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ESTIMATING FISHER INFORMATION IN NORMAL POPULATION WITH PRIOR INFORMATION

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1. INTRODUCTION

The normal distribution is perhaps the most important distribution as it is undeniable that, in a large number of important applications, we meet distributions, which are at least approximately normal. Such is the case, e.g., with the distributions of errors of physical and astronomical measurements, a great number of demographical, agricultural and biological distributions, etc. The central limit theorem affords a theoretical explanation of these empirical facts, for instance see Cramer (1974). Thus the estimation of parameters of a normal distribution assumes significance.

Let $x_1, x_2, ..., x_n$ be a random sample of size n drawn from a normal population, probability density function of which is given by:

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\};\ -\infty<x<\infty,\ \ -\infty<\mu<\infty,\ \ \sigma>0 \quad (1)$$

where μ is the population mean, σ^2 is the population variance and $\theta=1/\sigma^2$ is the amount of information provided by each x_i; $i=1,2,...,n$. Here the parameter under investigation is θ.

Fisher (1936) was the first who obtained an estimator for θ using Student's t-distribution, when σ^2 is unknown, as follows:

$$\hat{\theta}_F=\left(\frac{n}{n+2}\right)\frac{1}{s^2} \quad (2)$$

where $s^2=\{1/(n-1)\}\sum_{i=1}^{n}(x_i-\bar{x})^2$ is the minimum variance unbiased estimator

(MVUE) of σ^2 and $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$ is the sample mean. The bias and mean squared error (MSE) of $\hat{\theta}_F$ are respectively given by

$$\text{Bias}(\hat{\theta}_F) = \frac{6\theta}{(n+2)(n-3)} \tag{3}$$

and $$\text{MSE}(\hat{\theta}_F) = \frac{2\theta^2(n+3)(n^2-2n+10)}{(n+2)^2(n-3)(n-5)}. \tag{4}$$

It is obvious from (3) that Fisher's estimator $\hat{\theta}_F$ is biased. This led Mishra (1985) to suggest an unbiased estimator of θ as

$$\hat{\theta}_U = \left(\frac{n-3}{n-1}\right)\frac{1}{s^2} \quad \text{for } n > 3 \tag{5}$$

with $$\text{Var}(\hat{\theta}_U) = \frac{2\theta^2}{(n-5)}. \tag{6}$$

Further, motivated by Goodman (1953), Searls (1964) and Mishra (1985) obtained the minimum mean squared error (MMSE) estimator

$$\hat{\theta}_M = \left(\frac{n-5}{n-1}\right)\frac{1}{s^2} \quad \text{for } n > 5 \tag{7}$$

of θ in the class of estimators $C(1/s^2)$, C being a suitably chosen constant such that MSE of $C(1/s^2)$ is minimum. The bias and MSE of $\hat{\theta}_M$ are respectively given by

$$\text{Bias}(\hat{\theta}_M) = \frac{2\theta}{(n-3)} \tag{8}$$

and $$\text{MSE}(\hat{\theta}_M) = \frac{2\theta^2}{(n-3)}. \tag{9}$$

In many situations of practical importance, the guessed value of the parameter under study may be available either from past data or the experience gathered in due course of time. Davis and Arnold (1970) have shown that, in terms of squared error risk, the usual unbiased estimator should not necessarily be considered. They have exhibited that one can improve upon the unique best MSE estimator. In this context, Thompson (1968) considered the problem of shrinking the uniformly minimum variance unbiased estimator (UMVUE) $\hat{\xi}$ of the parameter ξ towards a natural origin ξ_0 and suggested a shrunken estimator $K\hat{\xi}+(1-K)\xi_0$, where $0 < K < 1$ is a constant. The relevance of such kind of shrunken estimators lies in the fact that, though perhaps they are biased, has smaller MSE than $\hat{\xi}$ for ξ in some interval around ξ_0. In addition, the problem of shrinking the maximum likelihood estimator (MLE) $\hat{\mu}$ of the mean μ of various populations towards a natural origin μ_0 was also studied by Mehta and Srinivasan (1971). They found their estimators to be better in an interval around μ_0.

This paper is an effort in the direction of obtaining an efficient class of shrunken estimators for the amount of information θ when a guessed value $\theta_0\ (=1/\sigma_0^2)$ of $\theta\ (=1/\sigma^2)$ is available. However, this guessed value θ_0 might not be cogent. The further object of this paper is to resolve this problem to an appreciable extent by applying preliminary testing procedure. Moreover, the properties of the suggested class of shrunken estimators and preliminary test estimators are discussed theoretically and empirically. In particular, some estimators of the suggested class are used in estimating the precision of sample mean.

2. THE SUGGESTED CLASS OF SHRUNKEN ESTIMATORS

Motivated by Jani (1991), Kourouklis (1994) and Singh and Saxena (2002), we define a class of estimators $\theta^*_{(p,q)}$ for θ in model (1) as

$$\theta^*_{(p,q)}=\theta_0[q+w(\theta_0 s^2)^p], \tag{10}$$

where p and q are real numbers such that $p\neq 0$, $0<q<\infty$, and w is a stochastic variable which may in particular be a scalar to be chosen such that MSE$(\theta^*_{(p,q)})$ is minimum.

Assuming w as a scalar, the MSE of $\theta^*_{(p,q)}$ is obtained by using the result: $E((s^2)^{jp})=K_{jp}\,\theta^{-jp}\ (j=1,2)$, as

$$\text{MSE}(\theta^*_{(p,q)})=\theta^2[(q\lambda-1)^2+w^2\lambda^{2(p+1)}K_{2p}+2(q\lambda-1)w\lambda^{(p+1)}K_{1p}] \tag{11}$$

where $\lambda=(\theta_0/\theta)$ and $K_{jp}=\left(\dfrac{2}{n-1}\right)^{jp}\left\{\Gamma\left(\dfrac{n+2jp-1}{2}\right)\Big/\Gamma\left(\dfrac{n-1}{2}\right)\right\}$.

