURES

In this section, we present some results including inequalities and comparisons of dynamic generalized information measures for weighted and parent (unweighted) distributions. Under some mild constraints, bounds for these measures are also presented.

Theorem 3. If the weight function $w(t)$ is increasing (decreasing) in $t > 0$, then

(*a*) $\bar{I}_{X,X_w}(t) \geq (\leq) \log\left(\dfrac{\lambda(t)}{\lambda_w(t)}\right)$

(*b*) $\bar{I}_{X,X_w}(\alpha,t) \geq (\leq) \dfrac{\alpha}{(\alpha-1)} \log\left(\dfrac{\lambda(t)}{\lambda_w(t)}\right)$, $\alpha \neq 1$.

Proof. Suppose $w(t)$ is increasing, from (6) we get $\dfrac{f(t)}{f^w(t)}$ is decreasing, implies that

$$\frac{f(t)}{f_w(t)} \leq \frac{f(x)}{f_w(x)}.$$

Now from the definition (12) we have

$$\bar{I}_{X,X_w}(t) = \int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(x)/F(t)}{f_w(x)/F_w(t)}\right) dx \geq \int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(t)/F(t)}{f_w(t)/F_w(t)}\right) dx$$

$$= \log\left(\frac{\lambda(t)}{\lambda_w(t)}\right) \text{ for all } x \leq t.$$

When $w(t)$ is decreasing then the inequality is reversed.

Proof. of (*b*) is similar to that of (*a*).

Example 1. Suppose X follows a power distribution with df (26) and assume that $w(t)=t, t>0$, an increasing function of t, then a straightforward computation yield $\overline{I}_{X,X_w}(t)=\log\left(\frac{c}{c+1}\right)+\frac{1}{c}$ and $\log\left(\frac{\lambda(t)}{\lambda_w(t)}\right)=\log\left(\frac{c}{c+1}\right)$, which proves result (*a*) of Theorem 3. In a similar calculation using the same df (26) and weight function, we get $\overline{I}_{X,X_w}(\alpha,t)=$ $\log\left(\frac{c}{c+1}\right)+\frac{1}{(\alpha-1)}\log\left(\frac{c}{c+1-\alpha}\right)$ and find that it also satisfies the result (*b*) of Theorem 3.

Theorem 4. When (*i*) $w(t)$ is non-decreasing (non-increasing) and (*ii*) $X \overset{RHR}{\leq} \left(\overset{RHR}{\geq}\right) X_w$, then $\overline{I}_{X,X_w}(t)$ is non-decreasing (non-increasing) for all $t>0$.

Proof. From the definition (12)

$$\overline{I}_{X,X_w}(t)=\log\left(\frac{f_w(t)}{F(t)}\right)+\int_0^t \frac{f(x)}{F(t)}\log\left(\frac{f(x)}{f_w(x)}\right)dx . \tag{30}$$

The first term of (30) is increasing using (*ii*) (see Sunoj and Maya (2006)) and

$$\int_0^t \frac{f(x)}{F(t)}\log\left(\frac{f(x)}{f_w(x)}\right)dx=\log\mu_w-\frac{1}{F(t)}\int_0^t f(x)\log w(x)dx . \tag{31}$$

Now the second term of (31) is given by

$$-\frac{1}{F(t)}\int_0^t f(x)\log w(x)dx=-\log(w(t))+\frac{1}{F(t)}\int_0^t \frac{w'(x)}{w(x)}F(x)dx \tag{32}$$

Differentiating (32) with respect to t and on simplification we get

$$\frac{d}{dt}\left(-\int_0^t \frac{f(x)\log w(x)}{F(t)}dx\right)=-\frac{\lambda(t)}{F(t)}\int_0^t \frac{w'(x)}{w(x)}F(x)dx \geq 0 .$$

and when $w(t)$ is increasing, then (30) is the sum of two increasing functions. It implies that $\overline{I}_{X,X_w}(t)$ is also increasing. Using the similar steps as above, the inequality in the reverse direction can be proved.

Example 2. Let X be a nonnegative rv having df

$$F(t)=\begin{cases}\dfrac{t^2}{2} & ; \quad 0\le t<1\\ \dfrac{(t^2+2)}{6} & ; \quad 1\le t<2\\ 1 & ; \quad t\ge 2\end{cases}$$

When $w(t)=t, t>0$, an increasing function of t then

$$\lambda_W(t)=\begin{cases}\dfrac{3}{t} & ; \quad 0\le t<1\\ \dfrac{3t^2}{(t^3-1)} & ; \quad 1\le t<2\\ 0 & ; \quad t\ge 2\end{cases}$$

proves the condition (2). Now using equation (14), we get

$$\overline{I}_{X,X_w}(t)=\begin{cases}\log\left(\dfrac{2}{3}\right)-\dfrac{1}{2} & ; \quad 0\le t<1\\ \log\left[\dfrac{2(t^3-1)}{3(t^2+2)}\right]-\dfrac{[2t^2\log t-(t^2-1)]}{2(t^2+2)} & ; \quad 1\le t<2\\ 0 & ; \quad t\ge 2\end{cases}$$

is an non-decreasing function in t, which can be easily checked.

