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SUPPLEMENTARY MATERIALS: COMPARISON BETWEEN THE EXACT LIKELIHOOD AND WHITTLE LIKELIHOOD FOR MOVING AVERAGE PROCESSES

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Supplementary materials provide the simulation results of parameter estimation accuracy for the MLE ($\hat{\theta}$) and Whittle estimator ($\tilde{\theta}_W$) for the MA(1) process. Tables A and B report the parameter estimates, the estimation bias (Bias), the mean of absolute error (MAE) and mean squared error (MSE) results with T=30 and 100 respectively.

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TABLE A
Accuracy performance of MLE and Whittle likelihood estimator for T =30. The better performances with smaller absolute value of Bias, smaller MAE and MSE are marked in bold.

	Estimate		Bias		MAE		MSE	
θ_0	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$
0.1	0.147	0.148	**0.047**	0.048	**0.124**	0.126	**0.027**	0.029
0.2	0.241	0.234	0.041	**0.034**	0.150	**0.144**	0.038	**0.037**
0.3	0.329	0.319	0.029	**0.019**	0.169	**0.162**	0.046	**0.043**
0.4	0.427	0.420	0.027	**0.020**	**0.160**	0.162	**0.041**	0.043
0.5	0.530	0.501	0.030	**0.001**	**0.155**	0.163	**0.039**	0.043
0.6	0.641	0.600	0.041	**-0.001**	**0.149**	0.151	**0.037**	0.038
0.7	0.722	0.681	0.022	**-0.020**	**0.124**	0.135	**0.025**	0.029
0.8	0.832	0.750	**0.032**	-0.050	**0.112**	0.122	**0.018**	0.025
0.9	0.898	0.790	**-0.002**	-0.110	**0.087**	0.143	**0.015**	0.040
0.91	0.911	0.807	**0.001**	-0.104	**0.078**	0.124	**0.009**	0.023
0.92	0.913	0.820	**-0.007**	-0.100	**0.074**	0.121	**0.009**	0.023
0.93	0.928	0.822	**-0.002**	-0.108	**0.066**	0.124	**0.007**	0.024
0.94	0.925	0.810	**-0.015**	-0.130	**0.067**	0.141	**0.009**	0.030
0.95	0.928	0.821	**-0.022**	-0.129	**0.065**	0.140	**0.009**	0.031
0.96	0.934	0.825	**-0.026**	-0.136	**0.059**	0.141	**0.008**	0.031
0.97	0.937	0.824	**-0.033**	-0.146	**0.056**	0.149	**0.008**	0.034
0.98	0.935	0.816	**-0.045**	-0.164	**0.060**	0.165	**0.010**	0.039
0.99	0.946	0.819	**-0.044**	-0.171	**0.051**	0.171	**0.007**	0.042
-0.1	-0.098	-0.094	**0.002**	0.006	0.168	**0.165**	0.050	**0.047**
-0.2	-0.212	-0.204	-0.012	**-0.004**	0.179	**0.172**	0.055	**0.051**
-0.3	-0.295	-0.296	0.005	**0.004**	**0.160**	0.163	**0.044**	0.045
-0.4	-0.415	-0.396	-0.015	**0.004**	0.150	**0.146**	0.039	**0.035**
-0.5	-0.522	-0.499	-0.022	**0.001**	0.154	**0.154**	**0.038**	0.039
-0.6	-0.618	-0.577	**-0.018**	0.023	0.156	**0.148**	0.044	**0.040**
-0.7	-0.723	-0.666	**-0.023**	0.034	**0.137**	0.139	**0.031**	0.033
-0.8	-0.817	-0.731	**-0.017**	0.069	**0.112**	0.129	**0.025**	0.030
-0.9	-0.898	-0.776	**0.002**	0.124	**0.089**	0.148	**0.013**	0.040
-0.91	-0.915	-0.793	**-0.005**	0.117	**0.080**	0.139	**0.011**	0.036
-0.92	-0.919	-0.792	**0.001**	0.128	**0.074**	0.146	**0.009**	0.040
-0.93	-0.925	-0.797	**0.005**	0.133	**0.071**	0.146	**0.009**	0.042
-0.94	-0.929	-0.806	**0.011**	0.134	**0.068**	0.145	**0.009**	0.038
-0.95	-0.930	-0.793	**0.020**	0.157	**0.065**	0.164	**0.010**	0.050
-0.96	-0.933	-0.807	**0.027**	0.153	**0.062**	0.158	**0.013**	0.049
-0.97	-0.939	-0.797	**0.031**	0.173	**0.055**	0.176	**0.008**	0.054
-0.98	-0.931	-0.801	**0.049**	0.179	**0.065**	0.180	**0.018**	0.059
-0.99	-0.938	-0.784	**0.052**	0.206	**0.059**	0.207	**0.013**	0.068

TABLE B
Accuracy performance of MLE and Whittle likelihood estimator for T =100. The better performances with smaller absolute value of Bias, smaller MAE and MSE are marked in bold.