Now, minimizing (11) with respect to w and replacing θ by its unbiased estimator $\hat{\theta}_U$, we get

$$\hat{w} = \frac{-[q\theta_0 s^2(n-1)-(n-3)]\,(n-3)^p}{[\theta_0 s^2(n-1)]^{p+1}} w_{(n,p)} \tag{12}$$

where $w_{(n,p)} = \dfrac{K_{1p}}{K_{2p}} = \left(\dfrac{n-1}{2}\right)^p \left\{\Gamma\left(\dfrac{n+2p-1}{2}\right)\Big/\Gamma\left(\dfrac{n+4p-1}{2}\right)\right\}$.

Substitution of (12) in (10) yields a class of shrunken estimators for θ as

$$\hat{\theta}_{(p,q)} = w'_{(n,p)}\, s^{-2}\left(\frac{n-3}{n-1}\right) + \{1-w'_{(n,p)}\}q\theta_0 \quad \text{for } n>3 \tag{13}$$

where $w'_{(n,p)} = \left[\left(\dfrac{n-3}{n-1}\right)^p w_{(n,p)}\right] = \left(\dfrac{n-3}{2}\right)^p \left\{\Gamma\left(\dfrac{n+2p-1}{2}\right)\Big/\Gamma\left(\dfrac{n+4p-1}{2}\right)\right\}$

lies between 0 and 1 {i.e., $0 < w'_{(n,p)} \le 1$} provided gamma functions exist, i.e., $p > (1-n)/2$.

It is observed from (13) that the proposed class of estimators $\hat{\theta}_{(p,q)}$ is the convex combination of $(n-3)/(n-1)s^2$ and $q\theta_0$, and hence $\hat{\theta}_{(p,q)}$ is non-negative as $n > 3,\ q > 0,\ s^{-2} > 0$ and $\theta_0 > 0$.

2.1. *Choices of scalars p and q*

The convex nature of proposed statistic $\hat{\theta}_{(p,q)}$ provides the criterion of selection of the scalar *p*. Therefore; the acceptable range of value of *p* is given by

$$\{p \mid 0 < w'_{(n,p)} \le 1 \ \textit{and} \ \ p > (1-n)/2\}\ \forall\ n. \tag{14}$$

If $w'_{(n,p)} = 1$, the proposed class of shrunken estimators turns out to be the unbiased estimator $\hat{\theta}_U$, otherwise it is biased with

$$\text{Bias}\{\hat{\theta}_{(p,q)}\} = \theta(q\lambda-1)(1-w'_{(n,p)}), \tag{15}$$

and thus the absolute relative bias of $\hat{\theta}_{(p,q)}$ is given by

$$\text{ARB}\{\hat{\theta}_{(p,q)}\} = \left|(q\lambda-1)(1-w'_{(n,p)})\right|. \tag{16}$$

The condition for unbiasedness that $w'_{(n,p)} = 1$, holds iff, sample size *n* is indefinitely large, i.e., $n \to \infty$. Moreover, if the proposed class of estimators $\hat{\theta}_{(p,q)}$

turns into $\hat{\theta}_U$ then this case does not deal with the use of prior information. A more realistic condition for unbiasedness without damaging the basic structure of $\hat{\theta}_{(p,q)}$ and utilizes prior information intelligibly can be obtained by (16). The ARB of $\hat{\theta}_{(p,q)}$ is zero when $q = \lambda^{-1}$ (or $\lambda = q^{-1}$).

The relative mean squared error (RMSE) of $\hat{\theta}_{(p,q)}$ is derived as

$$\text{RMSE}\{\hat{\theta}_{(p,q)}\} = (q\lambda - 1)^2 (1 - w'_{(n,\,p)})^2 + \frac{2}{(n-5)} w'^2_{(n,\,p)} . \tag{17}$$

It is obvious from (17) that the RMSE of $\hat{\theta}_{(p,q)}$ is minimum when $q = \lambda^{-1}$ (or $\lambda = q^{-1}$). Thus we see that at $q = \lambda^{-1}$, the suggested class of estimators is not only unbiased but renders maximum gain in efficiency, which is a remarkable property of the suggested class of estimators. Consequently, to obtain a substantial gain in efficiency as well as proportionately small magnitude of bias for fixed λ, one should select q in the vicinity of $q = \lambda^{-1}$. It is interesting to note that if one selects smaller values of *q* then higher values of λ will lead to a large gain in efficiency along with appreciable smaller magnitude of bias and *vice-versa.* This implies that for smaller values of *q*, the proposed class of estimators allows to choose guessed value much away from the true value, i.e., even if the experimenter has less confidence in the guessed value, the risk of using the suggested class of estimators is not higher. This is legitimate for all values of *p*.

The quantity $\lambda = (\theta_0 / \theta)$ represents the departure of natural origin θ_0 from the true value θ. But in practical situation it is hardly possible to get any idea about λ. As a result, an unbiased estimator of λ is proposed, namely

$$\hat{\lambda} = \theta_0\, s^2 \left(\frac{n-1}{2} \right) \frac{\Gamma[(n-1)/2]}{\Gamma[(n+1)/2]} . \tag{18}$$

It is being observed that if $q = \lambda^{-1}$, the suggested class of estimators yields favourable results. Keeping in view of this concept, one may select *q* as

$$q = \hat{\lambda}^{-1} = \theta_0^{-1}\, s^{-2} \left(\frac{2}{n-1} \right) \frac{\Gamma[(n+1)/2]}{\Gamma[(n-1)/2]} . \tag{19}$$

Here this is fit for being quoted that this is the criterion of selecting *q* numerically and one should carefully notice that this does not mean *q* is replaced by (19) in $\hat{\theta}_{(p,q)}$.

