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TRANSIENT BEHAVIOUR OF THE *M/M/*2 QUEUE WITH CATASTROPHES

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1. INTRODUCTION

Queueing models continue to be one of the most important areas of computer networks and have played a vital role in performance evaluation of computer systems. Computer systems typically comprise of a set of discrete resources- processors, discs, etc. Any transaction that can not immediately get hold of the required resource is usually queued up in buffer until the resource becomes available. This characteristic makes computer systems amenable for analysis using queueing models. A brief history of queueing analysis of computer systems can be found in Lavenberg (1988) and Takagi (1993).

Most real queueing systems exhibit time-dependent behaviour to a greater or lesser extent, either whilst setting to steady-state or because parameters change over time. In many cases this time-dependence is relatively unimportant and can be ignored by the operational researcher. However, in some cases, it is important to take care of time-dependence.

In many potential applications of queueing theory, the practitioner needs to know how the system will operate up to some instant t. Many systems begin operation and are stopped at some specific time t. Business or service operations such as rental agencies or physician's offices which open and close, never operate under steady-state conditions. Furthermore, if the system is empty initially, the fraction of time the server is busy and the initial rate of output etc., will be below the steady-state values and hence the use of steady-state results to obtain these measures is not appropriate. Thus the investigation of the transient behaviour of the yueueing processes is also important from the point of view of the theory and applications.

The notion of catastrophes occurring at random, leading to annihilation of all the customers there and the momentary inactivation of the service facilities until a new arrival of customer is not uncommon in many practical problems. The catastrophes may come either from outside the system or from another service station. In computer systems, if a job is infected, this job may transmit virus which may be transferred to other processors (CPU, I/O, Diskettes, etc.). Infected files in floppy diskettes, for instance, may also arrive at the processors according to some random process (Chao, 1995; Gelenbe *et* al., 1991). These infected jobs may be modeled by the catastrophes. Hence, computer networks with virus may be modeled by queueing networks with catastrophes.

Recently, Krishna Kumar *et al.* (1993) and Krishna Kumar (1996) have demonstrated, how the transient solution for the state probabilities and busy period in single server Poisson queue with balking can be obtained in a simple and direct way Exploiting this methodology, we obtain the transient solution for the probabilities in $M/M/2$ queue with catastrophes

2. MODEL DESCRIPTION AND ANALYSIS

Consider the $M/M/2$ queueing system having uniform mean interarrival λ^{-1} and service rate μ for channels with the possibility of catastrophes. The service discipline is first come first served, starting with an arbitrary number of customers. Apart from arrival and service processes, the catastrophes also occur at the service facilities as a Poisson process with rate γ. Whenever a catastrophe occurs at the system, all the customers there are destroyed immediately, both the servers get inactivated momentarily and the servers are ready for service when a new arrival occurs. Let $\{X(t);\ t \in R_+\}$ be the number of customers in the system at time t. Let $P_n(t) = P(X(t) = n), n = 0, 1, 2, \ldots$, denote the probability that there are n customers in the system at time t, $P(s,t) = \sum_{n=0}^{\infty} P_n(t) s^n$, its probability generating function, and $m(t)$, its mean. From the above assumptions, the state probabilities $P_n(t)$, $n = 0, 1, 2, \ldots$, can be described by the differential difference equations governing the system as follows:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t) + \gamma(1 - P_0(t)) \tag{2.1}$$

$$\frac{dP_1(t)}{dt} = -(\lambda + \mu + \gamma) P_1(t) + 2\mu P_2(t) + \lambda P_0(t) \tag{2.2}$$

$$\frac{dP_n(t)}{dt} = -(\lambda + 2\mu + \gamma) P_n(t) + 2\mu P_{n+1}(t) + \lambda P_{n-1}(t), \qquad n = 2,3,4,\ldots \tag{2.3}$$

