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EFFICIENT CLASSES OF ESTIMATORS OF POPULATION VARIANCE IN TWO-PHASE SUCCESSIVE SAMPLING UNDER RANDOM NON-RESPONSE

Zeeshan Basit
Quality Enhancement Cell, Government College Women University Sialkot, Pakistan

M. Ishaq Bhatti[1]
S P Jain School of Global Management, Sydney, Australia & UBD School of Business and Economics, UBD, Brunei

SUMMARY

This paper presents some efficient classes of estimators of population variance in two-phase successive sampling under random non-response. The suggested classes of estimators are for simple random sampling and for different situations of non-response. Up to first-order approximation MSE's of suggested classes of estimators are derived. The efficiency of the presented estimators is contrasted with the estimators for the complete response scenarios. Usefulness of the presented classes of estimators is checked. To test the efficiency real data sets are used. The proposed classes of estimators are more efficient. Results are interpreted.

Keywords: Successive Sampling; Auxiliary variable; Random non-response; Variance estimator; Bias; Mean square error.

1. INTRODUCTION

In statistical research, an auxiliary variable is any variable about which information is available prior to data collection and where this information is known for all population units. Auxiliary variables are used to optimize the sample, or to compile detailed tabulation when a frame is used for producing statistics directly, or to enhance other processes like editing and imputation. Information on auxiliary variable(s) is used to increase the accuracy of sample estimates. Auxiliary information is very useful to find estimators that are more reliable and efficient. Without spending more money on the survey, this variable could provide the surveyor extra information about the variable under study. The correlation between auxiliary and study variable could be negative or

[1] Corresponding Author. E-mail: i.bhatti@latrobe.edu.au

positive. Singh *et al.* (2017a), Singh *et al.* (2017b), Singh and Khalid (2018) and Shabbir and Gupta (2006) used auxiliary information for estimation of finite population variance.

Two-phase sampling is usually used where the collection of data on variables of interest is very costly. This method of sampling is a less time consuming and more accurate estimation method. This technique is also cost-effective for the estimation of unknown population parameters. Singh and Khalid (2022) are amongst those who suggested the estimation strategy for estimation of current population parameters in two-occasions successive sampling.

Mostly, information on an auxiliary variable may be attainable on the first (second) occasion. Among others, Singh and Pal (2016) and Beevi and Chandran (2017) used the auxiliary information on first (second) occasion for the estimation of the current population mean in two-occasion successive sampling. Further, Riaz *et al.* (2020) suggested a generalized class of estimators for estimation of the finite population mean under the existence of non-response. Singh *et al.* (2021), Singh *et al.* (2023) and Sharma and Singh (2020) presented an estimation strategy for the population variance in two-phase sampling.

Non-response occurs where there is a huge contrast between those who respond and those who do not respond. Non-response takes place when those who respond and those who do not respond significantly differ from each other. This takes place for many different reasons. They include the refusal of some people to take part in the survey, poor conduction of the survey, the survey not being submitted by some participants due to forgetfulness, failing of the survey to reach all the targeted participants in the sample, and the greater tendency of some participants to give answers as compared to others. In the first case, some people decline to participate in the survey. This may happen because the researcher asks the participants for information that may cause embarrassment for them, or because the information is about activities that are not legal.

In the second case, poor conduction of the survey may result in non-response. For instance, if the researcher has a snail mail survey for young adults or a smart phone survey for old adults, both cases will probably result in non-response from the intended population. Thirdly, some participants simply do not remember to submit the survey after conduction. The fourth reason can be that all the members in the sample did not receive the survey (questionnaire). As an example, the email containing the survey may remain unsent or in the Spam folder or the data may not get presented or displayed properly on certain devices. The next reason can be that some people have more inclination to give an answer. For instance, people who are all-time readers are more likely and interested to give answer about reading than those who do not read or read less. In the historical context, some researchers have observed and even experienced that those members of the population who have less income are less likely to respond to surveys. Similarly, one other researcher has noted that unmarried males are another group who are less likely or unlikely to respond. Non-response, which results from a contrast between those who respond and those who do not, is considered as bias in statistics and it can make the results invalid. In addition, it can bring about variance for the estimates because the resulting sample size is smaller than the size intended bt the researcher. When non-response occurs, surveys fail to get information on one or more study variables. Singh and Joarder (1998), Singh *et al.* (2017c), Singh and Khalid (2019, 2022), Singh *et al.* (2021, 2023), Sharma (2017) and Sharma and Singh (2020) proposed some estimators for estimating the population variance under random non-response.

Non-response cannot be completely erased in practice, but it can be overcome by making much effort to get information. Successive (rotation) sampling is more susceptible to the non-response due to its repetitiveness. In many real-life situations, measures of population variance are hugely valuable for example, physicians allow their patients to have regular access to prepare reports of fluctuations for body temperature, pulse rates, and blood pressure, etc., in order to recommend suitable medication to their patients. Motivated by the above work, and considering the importance of resolving random non-response, some efficient estimators of the population variance in two-occasion successive sampling under the two phase set-up are proposed. Revised ratio, product, exponential, and regression type estimators for estimation of population variance are recommended. The properties of the suggested classes of estimators have also been studied on some data available in the survey literature.