3. THEORITICAL COMPARISION OF THE PROPOSED CLASS OF ESTIMATORS AND NUMERICAL INVESTIGATION

James and Stein (1961) pointed out that minimum MSE is a highly desirable property and it is therefore used as a criterion to compare different estimators with each other. The conditions under which the suggested class of estimators is better than the unbiased estimator and the MMSE estimator are given below:

It follows from (6) and (17) that $\mathrm{MSE}(\hat{\theta}_{(p,q)})$ does not exceed $\mathrm{Var}(\hat{\theta}_U)$ if

$$(1-\sqrt{M})\, q^{-1} < \lambda < (1+\sqrt{M})\, q^{-1} \tag{20}$$

or equivalently,

$$(1-\sqrt{M})\, \lambda^{-1} < q < (1+\sqrt{M})\, \lambda^{-1} \tag{21}$$

where $M = \dfrac{2}{(n-5)}\left\{\dfrac{1+w'_{(n,p)}}{1-w'_{(n,p)}}\right\}$ for $n > 5$.

In a similar fashion we note from (9) and (17) that $\hat{\theta}_{(p,q)}$ has smaller MSE than that of MMSE estimator $\hat{\theta}_M$ if

$$(1-\sqrt{L})\, q^{-1} < \lambda < (1+\sqrt{L})\, q^{-1} \tag{22}$$

or equivalently,

$$(1-\sqrt{L})\, \lambda^{-1} < q < (1+\sqrt{L})\, \lambda^{-1} \tag{23}$$

where $L = \dfrac{2}{(1-w'_{(n,p)})^2}\left\{\dfrac{1}{(n-3)} - \dfrac{w'^2_{(n,p)}}{(n-5)}\right\}$ provided $n > 5$.

Besides minimum MSE criterion, minimum bias is also important and therefore should be considered under study. Thus we note from (8) and (16) that $\mathrm{ARB}(\hat{\theta}_{(p,q)})$ is less than $\mathrm{ARB}(\hat{\theta}_M)$ if

$$\left[\frac{(n-3)(1-w'_{(n,p)})-2}{q(n-3)(1-w'_{(n,p)})}\right] < \lambda < \left[\frac{(n-3)(1-w'_{(n,p)})+2}{q(n-3)(1-w'_{(n,p)})}\right]. \tag{24}$$

In order to have a tangible idea about the performance of the proposed class of estimators $\hat{\theta}_{(p,q)}$ against the MMSE estimator $\hat{\theta}_M$, Percent Relative Efficiencies (PREs) of $\hat{\theta}_{(p,q)}$ with respect to $\hat{\theta}_M$ have been computed by the formula:

$$\mathrm{PRE}(\hat{\theta}_{(p,q)}, \hat{\theta}_M) = \frac{(n-5)}{(n-3)[(n-5)(q\lambda-1)^2(1-w'_{(n,p)})^2 + 2w'^2_{(n,p)}]} \times 100\,. \tag{25}$$

TABLE 1

PREs of Proposed Class of Estimators $\hat{\theta}_{(p,q)}$ with respect to MMSE Estimator $\hat{\theta}_M$

$q\downarrow$	$p\rightarrow$	-2				-1			
	$n\rightarrow$	6	12	18	24	6	12	18	24
	$w'_{(n,p)}\rightarrow$ $\lambda\downarrow$	0.0319	0.1837	0.4400	0.5782	0.4180	0.7777	0.8667	0.9047
0.05	0.5	74.65	34.56	40.66	46.62	99.29	101.11	100.66	100.47
	5	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	10	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	15	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	20*	32780.5	2303.73	447.64	270.60	190.75	128.61	115.38	110.53
	25	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	30	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	35	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	40	70.98	32.88	38.83	44.69	96.87	100.00	100.00	100.00
	Range of λ	*(3.1, 36.8)*	*(8.7, 31.3)*	*(8.5, 31.4)*	*(8.3, 31.6)*	*(0.6, 39.3)*	*(0.0, 40.0)*	*(0.0, 40.0)*	*(0.0, 40.0)*
0.25	0.5	92.64	42.75	49.41	55.57	109.50	105.50	103.23	102.28
	1	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	2	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	3	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	4*	32780.5	2303.73	447.64	270.60	190.75	128.61	115.38	110.53
	5	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	6	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	7	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	8	70.98	32.88	38.83	44.69	96.87	100.00	100.00	100.00
	Range of λ	*(0.6, 7.3)*	*(1.7, 6.2)*	*(1.7, 6.3)*	*(1.6, 6.3)*	*(0.1, 7.8)*	*(0.0, 8.0)*	*(0.0, 8.0)*	*(0.0, 8.0)*
0.50	0.05	74.65	34.56	40.66	46.62	99.29	101.11	100.66	100.47
	0.5	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	1	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	1.5	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	2*	32780.5	2303.73	447.64	270.60	190.75	128.61	115.38	110.53
	2.5	1099.91	433.28	269.99	205.64	179.86	126.35	114.28	109.81
	3	282.07	126.11	123.25	119.54	153.55	120.03	111.11	107.69
	3.5	125.97	57.81	64.67	70.41	123.46	110.78	106.19	104.35
	4	70.98	32.88	38.83	44.69	96.87	100.00	100.00	100.00
	Range of λ	*(0.3, 3.6)*	*(0.8, 3.1)*	*(0.8, 3.1)*	*(0.8, 3.1)*	*(0.0, 3.9)*	*(0.0, 4.0)*	*(0.0, 4.0)*	*(0.0, 4.0)*

* Point of attaining maximum PRE, i.e., $\lambda = q^{-1}$.