We assume that the number of customers present initially is random and has probability generating function

$$h(s) = \sum_{k=0}^{\infty} p_k s^k.$$

It is easily seen that the probability generating function $P(s, t)$ satisfies the partial dilferential equation,

$$\frac{\partial P(s,t)}{\partial t} = \left[\lambda s - (\lambda + 2\mu + \gamma) + \frac{2\mu}{s}\right] P(s,t) + 2\mu\left(1 - \frac{1}{s}\right) P_0(t) + \mu(s-1) P_1(t) + \gamma \tag{2.4}$$

with the initial condition

$$P(s, 0) = h(s)\,. \tag{2.5}$$

The solution of this partial differential equation is obtained as

$$P(s,t) = \sum_{k=0}^{\infty} p_k s^k e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right] t} + \gamma \int_0^t e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right] u} du$$

$$+ \int_0^t \left[2\mu \left(1 - \frac{1}{s}\right) P_0(u) + \mu (s-1) P_1(u) \right] e^{\left[\lambda s + \frac{2\mu}{s} - (\lambda + 2\mu + \gamma)\right](t-u)} du\,. \tag{2.6}$$

It is well known that (see Watson, 1962), if $\alpha = 2\sqrt{2\lambda\mu}$ and $\beta = \sqrt{\frac{A}{2\mu}}$, then

$$e^{\left(\lambda s + \frac{2\mu}{s}\right) t} = \sum_{n=-\infty}^{\infty} (\beta s)^n I_n(\alpha t)$$

where $I_n(\cdot)$ is the modified Bessel [unction of order n. Using this in *(2.6)* and comparing the coefficient of s^n on either side, we get, for $n = 1, 2, 3, \ldots$,

$$P_n(t) = \sum_{k=0}^{\infty} p_k I_{n-k}(\alpha t) \beta^{n-k} e^{-(\lambda+2\mu+\gamma)t} + \gamma \beta^n \int_0^t e^{-(\lambda+2\mu+\gamma)u} I_n(\alpha u)\, du +$$

$$2\mu \int_0^t e^{-(\lambda+2\mu+\gamma)(t-u)} [I_n(\alpha(t-u))\beta^n - I_{n+1}(\alpha(t-u))\beta^{n+1}] P_0(u)\, du +$$

$$\mu \int_0^t e^{-(\lambda+2\mu+\gamma)(t-u)} [I_{n-1}(\alpha(t-u))\beta^{n-1} - I_n(\alpha(t-u))\beta^n] P_1(u)\, du\,. \tag{2.7}$$

As $P(s, t)$ does not contain terms with negative powers of s, the right hand side of *(2.7)* with n replaced by $-n$ must be zero. Thus,

$$0 = \sum_{k=0}^{\infty} p_k I_{n+k}(\alpha t) \beta^{-k} e^{-(\lambda+2\mu+\gamma)t} + \gamma \int_0^t e^{-(\lambda+2\mu+\gamma)u} I_n(\alpha u)\, du +$$

$$2\mu \int_0^t e^{-(\lambda+2\mu+\gamma)(t-u)} [I_n(\alpha(t-u)) - I_{n-1}(\alpha(t-u))\beta] P_0(u)\, du +$$

$$\mu \int_0^t e^{-(\lambda+2\mu+\gamma)(t-u)} [I_{n+1}(\alpha(t-u))\beta^{-1} - I_n(\alpha(t-u))] P_1(u)\, du \tag{2.8}$$

where we have used $I_{-k}(\cdot) = I_k(\cdot)$.

Using *(2.8)* in (2.7), we get, for $n = 1, 2, 3, \ldots$,

$$P_n(t) = \sum_{k=0}^{\infty} p_k \beta^{n-k} [I_{n-k}(\alpha t) - I_{n+k}(\alpha t)] e^{-(\lambda+2\mu+\gamma)t} +$$

$$n\beta^n \int_0^t P_0(u) \frac{I_n(\alpha(t-u))}{t-u} e^{-(\lambda+2\mu+\gamma)(t-u)} du +$$

$$\frac{n\beta^{n-2}}{2} \int_0^t P_1(u) \frac{I_n(\alpha(t-u))}{t-u} e^{-(\lambda+2\mu+\gamma)(t-u)} du\,. \tag{2.9}$$

In the sequel, let $P_n^*(z)$ denote the Laplace transform of $P_n(t)$. Taking $n = 1$ in (2.9), and transforming, we get, after some algebraic manipulations,

$$P_1^*(z)\left\{1-\frac{\left[(z+\lambda+2\mu+\gamma)-\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}\right]}{2\alpha\beta}\right\}=$$

$$P_0^*(z)\frac{\beta}{\alpha}\left[(z+\lambda+2\mu+\gamma)-\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}\right]+$$

$$\sum_{k=1}^{\infty}p_k\beta^{-k+1}\left\{\frac{\left[(z+\lambda+2\mu+\gamma)-\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}\right]^{k-1}}{\alpha^{k-1}\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}}-\right.$$

$$\left.\frac{\left[(z+\lambda+2\mu+\gamma)-\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}\right]^{k+1}}{\alpha^{k+1}\sqrt{(z+\lambda+2\mu+\gamma)^2-\alpha^2}}\right\}. \tag{2.10}$$