	Estimate		Bias		MAE		MSE	
θ_0	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$	$\hat{\theta}$	$\tilde{\theta}_W$
0.1	0.113	0.111	0.013	**0.011**	**0.075**	**0.075**	**0.008**	**0.008**
0.2	0.203	0.200	0.003	**0.000**	**0.074**	**0.074**	**0.009**	**0.009**
0.3	0.305	0.302	0.005	**0.002**	0.078	**0.077**	0.010	**0.009**
0.4	0.400	0.395	**0.000**	-0.005	**0.078**	**0.078**	**0.009**	**0.009**
0.5	0.505	0.497	0.005	**-0.003**	0.072	**0.070**	0.009	**0.008**
0.6	0.609	0.598	0.009	**-0.002**	**0.069**	0.070	**0.008**	**0.008**
0.7	0.708	0.690	**0.008**	-0.010	0.066	**0.065**	**0.007**	**0.007**
0.8	0.815	0.782	**0.015**	-0.018	**0.054**	0.065	**0.005**	0.007
0.9	0.909	0.858	**0.009**	-0.043	**0.044**	0.068	**0.003**	0.008
0.91	0.920	0.865	**0.010**	-0.045	**0.043**	0.067	**0.003**	0.008
0.92	0.930	0.864	**0.010**	-0.056	**0.039**	0.075	**0.002**	0.009
0.93	0.937	0.873	**0.007**	-0.057	**0.038**	0.072	**0.002**	0.009
0.94	0.946	0.875	**0.006**	-0.065	**0.036**	0.078	**0.002**	0.010
0.95	0.956	0.883	**0.006**	-0.067	**0.034**	0.076	**0.002**	0.010
0.96	0.964	0.883	**0.004**	-0.078	**0.031**	0.084	**0.001**	0.012
0.97	0.972	0.890	**0.002**	-0.080	**0.025**	0.086	**0.001**	0.013
0.98	0.979	0.887	**-0.001**	-0.093	**0.022**	0.095	**0.001**	0.015
0.99	0.983	0.894	**-0.007**	-0.096	**0.017**	0.096	**0.001**	0.015
-0.1	-0.102	-0.102	-0.002	**-0.002**	**0.078**	0.079	**0.010**	0.010
-0.2	-0.199	-0.197	**0.001**	0.003	0.080	**0.080**	0.011	**0.011**
-0.3	-0.302	-0.296	**-0.002**	0.004	**0.072**	0.072	**0.008**	0.008
-0.4	-0.405	-0.401	-0.005	**-0.001**	0.078	**0.077**	0.010	**0.009**
-0.5	-0.512	-0.504	-0.012	**-0.004**	**0.075**	0.077	**0.009**	0.009
-0.6	-0.603	-0.590	**-0.003**	0.010	**0.068**	0.069	**0.007**	0.008
-0.7	-0.710	-0.691	-0.010	**0.009**	**0.062**	0.062	0.006	**0.006**
-0.8	-0.807	-0.777	**-0.007**	0.023	**0.051**	0.061	**0.004**	0.006
-0.9	-0.913	-0.855	**-0.013**	0.045	**0.047**	0.069	**0.003**	0.008
-0.91	-0.920	-0.867	**-0.010**	0.043	**0.042**	0.066	**0.003**	0.007
-0.92	-0.929	-0.871	**-0.009**	0.049	**0.040**	0.071	**0.002**	0.008
-0.93	-0.938	-0.877	**-0.008**	0.053	**0.037**	0.068	**0.002**	0.008
-0.94	-0.948	-0.880	**-0.008**	0.060	**0.037**	0.075	**0.002**	0.010
-0.95	-0.957	-0.877	**-0.007**	0.073	**0.034**	0.084	**0.002**	0.013
-0.96	-0.964	-0.889	**-0.004**	0.071	**0.029**	0.080	**0.001**	0.011
-0.97	-0.974	-0.891	**-0.004**	0.079	**0.025**	0.082	**0.001**	0.012
-0.98	-0.979	-0.884	**0.001**	0.096	**0.021**	0.098	**0.001**	0.016
-0.99	-0.985	-0.897	**0.005**	0.093	**0.014**	0.094	**0.001**	0.014