TABLE 1 (continue)

$q\downarrow$	$p\rightarrow$	1				2			
	$\lambda\downarrow$ $n\rightarrow$	6	12	18	24	6	12	18	24
	$w'_{(n,p)}\rightarrow$	0.4286	0.6923	0.7895	0.8424	0.0909	0.3176	0.4660	0.5853
0.05	0.5	98.36	97.92	96.61	96.88	83.11	47.14	43.78	47.72
	5	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	10	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	15	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	20*	181.42	162.28	139.05	127.49	4033.53	770.84	399.17	264.13
	25	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	30	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	35	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	40	96.07	95.94	95.10	95.68	79.08	44.94	41.85	45.77
	Range of λ	*(0.8, 39.1)*	*(1.0, 38.9)*	*(1.6, 38.3)*	*(1.8, 38.1)*	*(2.2, 37.7)*	*(7.1, 32.8)*	*(8.1, 31.8)*	*(8.2, 31.7)*
0.25	0.5	107.98	106.11	102.71	101.63	102.68	57.67	52.96	56.78
	1	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	2	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	3	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	4*	181.42	162.28	139.05	127.49	4033.53	770.84	399.17	264.13
	5	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	6	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	7	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	8	96.07	95.94	95.10	95.68	79.08	44.94	41.85	45.77
	Range of λ	*(0.1, 7.8)*	*(0.2, 7.8)*	*(0.3, 7.6)*	*(0.3, 7.6)*	*(0.4, 7.5)*	*(1.4, 6.5)*	*(1.6, 6.3)*	*(1.6, 6.3)*
0.50	0.05	98.36	97.92	96.61	96.88	83.11	47.14	43.78	47.72
	0.5	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	1	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	1.5	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	2*	181.42	162.28	139.05	127.49	4033.53	770.84	399.17	264.13
	2.5	171.88	155.56	135.14	124.90	977.79	383.61	260.28	203.47
	3	148.45	138.36	124.65	117.71	298.77	153.01	127.34	120.47
	3.5	120.97	116.84	110.36	107.41	138.48	76.44	68.79	71.71
	4	96.07	95.94	95.10	95.68	79.08	44.94	41.85	45.77
	Range of λ	*(0.0, 3.9)*	*(0.1, 3.9)*	*(0.1, 3.8)*	*(0.1, 3.8)*	*(0.2, 3.7)*	*(0.7, 3.2)*	*(0.8, 3.1)*	*(0.8, 3.1)*

TABLE 2

ARBs of proposed estimator $\hat{\theta}_{(p,q)}$ and MMSE estimator $\hat{\theta}_M$

$q\downarrow$	$p\rightarrow$	-2				-1			
	$n\rightarrow$	6	12	18	24	6	12	18	24
0.05	0.5	0.94	0.80	0.55	0.41	0.57	0.22	0.13	0.09
	5	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	10	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	15	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	20**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	25	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	30	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	35	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	40	0.97	0.82	0.56	0.42	0.58	0.22	0.13	0.10
	Range of λ	*(6.2, 33.7)*	*(14.5, 25.4)*	*(15.2, 24.7)*	*(15.4, 24.5)*	*(0, 42.91)*	*(0.01, 39.9)*	*(0, 40)*	*(0, 40)*
0.25	0.5	0.85	0.71	0.49	0.37	0.51	0.19	0.12	0.08
	1	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	2	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	3	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	4**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	6	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	7	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	8	0.97	0.82	0.56	0.42	0.58	0.22	0.13	0.10
	Range of λ	*(1.25, 6.75)*	*(2.91, 5.09)*	*(3.05, 4.95)*	*(3.10, 4.90)*	*(0, 8.58)*	*(0, 8)*	*(0, 8)*	*(0, 8)*
0.50	0.05	0.94	0.80	0.55	0.41	0.57	0.22	0.13	0.09
	0.5	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	1	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	1.5	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	2**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2.5	0.24	0.20	0.14	0.11	0.15	0.06	0.03	0.02
	3	0.48	0.41	0.28	0.21	0.29	0.11	0.07	0.05
	3.5	0.73	0.61	0.42	0.32	0.44	0.17	0.10	0.07
	4	0.97	0.82	0.56	0.42	0.58	0.22	0.13	0.10
	Range of λ	*(0.62, 3.38)*	*(1.46, 2.54)*	*(1.54, 2.48)*	*(1.55, 2.45)*	*(0, 4.29)*	*(0, 4)*	*(0, 4)*	*(0, 4)*
ARB of $\hat{\theta}_M \rightarrow$		0.67	0.22	0.13	0.10	0.67	0.22	0.13	0.10

** Point of attaining unbiasedness, i.e., $\lambda = q^{-1}$

TABLE 2 (continue)

$q\downarrow$	$p\rightarrow$	1				2			
	$n\rightarrow$	6	12	18	24	6	12	18	24
0.05	0.5	0.56	0.30	0.21	0.15	0.89	0.67	0.52	0.40
	5	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	10	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	15	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	20**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	25	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	30	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	35	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	40	0.57	0.31	0.21	0.16	0.91	0.68	0.53	0.41
	Range of λ	*(0, 43.34)*	*(5.56, 34.4)*	*(7.33, 32.6)*	*(7.91, 32.0)*	*(5.33, 34.6)*	*(13.4, 26.5)*	*(15.0, 24.9)*	*(15.4, 24.5)*
0.25	0.5	0.50	0.27	0.18	0.14	0.80	0.60	0.47	0.36
	1	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	2	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	3	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	4**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	6	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	7	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	8	0.57	0.31	0.21	0.16	0.91	0.68	0.53	0.41
	Range of λ	*(0, 8.67)*	*(1.11, 6.89)*	*(1.47, 6.53)*	*(1.58, 6.42)*	*(1.07, 6.93)*	*(2.7, 5.3)*	*(3, 5)*	*(3.08, 4.92)*
0.50	0.05	0.56	0.30	0.21	0.15	0.89	0.67	0.52	0.40
	0.5	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	1	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	1.5	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	2**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2.5	0.14	0.08	0.05	0.04	0.23	0.17	0.13	0.10
	3	0.29	0.15	0.11	0.08	0.45	0.34	0.27	0.21
	3.5	0.43	0.23	0.16	0.12	0.68	0.51	0.40	0.31
	4	0.57	0.31	0.21	0.16	0.91	0.68	0.53	0.41
	Range of λ	*(0, 4.33)*	*(0.56, 3.44)*	*(0.73, 3.27)*	*(0.79, 3.21)*	*(0.53, 3.47)*	*(1.35, 2.65)*	*(1.5, 2.5)*	*(1.54, 2.46)*
ARB of $\hat{\theta}_M \rightarrow$		0.67	0.22	0.13	0.10	0.67	0.22	0.13	0.10