On taking Laplace transform, equation *(2.1)* becomes

$$P_0^*(z)=\frac{p_0}{z+\lambda+\gamma}+\frac{\gamma}{z(z+\lambda+\gamma)}+\frac{\mu P_1^*(z)}{z+\lambda+\gamma}. \tag{2.11}$$

Using *(2.11)* in *(2.10)* and considerably simplifying the working, we get an expression for $P_1^*(z)$ as

$$P_1^*(z)=$$

$$\frac{\left\{\left(\frac{p_0z+\gamma}{z(z+\lambda+\gamma)}\right)\frac{\beta}{\alpha}\left[w-\sqrt{w^2-\alpha^2}\right]+\sum_{k=1}^{\infty}p_k\beta^{-k+1}\left[\frac{\left(w-\sqrt{w^2-\alpha^2}\right)^{k-1}}{\alpha^{k-1}\sqrt{w^2-\alpha^2}}-\frac{\left(w-\sqrt{w^2-\alpha^2}\right)^{k+1}}{\alpha^{k+1}\sqrt{w^2-\alpha^2}}\right]\right\}}{\left(1-\frac{z+\gamma+2\lambda}{4\lambda(z+\lambda+\gamma)}\right)\left[w-\sqrt{w^2-\alpha^2}\right]} \tag{2.12}$$

where $w=z+\lambda+2\mu+\gamma$.

The above equation can be expressed as

$$P_1^*(z)=\frac{p_0\beta}{\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}\frac{\lambda^m}{(z+\lambda+\gamma)^{m+1}}\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1}+$$

$$\frac{\gamma\beta}{z\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}\frac{\lambda^m}{(z+\lambda+\gamma)^{m+1}}\left[w-\sqrt{w^2-\alpha^2}\right]^{n+1}+$$

$$\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{p_k}{\beta^{k-1}}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}\frac{\lambda^m}{(z+\lambda+\gamma)^m}\times$$

$$\left\{\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+k-1}}{\alpha^{k-1}\sqrt{w^2-\alpha^2}}-\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{n+k+1}}{\alpha^{k+1}\sqrt{w^2-\alpha^2}}\right\} \tag{2.13}$$

which on inversion yields the explicit expression for $P_1(t)$ as

$$
\begin{aligned}
&P_1(t)= \\
&\frac{p_0\beta}{\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}(n+1)\alpha^{n+1}\lambda^m\int_0^t\frac{e^{-(\lambda+\gamma)u}u^m}{m!}\frac{I_{n+1}(\alpha(t-u))}{t-u}e^{-(\lambda+2\mu+\gamma)(t-u)}du+ \\
&\frac{\gamma\beta}{\alpha}\int_0^t\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}(n+1)\alpha^{n+1}\lambda^m\int_0^u\frac{e^{-(\lambda+\gamma)v}v^m}{m!}\frac{I_{n+1}(\alpha(u-v))}{u-v}e^{-(\lambda+2\mu+\gamma)(u-v)}dvdu+ \\
&\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{p_k}{\beta^{k-1}}\frac{1}{(4\lambda)^n}\sum_{m=1}^{n}\binom{n}{m}\lambda^m\alpha^n\int_0^t\frac{e^{-(\lambda+\gamma)u}u^{m-1}}{(m-1)!}e^{-(\lambda+2\mu+\gamma)(t-u)}[I_{n+k-1}(\alpha(t-u))- \\
&I_{n+k+1}(\alpha(t-u))]du+\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{p_k\beta^{1-k}}{(2\beta)^n}[I_{n+k-1}(\alpha t)-I_{n+k+1}(\alpha t)]e^{-(\lambda+2\mu+\gamma)t} \qquad (2.14)
\end{aligned}
$$

Also, we have, from (2.1),

$$P_0(t) = \mu\int_0^t P_1(u)e^{-(\lambda+\gamma)(t-u)}du + \frac{\gamma}{\lambda+\gamma}[1-e^{-(\lambda+\gamma)t}] + p_0e^{-(\lambda+\gamma)t}\,. \qquad (2.15)$$

Using *(2.14)* and (2.15) in (2.9), we completely determine all the state probabilities of the system size.

