



the catastrophes. Hence, computer networks with virus may be modeled by queueing networks with catastrophes.

Recently, Krishna Kumar *et al.* (1993) and Krishna Kumar (1996) have demonstrated, how the transient solution for the state probabilities and busy period in single server Poisson queue with balking can be obtained in a simple and direct way. Exploiting this methodology, we obtain the transient solution for the probabilities in  $M/M/2$  queue with catastrophes.

## 2. MODEL DESCRIPTION AND ANALYSIS

Consider the  $M/M/2$  queueing system having uniform mean interarrival  $A^{-1}$  and service rate  $\mu$  for channels with the possibility of catastrophes. The service discipline is first come first served, starting with an arbitrary number of customers. Apart from arrival and service processes, the catastrophes also occur at the service facilities as a Poisson process with rate  $\gamma$ . Whenever a catastrophe occurs at the system, all the customers there are destroyed immediately, both the servers get inactivated momentarily and the servers are ready for service when a new arrival occurs. Let  $\{X(t); t \in R_+\}$  be the number of customers in the system at time  $t$ . Let  $P_n(t) = P(X(t) = n)$ ,  $n = 0, 1, 2, \dots$ , denote the probability that there are  $n$  customers in the system at time  $t$ ,  $P(s, t) = \sum_{n=0}^{\infty} P_n(t) s^n$ , its probability generating function, and  $m(t)$ , its mean. From the above assumptions, the state probabilities  $P_n(t)$ ,  $n = 0, 1, 2, \dots$ , can be described by the differential difference equations governing the system as follows:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t) + \gamma(1 - P_0(t)) \quad (2.1)$$

$$\frac{dP_1(t)}{dt} = -(\lambda + \mu + \gamma) P_1(t) + 2\mu P_2(t) + \lambda P_0(t) \quad (2.2)$$

$$\frac{dP_n(t)}{dt} = -(\lambda + 2\mu + \gamma) P_n(t) + 2\mu P_{n+1}(t) + \lambda P_{n-1}(t), \quad n = 2, 3, 4, \dots \quad (2.3)$$

We assume that the number of customers present initially is random and has probability generating function

$$h(s) = \sum_{k=0}^{\infty} p_k s^k.$$

It is easily seen that the probability generating function  $P(s, t)$  satisfies the partial differential equation,

$$\frac{\partial P(s, t)}{\partial t} = \left[ \lambda s - (\lambda + 2\mu + \gamma) + \frac{2\mu}{s} \right] P(s, t) + 2\mu \left( 1 - \frac{1}{s} \right) P_0(t) + \mu(s - 1) P_1(t) + \gamma \quad (2.4)$$

with the initial condition

$$P(s, 0) = h(s) . \tag{2.5}$$

The solution of this partial differential equation is obtained as

$$P(s, t) = \sum_{k=0}^{\infty} p_k s^k e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right]t} + \gamma \int_0^t e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right]u} du + \int_0^t \left[2\mu \left(1 - \frac{1}{s}\right) P_0(u) + \mu(s - 1) P_1(u)\right] e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right](t-u)} du . \tag{2.6}$$

It is well known that (see Watson, 1962), if  $\alpha = 2\sqrt{2\lambda\mu}$  and  $\beta = \sqrt{\frac{A}{2\mu}}$ , then

$$e^{\left(\lambda s + \frac{2\mu}{s}\right)t} = \sum_{n=-\infty}^{\infty} (\beta s)^n I_n(\alpha t)$$

where  $I_n(\cdot)$  is the modified Bessel [unction of order n. Using this in (2.6) and comparing the coefficient of  $s^n$  on either side, we get, for  $n = 1, 2, 3, \dots$ ,

$$P_n(t) = \sum_{k=0}^{\infty} p_k I_{n-k}(\alpha t) \beta^{n-k} e^{-(\lambda + 2\mu + \gamma)t} + \gamma \beta^n \int_0^t e^{-(\lambda + 2\mu + \gamma)u} I_n(\alpha u) du + 2\mu \int_0^t e^{-(\lambda + 2\mu + \gamma)(t-u)} [I_n(\alpha(t-u)) \beta^n - I_{n+1}(\alpha(t-u)) \beta^{n+1}] P_0(u) du + \mu \int_0^t e^{-(\lambda + 2\mu + \gamma)(t-u)} [I_{n-1}(\alpha(t-u)) \beta^{n-1} - I_n(\alpha(t-u)) \beta^n] P_1(u) du . \tag{2.7}$$

As  $P(s, t)$  does not contain terms with negative powers of  $s$ , the right hand side of (2.7) with  $n$  replaced by  $-n$  must be zero. Thus,

$$0 = \sum_{k=0}^{\infty} p_k I_{n+k}(\alpha t) \beta^{-k} e^{-(\lambda + 2\mu + \gamma)t} + \gamma \int_0^t e^{-(\lambda + 2\mu + \gamma)u} I_n(\alpha u) du + 2\mu \int_0^t e^{-(\lambda + 2\mu + \gamma)(t-u)} [I_n(\alpha(t-u)) - I_{n-1}(\alpha(t-u)) \beta] P_0(u) du + \mu \int_0^t e^{-(\lambda + 2\mu + \gamma)(t-u)} [I_{n+1}(\alpha(t-u)) \beta^{-1} - I_n(\alpha(t-u))] P_1(u) du \tag{2.8}$$

where we have used  $I_{-k}(\cdot) = I_k(\cdot)$ .

Using (2.8) in (2.7), we get, for  $n = 1, 2, 3, \dots$ ,

$$P_n(t) = \sum_{k=0}^{\infty} p_k \beta^{n-k} [I_{n-k}(\alpha t) - I_{n+k}(\alpha t)] e^{-(\lambda + 2\mu + \gamma)t} + n\beta^n \int_0^t P_0(u) \frac{I_n(\alpha(t-u))}{t-u} e^{-(\lambda + 2\mu + \gamma)(t-u)} du + \frac{n\beta^{n-2}}{2} \int_0^t P_1(u) \frac{I_n(\alpha(t-u))}{t-u} e^{-(\lambda + 2\mu + \gamma)(t-u)} du . \tag{2.9}$$

In the sequel, let  $P_n^*(z)$  denote the Laplace transform of  $P_n(t)$ . Taking  $n = 1$  in (2.9), and transforming, we get, after some algebraic manipulations,