The PREs of $\hat{\theta}_{(p,q)}$ with respect to $\hat{\theta}_M$ and ARBs of $\hat{\theta}_{(p,q)}$ and $\hat{\theta}_M$ have been computed for n = 6(6)24, $p = \pm 1, \pm 2$, q = 0.05, 0.25, 0.50 and different values of λ and compiled in tables 1 and 2 with corresponding values of $w'_{(n,p)}$ and range of dominance of λ. For calculating gamma functions contained in $w'_{(n,p)}$, Gauss-Laguerre 5-point integration method has been used. It has been observed from tables 1 and 2 that if n, p, q are fixed at some values, the relative efficiency of proposed class of shrunken estimators increases up to $\lambda = q^{-1}$, attains its maximum at this point and then decreases symmetrically in magnitude, as λ increases either sides in its range of dominance for all *n*, *p* and *q*. On the other hand, the ARBs of the proposed class of estimators decreases up to $\lambda = q^{-1}$, the proposed estimator becomes unbiased at this point and then the ARBs increases as λ increases in its range of dominance for all *n*, *p* and *q*, see figure 1.

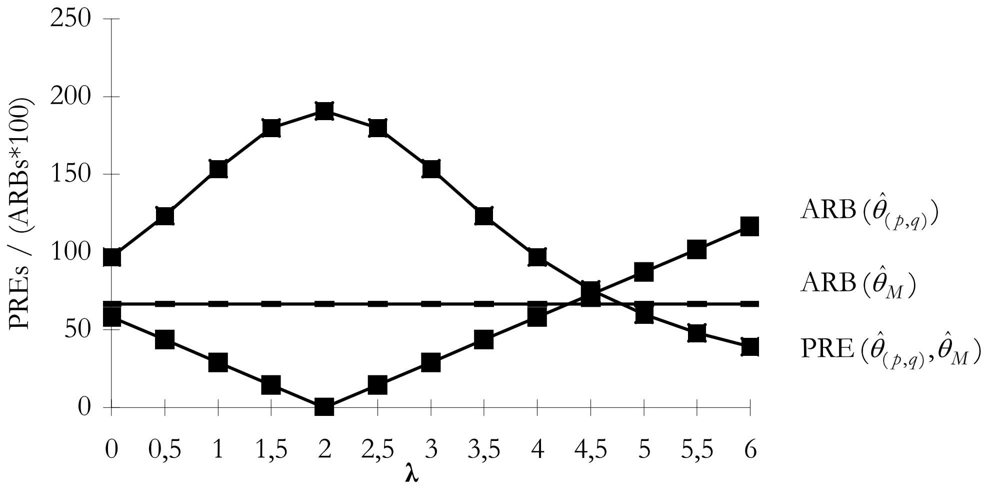

Figure 1 – Effect of departure of guessed value from true value on Percent Relative Efficiency and Absolute Relative Bias.

The effect of change in sample size *n* is also a matter of great interest. For fixed *p*, *q* and λ the gain in relative efficiency decreases with increment in *n*, i.e., the proposed class of shrunken estimators is beneficial especially for small samples. Besides, it appears that for getting better estimators in the class, the value of $w'_{(n,p)}$ should be as small as possible in the interval (0,1]. Thus, for choosing *p* one should not consider the smaller values of $w'_{(n,p)}$ in isolation, but also the wider length of effective interval of λ, see figure 2.

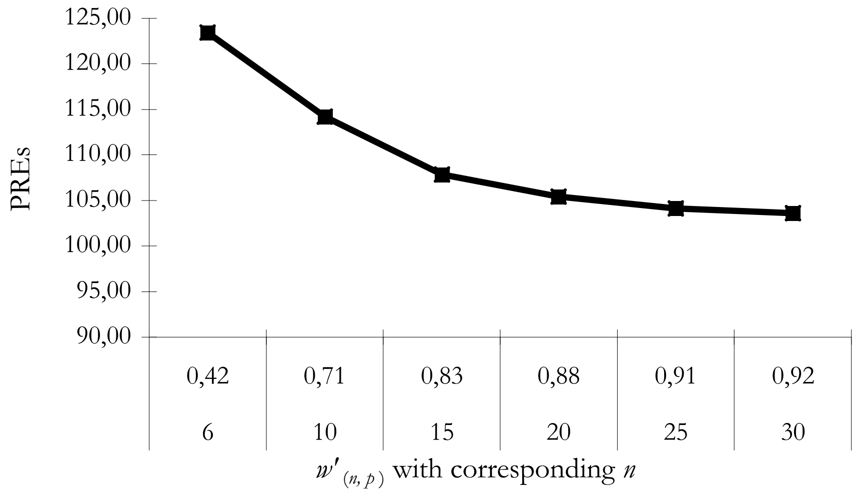

Figure 2 – Effect of sample size on gain in efficiency.

Figures 1 and 2 show the general characteristics of the proposed class of shrunken estimators. The trends revealed by these graphs are agreed for all values of scalars comprehend in the proposed class and hence can be extrapolated in general.

The proposed class of shrunken estimators $\hat{\theta}_{(p,q)}$ so far discussed above is entirely based on the validity of the prior or guessed value θ_0. Nevertheless, situations frequently arise when the available information about the parameter is inadequate or of uncertain validity. In such a case the problem is to decide whether to use this information or not. Thus, to test the accuracy of the guessed value, we applied preliminary testing procedure pioneered by Bancroft (1944) in the next section.

4. FORMULATION OF A CLASS OF SHRUNKEN TESTIMATORS

The preliminary test estimators were first ascribed as 'testimators' by Scolve *et al.* (1972). Here we propose a two-tailed preliminary test of significance by employing the suggested class of shrunken estimators $\hat{\theta}_{(p,q)}$. The competing hypotheses are $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ and the appropriate test statistic under H_0 is given by $t = (n-1)s^2\theta_0 \sim \chi^2_{(n-1)}$. Clearly, t is a chi-square variate with (n-1) degrees of freedom having density

$$f(t,\delta) = \frac{\delta^{(n-1)/2}}{2^{(n-1)/2}\Gamma[(n-1)/2]} e^{-t\delta/2} t^{(n-3)/2}; \quad 0 \leq t \leq \infty \tag{26}$$

where $\delta = \lambda^{-1} (= \theta/\theta_0) > 0$.