Remark: If $\gamma=0$, we will obtain the probabilities $P_n(t)$ for $M/M/2$ queue result as a special case without using Rouche's theorem. In this case, we have

$$
\begin{aligned}
&P_1(t) = \\
&\frac{p_0\beta}{\alpha}\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\sum_{m=0}^{n}\binom{n}{m}(n+1)\alpha^{n+1}\lambda^m\int_0^t\frac{e^{-\lambda u}u^m}{m!}\frac{I_{n+1}(\alpha(t-u))}{t-u}e^{-(\lambda+2\mu)(t-u)}du+ \\
&\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{p_k}{\beta^{k-1}}\frac{1}{(4\lambda)^n}\sum_{m=1}^{n}\binom{n}{m}\lambda^m\alpha^n\int_0^t\frac{e^{-\lambda u}u^{m-1}}{(m-1)!}e^{-(\lambda+2\mu)(t-u)}[I_{n+k-1}(\alpha(t-u))- \\
&I_{n+k+1}(\alpha(t-u))]du+\sum_{k=1}^{\infty}p_k\beta^{-k+1}\sum_{n=0}^{\infty}\frac{1}{(4\lambda)^n}\alpha^n[I_{n+k-1}(\alpha t)-I_{n+k+1}(\alpha t)]e^{-(\lambda+2\mu)t} \\
&\qquad (2.16)
\end{aligned}
$$

$$P_0(t) = \mu\int_0^t P_1(u)e^{-\lambda(t-u)}du + p_0e^{-\lambda t} \qquad (2.17)$$

and

$$
\begin{aligned}
P_n(t) = {}& \sum_{k=0}^{\infty}p_k\beta^{n-k}[I_{n-k}(\alpha t)-I_{n+k}(\alpha t)]e^{-(\lambda+2\mu)t} + \frac{2n\beta^{n-1}}{\alpha}\int_0^t[\lambda P_0(u)+\mu P_1(u)]\times \\
& e^{-(\lambda+2\mu)(t-u)}\frac{I_n(\alpha(t-u))}{t-u}du\,. \qquad (2.18)
\end{aligned}
$$

Miyazaki *et* al. *(1992)* have considered the $M/M/2$ queueing model without disasters and obtained the transient solution using extensive complex analysis. Our approach provides the solution for a more general model incorporating disasters in an elegant way.

The following theorems provide the asymptotic behaviour of tile probability of the server being idle and the mean system size.

Theorem 2.1. If $\gamma > 0$, the asymptotic behaviour of the probability of the server being idle is

$$P_0(t) \to \frac{\gamma\left[4\lambda - \left((\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2 - \alpha^2}\right)\right]}{\left[4\lambda(\lambda+\gamma) - (2\lambda+\gamma)\left((\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2-\alpha^2}\right)\right]},$$

as $t \to \infty$. (2.19)

Proof. From (2.10) and (2.11), we have

$$P_0^*(z) = \frac{1}{1 - \frac{z+\gamma+2\lambda}{4\lambda(z+\lambda+\gamma)}\left[w - \sqrt{w^2-\alpha^2}\right]} \times$$

$$\left\{\frac{\mu}{z+\lambda+\gamma}\left\{\left[\frac{p_0}{z+\lambda+\gamma} + \frac{\gamma}{z(z+\lambda+\gamma)}\right]\frac{\beta}{\alpha}\left[w - \sqrt{w^2-\alpha^2}\right] + \right.\right.$$

$$\left.\left.\sum_{k=1}^{\infty} p_k \beta^{-k+1}\left[\frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{k-1}}{\alpha^{k-1}\sqrt{w^2-\alpha^2}} - \frac{\left[w-\sqrt{w^2-\alpha^2}\right]^{k+1}}{\alpha^{k+1}\sqrt{w^2-\alpha^2}}\right]\right\}\right\} +$$

$$\frac{\gamma}{z(z+\lambda+\gamma)} + \frac{p_0}{z++\lambda+\gamma} \tag{2.20}$$

and a little algebra shows that this reduces to

$$P_0^*(z) \sim \frac{\gamma\left\{1 - \left[(\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2-\alpha^2}\right]\left(\frac{1}{4\lambda} + \frac{\beta\mu}{\alpha(\lambda+\gamma)}\right)\right\}}{\left\{1 - \left[(\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2-\alpha^2}\right]\left(\frac{1}{4\lambda} + \frac{\beta\mu}{\alpha(\lambda+\gamma)}\right)\right\}} +$$