2. Sample Structures on Two Occasions

Let us consider a finite population of size N, which has been sampled over two occasions. The character under study is denoted by $x(y)$ on the first (second) occasion and it is assumed that random non-response occurs in study variables on both occasions. Let z be a stable auxiliary variable with unknown population mean. z has positive correlation with the study variable $x(y)$ on first (second) occasion. To assess the estimate of the population mean of the auxiliary variable z on the first occasion, a preliminary random sample (without replacement) $S_{n'}$ of n ' units is drawn on the first occasion and information on z is collected. Further, a second-phase sample of size $n\,(n < n')$ is drawn from the first-phase (preliminary) sample by the method of simple random sampling without replacement (SRSWOR), and subsequently the information on the study variable x is gathered. A random sub-sample of size m is retained (matched) from the responding units of the second-phase sample selected on the first occasion for its use on the current (second) occasion, under the assumption that these matched units will again respond on the second occasion. Again. to furnish a fresh estimate of population mean of the auxiliary variable z on the current (second) occasion, a preliminary (first-phase) sample of size u ' drawn from the non-sampled units of the population by the method of simple random sampling without replacement (SRSWOR) and information on z is collected. A second-phase sample of size $u = (n - m) = n\mu\,(u < u')$ is drawn from the first phase sample by method of simple random sampling without replacement (SRSWOR) and the information on study variable y is gathered. Here "μ" is the fraction of fresh samples on the current (second) occasion.

3. Non-response Probability Model and Notations

Let S_n and $S_{n'}$ be the samples of sizes n and n' on the second and first phase, respectively, on variable x. S_u is the random non-response in the second phase-sample. r_1 represents the sampled units on which information of x couldn't be taken. Similarly, r_2 is the number of nonresponding units on which information on y could not be gathered in the second-phase sample because of random non-response. The remaining $(u - r_2)$ units of the second-phase sample can be treated as a random sample drawn using simple random sampling without replacement scheme for the first-phase sample S_u. Let p_1 and p_2 be the probability of non-response among $(n-2)$ and $(u-2)$ possible values of non-responses. Consequently, r_1 and r_2' have the following discrete probability distributions:

$$P(r_1) = \frac{n-r_1}{nq_1+2p_1}\,{}^{n-2}C_{r_1} p_1^{r_1} q_1^{n-r_1-2};\, r_1 = 0,1,2,\ldots,n-2,$$

and

$$P(r_2) = \frac{u-r_2}{uq_2+2p_2}\,u-2C_{r_2} p_2^{r_2} q_2^{u-r_2-2};\, r_2 - 0,1,2,\ldots,u-2.$$

The following notations are used. Sample means and variances of the variables x, y, z based on samples of size n, u, n and u on the first (second) occasions and $s_{y_m}^2, s_{x_m}^2, s_{z_n}^2$ are the sample variances of the variables x, y and z, respectively, based on respective sample sizes shown in their subscripts:

$$\bar{x}_n^* = \frac{1}{n-r_1}\sum_{i=1}^{n-r_1} x_i,\ \bar{y}_u^* = \frac{1}{u-r_2}\sum_{i=1}^{u-r_2} y_i,\ \bar{z}_u^* = \frac{1}{u-r_2}\sum_{i=1}^{u-r_2} z_i,$$

$$\bar{z}_{n'} = \frac{1}{n'}\sum_{i=1}^{n'} z_i,\ \bar{z}_u' = \frac{1}{u'}\sum_{i=1}^{u'} z_i,$$

$$s_{z_{u'}}^2 = \frac{1}{u'-1}\sum_{i=1}^{u'} (z_i - \bar{z}_{u'})^2,\ s_{x_n}^{*2} = \frac{1}{n-r_1-1}\sum_{i=1}^{n-r_1} (x_i - \bar{x}_n^*)^2,$$

$$s_{z_{n'}}^2 = \frac{1}{n'-1}\sum_{i=1}^{n'} (z_i - \bar{z}_{n'})^2,$$

$$s_{y_n}^{*2} = \frac{1}{u-r_2-1}\sum_{i=1}^{u-r_2} (y_i - \bar{y}_u^*)^2 \text{ and } s_{z_n}^{*2} = \frac{1}{u-r_2-1}\sum_{i=1}^{u-r_2} (z_i - \bar{z}_u^*)^2.$$

4. Proposed Classes of Estimators

Singh and Joarder (1998) proposed a class of estimators of the population variance S_y^2; along these lines we propose:

$$T = \varphi T_u + (1-\varphi) T_m, \tag{1}$$

where T is the convex combination of the proposed estimators and $\varphi (0 \leq \varphi \leq 1)$ is an unknown constant (scalar) which could only be calculated by minimizing the mean square error of the estimator T.