The suggested class of preliminary test estimators is thus given by:

$$\tilde{\theta}_{(p,q)} = \begin{cases} w'_{(n,p)} s^{-2} \left(\dfrac{n-3}{n-1}\right) + (1 - w'_{(n,p)}) q\theta_0 & \text{if } t \in R \\ \left(\dfrac{n-5}{n-1}\right) s^{-2} & \text{if } t \in R^C \end{cases} \tag{27}$$

where R is called the 'Acceptance Region' defined as the region of accepting H_0 when H_0 is true, i.e., the region of the outcome set where H_0 is accepted if the sample point falls in that region. On the other hand, its complement, R^C, is called the 'Rejection Region' or 'Critical Region'.

It is implied from (27), if the null hypothesis H_0 is accepted we use the proposed class of shrunken estimators $\hat{\theta}_{(p,q)}$, otherwise we use the MMSE estimator $\hat{\theta}_M$ as an estimator of θ. If α is a pre-assigned level of significance, the hypothesis H$_0$ is accepted if $r_1 \le t \le r_2$, where r_1 and r_2 are such that $P[t < r_1] = \alpha/2$ and $P[t < r_2] = 1-(\alpha/2)$ respectively and thus may be defined as $r_1 = \chi^2[(n-1);(\alpha/2)]$ and $r_2 = \chi^2[(n-1);1-(\alpha/2)]$ respectively. The region R is then defined as $R = [r_1, r_2]$.

The MSE of the proposed class of shrunken testimators is given by

$$\begin{aligned}
\text{MSE}\{\tilde{\theta}_{(p,q)}\} &= \int_R \left\{ w'_{(n,p)}\, s^{-2}\left(\frac{n-3}{n-1}\right) + (1-w'_{(n,p)})q\theta_0 \right\}^2 f(t,\delta)\mathrm{d}t \\
&\quad + \int_{R^C} \left\{ \left(\frac{n-5}{n-1}\right) s^{-2} - \theta \right\}^2 f(t,\delta)\mathrm{d}t \\
&= \theta^2 \int_{r_1}^{r_2} \left\{ \frac{(n-3)w'_{(n,p)}}{\delta t} + \frac{(1-w'_{(n,p)})q}{\delta} - 1 \right\}^2 f(t,\delta)\mathrm{d}t \\
&\quad + MSE\{\hat{\theta}_M\} - \theta^2 \int_{r_1}^{r_2} \left\{ \frac{n-5}{\delta t} - 1 \right\}^2 f(t,\delta)\mathrm{d}t \\
&= \theta^2 \left[\frac{\delta^{(n-1)/2}}{2^{(n-1)/2}\Gamma[(n-1)/2]} \cdot \right. \\
&\quad \cdot \int_{r_1}^{r_2} \left\{ \frac{(n-3)w'_{(n,p)}}{\delta t} + \frac{(1-w'_{(n,p)})q}{\delta} - 1 \right\}^2 \exp(-t\delta/2)\, t^{(n-3)/2}\mathrm{d}t \\
&\quad + \frac{2}{(n-3)} - \frac{\delta^{(n-1)/2}}{2^{(n-1)/2}\Gamma[(n-1)/2]} \cdot \\
&\quad \left. \cdot \int_{r_1}^{r_2} \left\{ \frac{n-5}{\delta t} - 1 \right\}^2 \exp(-t\delta/2)\, t^{(n-3)/2}\mathrm{d}t \right]
\end{aligned} \tag{28}$$

5. COMPARISION OF SUGGESTED CLASS OF PRELIMINARY TEST ESTIMATORS AND NUMERICAL ILLUSTRATIONS

The suggested class of shrunken testimators $\tilde{\theta}_{(p,q)}$ performs better than the unbiased estimator $\hat{\theta}_U$ in terms of efficiency if

$$A(n-5) < 2, \tag{29}$$

where A is the quantity contained in (28) in square brackets. Similarly, $\tilde{\theta}_{(p,q)}$ will be more efficient than the MMSE estimator $\hat{\theta}_M$ if

$$A(n-3)<2\,. \tag{30}$$

To have a perceptible idea about the performance of the suggested class of shrunken testimators, the fraction relative improvement (FRI) of $\tilde{\theta}_{(p,q)}$ over $\hat{\theta}_M$ have been computed by the formula:

$$\text{FRI}\,(\tilde{\theta}_{(p,q)};\hat{\theta}_M)=1-\frac{\text{MSE}(\tilde{\theta}_{(p,q)})}{\text{MSE}(\hat{\theta}_M)}=\left[1-\frac{A(n-3)}{2}\right]. \tag{31}$$

The FRIs of $\tilde{\theta}_{(p,q)}$ with respect to $\hat{\theta}_M$ for n = 6(6)24, q = 2, p = $\pm 1,\pm 2$ and different values of δ at 0.01 and 0.05 level of significance have been computed and evinced in table 3. The value of the integrals contained in A can be evaluated by using a quadrature formula. We have used 5-point Gauss-Legender for the same. The value of gamma function can be assessed as before.

TABLE 3

Fraction Relative Improvement of $\tilde{\theta}_{(p,q)}$ over $\hat{\theta}_M$ (in percentage)