$$+ \frac{\frac{\beta\gamma\mu}{\alpha(\lambda+\gamma)}\left[(\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2-\alpha^2}\right]}{\left\{1 - \left[(\lambda+2\mu+\gamma) - \sqrt{(\lambda+2\mu+\gamma)^2-\alpha^2}\right]\left(\frac{1}{4\lambda} + \frac{\beta\mu}{\alpha(\lambda+\gamma)}\right)\right\}}$$

as $z \to 0$. (2.21)

By using the Tauberian theorem (see, Widder, 1946), the result (2.19) follows.

Theorem 2.2. If $\gamma > 0$, the asymptotic behaviour of the mean system size $m(t)$ is given by

$$m(t) \to \frac{(\lambda - 2\mu)}{\gamma} +$$

$$\left\{ \frac{8\lambda\mu + (\lambda - 2\mu)\left[(\lambda + 2\mu + \gamma) - \sqrt{(\lambda + 2\mu + \gamma)^2 - \alpha^2}\right]}{4\lambda(\lambda + \gamma) - (\gamma + 2\lambda)\left[(\lambda + 2\mu + \gamma) - \sqrt{(\lambda + 2\mu + \gamma)^2 - \alpha^2}\right]} \right\},$$

as $t \to \infty$. (2.22)

Proof Differentiating *12.4)* with respect to s at $s = 1$, we yet

$$\frac{dm(t)}{dt} + \gamma m(t) = (\lambda - 2\mu) + 2\mu P_0(t) + \mu P_1(t). \tag{2.23}$$

Solving the differential equation *(2.21)*for $m(t)$ with $m(0) = \sum_{k \; 1} kp_k$, we get

$$m(t) = \frac{(\lambda - 2\mu)}{\gamma}(1 - e^{-\gamma t}) + 2\mu \int_0^t P_0(u) e^{-\gamma(t-u)} du + \mu \int_0^t P_1(u) e^{-\gamma(t-u)} du +$$

$$\sum_{k=1}^{\infty} kp_k e^{-\gamma t}. \tag{2.24}$$

If m (z) is the Laplace transform of $m(t)$, from *(2.24)*and (2.11), we have

$$m^*(z) = \frac{(\lambda - 2\mu)}{z(z + \gamma)} + \frac{\sum_{k=1}^{\infty} kp_k}{z + \gamma} + \frac{2\mu}{z + \gamma} \left\{ \frac{\gamma + zp_0}{z(z + \lambda + \gamma)} \right\} +$$

$$\left\{ \frac{2\mu^2}{(z + \gamma)(z + \lambda + +\gamma)} + \frac{\mu}{z + \gamma} \right\} P_1^*(z). \tag{2.25}$$

Using *(2.12)*in (2.25), we obtain

$$\lim_{z \to 0} zm^*(z) = \frac{(\lambda - 2\mu)}{\gamma} +$$

$$\lim_{z \to 0} zP_0^* \frac{(z)}{(z + \gamma)} \left\{ 2\mu + \frac{\left[w - \sqrt{w^2 - \alpha^2}\right]}{4\left[1 - \frac{1}{4\lambda}\left[w - \sqrt{w^2 - \alpha^2}\right]\right]} \right\} \tag{2.26}$$

Then the result *(2.22)*follows from (2.26), by using the Tauberian theorem (see Widder, *1946).*

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RIASSUNTO

Comportamento transiente del modello M/M/2 con presenza di catastrofi

In questo lavoro viene presentata una soluzione transiente per la dimensione di un sistema M/M/2 con possibilitlt di catastrofi ai punti di servizio. Si otticnc la probabilitlt di stato della dimensione del sistema al tempo t in cui la coda inizia con un numero arbitrario di clienti. Infine viene studiato il comportamento asintotico della probabilitlt che il server sia inattivo e della dimensione media del sistema.

SUMMARY

Transient behaviour of the M/M/2 queue with catastrophes

This paper presents a transient solution for the system size in the $M/M/2$ queue with the possibility of catastrophes at the service stations. The state probability of the system size at time t, where the queue starts with any number of customers, is obtained. Asymptotic behaviour of the probability of the server being idle and mean system size are discussed.