Theorem 5. When $w(t)$ is increasing (decreasing) and $\dfrac{E(w(X)|X\le t)}{w(t)}$ is increasing (decreasing), then $\overline{I}_{X,X_w}(t)$ is increasing (decreasing) for all $t>0$.

Proof. When $w(t)$ is increasing, from Theorem 4,

$$\overline{I}_{X,X_w}(t)\ge\log\left(\frac{\lambda(t)}{\lambda^w(t)}\right).$$

Now using (14) and the condition given in theorem, we get $\log\left(\frac{\lambda(t)}{\lambda^{w}(t)}\right)$ increases, which imply the required result. Similarly one can prove the inequality in the reverse direction.

4. DYNAMIC GENERALIZED INFORMATION MEASURES FOR RESIDUAL LIFETIME MODELS

In this section, we study the comparison of weighted models and their unweighted counter part using the dynamic generalized information measures for residual lifetime models. For weighted distributions (6), the measures described by Ebrahimi and Kirmani (1996b) in (2) and Asadi *et al.*, 2005) in (4) are given by

$$I_{X,X_w}(t)=\int_t^{\infty}\frac{f(x)}{\overline{F}(t)}\log\left(\frac{f(x)/\overline{F}(t)}{f_w(x)/\overline{F}_w(t)}\right)dx \tag{33}$$

and

$$I_{X,X_w}(\alpha,t)=\frac{1}{(\alpha-1)}\log\int_t^{\infty}\frac{f^{\alpha}(x)f_w^{(1-\alpha)}(x)}{\overline{F}^{\alpha}(t)\overline{F}_w^{(1-\alpha)}(t)}dx \tag{34}$$

where $\overline{F}_w(t)=\frac{E(w(X)|X>t)F(t)}{\mu_w}$. Now using the definitions of weighted distribution, we can write (33) and (34) as

$$I_{X,X_w}(t)=\log[E(w(X)|X>t)]-E(\log w(X)|X>t)$$

and

$$I_{X,X_w}(\alpha,t)=\log[E(w(X)|X>t)]+\frac{1}{(\alpha-1)}\log[E(w^{(1-\alpha)}(X)|X>t)].$$

For the size-biased model, these measures can be written in terms of vitality and geometric vitality functions and are given by

$$I_{X,X_s}(t)=\log m_{\beta}(t)-\beta(\log G(t))$$

$$I_{X,X_s}(\alpha,t)=\log m_{\beta}(t)+\frac{1}{(\alpha-1)}\log m_{\beta(1-\alpha)}(t),$$

where $m(t)=E(X|X>t)$ and $\log G(t)=E(\log X|X>t)$.

Theorem 6. For the size-biased model, $I_{X,X_s}(t)=k$ characterizes Pareto distribution with

$$\bar{F}(t)=\left(\frac{a}{t}\right)^{c};\; t>a, a, c>0. \tag{35}$$

Remark 5. When the weight function is $w(x)=(\bar{F}(x))^{\theta-1}$, then the model becomes a proportional hazards model and in that case $I_{X,X_w}(t)$ is independent of t.

Theorem 7. The relationship

$$I_{X,X_s}(\alpha,t)=k \tag{36}$$

where k is constant is satisfied if and only if X follows Pareto I distribution (35).

Proof. Assume that X follows Pareto I distribution (35), then by direct calculation, we get (36) with $k=\dfrac{c^{\alpha}(1-\beta)^{1-\alpha}}{c+\beta\alpha-\beta}$, independent of t. Conversely assume (36) then by using the definition we obtain

$$\frac{1}{(\alpha-1)}\log\int_{0}^{t}\frac{f^{\alpha}(x)f_{w}^{(1-\alpha)}(x)}{\bar{F}^{\alpha}(t)\bar{F}_{w}^{(1-\alpha)}(t)}dx=k. \tag{37}$$

Differentiating with respect to t, (37) implies

$$f^{\alpha}(t)f_{w}^{(1-\alpha)}(t)=k(\alpha f(t)\bar{F}^{\alpha-1}(t)\bar{F}_{w}^{(1-\alpha)}(t)+(1-\alpha)f_{w}(t)\bar{F}_{w}^{\alpha}(t)\bar{F}^{\alpha}(t)). \tag{38}$$

Dividing each term by $(f^{\alpha}(t)f_{w}^{(1-\alpha)}(t))$ and rearranging the terms, (38) becomes

$$\frac{1}{k}=\alpha\psi^{\alpha-1}(t)+(1-\alpha)\psi^{\alpha}(t) \tag{39}$$

where $\psi(t)=\dfrac{h_w(t)}{h(t)}$. Differentiating (39) and on simplification we get $\psi(t)=k^{*}$, which on further simplification yields the required result.

Department of Statistics S.S. MAYA
Cochin University of Science and Technology, India S.M. SUNOJ

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SUMMARY

Some dynamic generalized information measures in the context of weighted models

In this paper, we study some dynamic generalized information measures between a true distribution and an observed (weighted) distribution, useful in life length studies. Further, some bounds and inequalities related to these measures are also studied.