$p\downarrow$	q = 2								
	$\delta\downarrow$	α = 0.01				α = 0.05			
	$n\rightarrow$	6	12	18	24	6	12	18	24
	1.35	47.46	7.33	12.97	15.93	49.37	5.92	19.96	27.89
	1.5	63.73	40.68	38.46	36.59	65.67	38.67	44.46	46.90
	2	77.59	65.57	54.33	46.55	79.15	62.67	52.64	42.74
	2.5	71.34	46.81	35.30	26.98	72.04	47.76	30.83	17.90
-2	3	62.38	26.22	19.15	12.52	61.36	30.54	14.01	5.20
	3.5	54.71	13.52	10.19	4.94	51.22	18.12	5.47	1.18
	4	48.77	7.16	5.24	1.62	42.54	10.32	1.90	0.22
	4.5	44.07	4.12	2.50	0.44	35.33	5.69	0.59	0.03
	5	40.08	2.54	1.09	0.11	29.36	3.04	0.17	0.00
	1	16.55	4.12	2.55	1.94	27.43	11.19	7.34	5.43
	1.5	63.84	31.20	23.86	21.41	72.86	43.25	35.40	31.70
	2	69.64	36.92	30.63	27.38	74.24	43.69	32.79	24.94
	2.5	67.53	35.06	27.33	20.58	67.18	34.89	20.29	10.88
-1	3	64.20	31.48	20.75	11.44	58.66	24.58	9.83	3.28
	3.5	61.11	27.56	13.67	4.86	50.52	15.95	4.01	0.76
	4	58.23	23.01	7.76	1.63	43.22	9.71	1.43	0.14
	4.5	55.24	18.05	3.85	0.45	36.82	5.59	0.45	0.02
	5	51.95	13.29	1.71	0.11	31.23	3.07	0.13	0.00
	1.25	50.57	26.39	20.15	17.30	61.47	36.08	30.77	28.34
	1.5	63.46	38.46	31.06	27.67	72.76	47.94	41.50	37.95
	2	69.23	43.22	36.06	31.69	73.99	47.86	37.34	28.94
	2.5	67.30	37.85	29.10	21.91	66.99	37.17	22.40	12.32
1	3	64.17	31.37	20.57	11.66	58.55	25.56	10.63	3.66
	3.5	61.23	25.95	13.07	4.88	50.48	16.30	4.28	0.84
	4	58.44	20.99	7.31	1.63	43.23	9.81	1.51	0.16
	4.5	55.52	16.20	3.61	0.45	36.86	5.61	0.48	0.02
	5	52.25	11.84	1.60	0.11	31.28	3.06	0.14	0.00
	1.3	44.33	12.56	4.65	6.84	41.11	13.65	11.80	18.55
	1.5	65.52	46.92	39.33	36.72	68.16	47.29	45.63	47.03
	2	77.25	62.20	53.33	46.22	78.95	60.44	51.81	42.44
	2.5	71.29	45.72	34.94	26.86	71.55	45.46	30.27	17.76
2	3	63.03	28.32	19.28	12.50	61.07	29.37	13.77	5.16
	3.5	55.96	17.26	10.41	4.94	51.18	17.68	5.38	1.17
	4	50.43	11.10	5.40	1.62	42.68	10.20	1.87	0.22
	4.5	45.94	7.48	2.58	0.44	35.58	5.68	0.59	0.03
	5	42.02	5.10	1.13	0.11	29.66	3.05	0.17	0.00

From table 3 it is seen that for fixed *n*, *p*, *q*, the fraction relative improvement of the proposed class of preliminary test estimators over the MMSE estimator increases up to a certain point lies in the closed proximity of $q = \delta$, i.e., $q = \lambda^{-1}$ attains its maximum at this point and then gradually decreases as the guessed value departs from the true value, i.e., as δ increases. On the other hand, when δ, *p*, *q* are fixed, the FRI abates as *n* multiplies, i.e., there is a substantial gain in efficiency for small sample sizes; while for large sample sizes, the gain in efficiency is marginal. But if δ is very small, the FRIs first decrease and after attaining its minimum at some *n*, starts increasing as n increases.

Thus we find that even after resolving uncertainty of guessed value, the condition of rendering massive gain in relative efficiency remains almost unchanged. The only difference observed in this case, is the point of procuring maximum relative efficiency shifts slightly from $q = \lambda^{-1}$ but still it lies in the closed proximity of $q = \lambda^{-1}$.

6. ESTIMATION OF PRECISION OF SAMPLE MEAN

Denoting precision by $\wp$, the precision of sample mean $\overline{x}$ is given by

$$\wp(\overline{x}) = 1\big/\mathrm{Var}(\overline{x}) = 1\big/(\sigma^2/n) = n\theta$$

Our problem is to estimate this precision using estimators for amount of information. Its estimate is therefore given by

$$\widehat{\wp(\overline{x})} = n\hat{\theta}.$$

We obtained the following estimators for precision of sample mean:

(i) Unbiased estimator

$$\widehat{\wp(\overline{x})_U} = n\left(\frac{n-3}{n-1}\right)\frac{1}{s^2} \text{ ; for } n > 3. \tag{32}$$

having $\mathrm{Var}\left\{\widehat{\wp(\overline{x})_U}\right\} = \dfrac{2n^2\theta^2}{(n-5)}$. (33)

(ii) MMSE estimator

$$\widehat{\wp(\overline{x})_M} = n\left(\frac{n-5}{n-1}\right)\frac{1}{s^2} \text{ for } n > 5. \tag{34}$$

with $\mathrm{Bias}\left\{\widehat{\wp(\overline{x})_M}\right\} = \dfrac{-2n\theta}{(n-3)}$ (35)

and $\text{MSE}\left\{\widehat{\wp(\bar{x})_M}\right\} = \dfrac{n^2(2n-6)\theta^2}{(n-3)^2}$. (36)

(iii) Shrunken estimator based on suggested class of estimators

$$\widehat{\wp(\bar{x})_{(p,q)}} = n\left[w'_{(n,p)}\, s^{-2}\left(\frac{n-3}{n-1}\right) + (1-w'_{(n,p)})q\theta_0\right] \text{ for } n > 3 \tag{37}$$

having

$$\text{Bias}\left\{\widehat{\wp(\bar{x})_{(p,q)}}\right\} = \theta\left[n(q\lambda-1)(1-w'_{(n,p)})\right] \tag{38}$$

and

$$\text{MSE}\left\{\widehat{\wp(\bar{x})_{(p,q)}}\right\} = \theta^2\left[n^2(q\lambda-1)^2(1-w'_{(n,p)})^2 + \frac{2n^2(w'_{(n,p)})^2}{(n-5)}\right]. \tag{39}$$

The effective interval of λ in which the proposed estimator $\widehat{\wp(\bar{x})_{(p,q)}}$ is more efficient than unbiased estimator $\widehat{\wp(\bar{x})_U}$ is the same as given in (20) and that of MMSE estimator $\widehat{\wp(\bar{x})_M}$ is given by

$$(1-\sqrt{D})q^{-1} < \lambda < (1+\sqrt{D})q^{-1} \tag{40}$$

where $D = \dfrac{1}{(1-w'_{(n,p)})^2}\left[\dfrac{2n-6}{(n-3)^2} - \dfrac{2w'^2_{(n,p)}}{(n-5)}\right]$.

7. APPLICATIVE EXAMPLE

To emphasize the application of the suggested class of shrunken estimators an example is considered below from Box and Tiao (1973) wherein our interest is to estimate the precision of sample mean.

A wheat researcher is studying the yield of a certain variety of wheat in the state of Madhya Pradesh, India. He has at his disposal 15 farms each of size 4 acres scattered throughout the state on which he can plant the wheat and observe the yield. The yield (in quintals) were recorded as order statistics as 13.4, 14.2, 28.8, 29.0, 29.8, 33.0, 37.8, 39.6, 43.4, 49.8, 54.8, 58.2, 67.4, 70.2 and 91.2. Assuming that the yields to be distributed normally $\text{N}(\mu, \sigma^2)$, where μ is the true average yield and σ^2 is specified from some similar study in the past as 350. Here n = 15, θ_0 = 0.0028571, $\bar{x}$ = 44.04, s^2 = 462.49. The departure of the guessed value from the true value is estimated as $\hat{\lambda} = 1.3213$. Now q has been chosen as $q = \hat{\lambda}^{-1} = 0.7567$. For this data set we have chosen $p = -2$ for which $w'_{(n,p)} =$ 0.3333. The effective intervals of λ in which the suggested estimator $\hat{\theta}_{(-2,\,0.7567)}$ is more efficient than the unbiased estimator and the MMSE estimator are comes out to be (0.4857, 2.1573) and (0.5681, 2.0748) respectively. Just for an illustration it is assumed that λ = 1.5. Now for preliminary testing procedure, t = 18.4993 and R = (4.0747, 31.3193). Clearly, $t \in R$ implying that θ_0 = 0.0028571 could be used to estimate θ. The final findings are displayed in table 4.

TABLE 4

Estimates, ARBs, RMSEs and PREs of competitive estimators for precision of average yield of wheat

Estimators→ *Characteristics*↓	Unbiased $\widehat{\wp(\overline{x})}_U$	MMSE $\widehat{\wp(\overline{x})}_M$	Proposed $\widehat{\wp(\overline{x})}_{(-2,\,0.7567)}$
Estimate	0.027799	0.023166	0.030885
ARB	0.0000	2.5000	1.3505
RMSE	45.0000	37.5000	6.8230
PRE of proposed estimator w.r.t.	659.53 %	549.61 %	100.00 %

It has been perceived from table 4 that

$$\text{ARB}\left\{\widehat{\wp(\overline{x})}_U\right\} < \text{ARB}\left\{\widehat{\wp(\overline{x})}_{(p,q)}\right\} < \text{ARB}\left\{\widehat{\wp(\overline{x})}_M\right\}$$

and

$$\text{RMSE}\left\{\widehat{\wp(\overline{x})}_{(p,q)}\right\} < \text{RMSE}\left\{\widehat{\wp(\overline{x})}_M\right\} < \text{RMSE}\left\{\widehat{\wp(\overline{x})}_U\right\}.$$

This implies that there is substantial gain in relative efficiency by using the proposed estimators $\widehat{\wp(\overline{x})}_{(p,q)}$ over unbiased estimator as well as MMSE estimator. It is also interesting to note that the suggested estimator is relatively less biased than the MMSE estimator. This escorts us to quote that the proposed class of estimators performs tremendously better than the conventional estimators.

8. CONCLUDING REMARKS AND RECOMMENDATIONS

It has been found out that the suggested class of shrunken estimators has considerable gain in efficiency for a number of choices of scalars comprehend in it, particularly for small samples, i.e., for small *n*. Even for large sample sizes, so far as the proper selection of scalars is concerned, a number of estimators from the suggested class of shrunken estimators are more efficient than the MMSE estimator. Accordingly, even if the experimenter has less confidence in the guessed value θ_0 of θ, the efficiency of the suggested class of shrunken estimators can be increased considerably by choosing the scalars p and q appropriately.

While dealing with the suggested class of shrunken estimators it is recommended that one should not consider the substantial gain in efficiency in isolation, but also the wider range of dominance of λ, because enough flexible range of dominance of λ leads to increase the possibility of getting better estimators from the proposed class. The suggested class of shrunken estimators is, therefore, recommended for its use in practice.

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RIASSUNTO

La stima delll'informazione di Fisher in una popolazione Normale con informazioni a priori

Il lavoro propone una classe di stimatori *shrunken* poi utilizzata per costruire una classe di stimatori preliminari per l'informazione in campioni completi da una popolazione Normale quando valori a priori sono ipotizzati. Le proprietà di tali stimatori vengono analizzate e confrontate con quelle degli stimatori non distorti e a minimo errore quadratico medio nonché illustrate attraverso un'analisi empirica. I risultati incoraggiano all'uso degli stimatori presentati. Si fornisce e discute infine un esempio di classe di stimatori impiegati per la stima della precisione della media campionaria.

SUMMARY

Estimating Fisher information in normal population with prior information

This paper is contemplated to propose a class of shrunken estimators which is further used in constructing a class of preliminary test estimators for amount of information in complete samples from normal population when some 'apriori' or guessed value of amount of information is available and analyses their characteristics. The proposed classes of shrunken estimators and preliminary test estimators are compared with the usual unbiased estimator and MMSE estimator. Eventually, empirical study is carried out to demonstrate the performance of some shrunken estimators and preliminary test estimators of the proposed classes over the MMSE estimator. The suggested class of estimators is found to give gratifying results. Subsequently, the usage of proposed classes of estimators in estimating the precision of sample mean has been exclusively discussed followed by an example.