4.1. *Formulation of the estimator T_n*

For population variance S_y^2 estimation on the current occasion, we propose an estimator T_u that is based on a fresh sample size u collected on the current occasion as:

$$T_u = F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right). \tag{2}$$

By considering the composite function $F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)$ as one to one function of $s_{y_u}^{*2}, s_{z_u}^2$ and $s_{z_u'}^2$ get

$$F\left(S_y^2, S_z^2, S_z^2\right) = S_y^2 \Rightarrow \left.\frac{\partial F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)}{\partial s_{y_u}^{*2}}\right|_{\left(S_y^2, S_z^2, S_z^2\right)} = 1. \tag{3}$$

Consider the following regularity conditions satisfied:

(i) Whatever the chosen samples, $\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)$ assumes values in a closed convex subspace R^3 of the three-dimensional real space containing the points $\left(S_y^2, S_z^2, S_z^2\right)$;

(ii) The function $F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)$ is continuous and bounded on R^3;

(iii) The first and second partial derivatives of $F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)$ exist and are continuous and bounded on R^3 .

The following estimators of S_y^2 are the members of the class defined by T_u:

$$t_1 = s_{y_u}^{*2} \frac{s_{z_u'}^2}{s_{z_u}^2},\ t_2 = s_{y_u}^{*2} \frac{s_{z_u}^2}{s_{z_u'}^2},$$

$$t_4 = s_{y_u}^{*2} \exp\left(\frac{s_{z_u'}^2 - s_{z_u}^2}{s_{z_u'}^2 + s_{z_u}^2}\right),\ t_5 = s_{y_u}^{*2} \exp\left(\frac{s_{z_u}^2 - s_{z_u'}^2}{s_{z_u}^2 + s_{z_u'}^2}\right),\ t_6 = s_{y_u}^{*2} \left(\frac{s_{z_u}^2}{s_{z_u'}^2}\right)^{\alpha_2},$$

$$t_7 = s_{y_u}^{*2}\left[2-\left(\frac{s_{z_u}^2}{s_{z_u'}^2}\right)^{\alpha_3}\right],\ t_8 = s_{y_u}^{*2}\left[\frac{s_{z_u'}^2}{s_{z_u'}^2+\alpha_4\left(s_{z_u}^2-s_{z_u'}^2\right)}\right],$$

$$t_9 = s_{y_u}^{*2}\left[\alpha_5\frac{s_{z_u'}^2}{s_{z_u}^2}+(1-\alpha_5)\left(\frac{s_{z_u'}^2}{s_{z_u}^2}\right)^2\right],\ t_{10} = s_{y_u}^{*2}\left[\alpha_6\frac{s_{z_u}^2}{s_{z_u'}^2}+(1-\alpha_6)\left(\frac{s_{z_u}^2}{s_{z_u'}^2}\right)^2\right],$$

$$t_{11} = s_{y_u}^{*2}\left[\alpha_7+(1-\alpha_7)\frac{s_{z_u}^2}{s_{z_u'}^2}\right],\ t_{12} = s_{y_u}^{*2}\left[\alpha_8+(1-\alpha_8)\frac{s_{z_u}^2}{s_{z_u'}^2}\right]$$

$$t_{13} = s_{y_u}^{*2}\left[\frac{\alpha_9 s_{z_u'}^2+(1-\alpha_9)s_{z_u}^2}{\alpha_9 s_{z_u}^2+(1-\alpha_9)s_{z_u'}^2}\right],\ldots,\text{etc.}$$

where α_i are scalar constants so that the MSE$'s$ of the estimators defined above could be minimized.

4.2. Formulation of the estimator T_m

Another estimator of the current variance of the population S_y^2, based on size m (matched sample size) is defined as:

$$T_m = G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right). \tag{4}$$

For any chosen sample, $T_m = G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right)$ assumes values in a bounded set, containing the points $\left(S_y^2, S_x^2, S_x^2, S_z^2, S_z^2\right)$, where $G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right)$ is a composite function of $\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right)$, such that

$$G\left(S_y^2, S_x^2, S_x^2, S_z^2, S_z^2\right) = S_y^2 \Rightarrow \left.\frac{\partial G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right)}{\partial s_{y_m}^2}\right|_{\left(S_y^2, S_x^2, S_x^2, S_z^2, S_z^2\right)} = 1. \tag{5}$$

Given product, ratio, regression, and exponential type estimators of S_y^2 are the members of the class defined by T_m,

$$k_1 = s_{y_m}^2\frac{s_{x_n}^{*2}}{s_{x_m}^2}\frac{s_{z_{n'}}^2}{s_{z_n}^2},\ k_2 = s_{y_m}^2\frac{s_{x_n}^{*2}}{s_{x_m}^2}\frac{s_{z_n}^2}{s_{z_{n'}}^2},\ k_3 = s_{y_m}^2\frac{s_{x_m}^2}{s_{x_n}^{*2}}\frac{s_{z_{n'}}^2}{s_{z_n}^2},$$

$$k_4 = \left[s_{y_m}^2 + b_1\left(s_{x_n}^{*2}-s_{x_m}^2\right)\right]\frac{s_{z_{n'}}^2}{s_{z_n}^2},\ k_5 = s_{y_m}^2\exp\left(\frac{s_{x_n}^{*2}-s_{x_m}^2}{s_{x_n}^{*2}+s_{x_m}^2}\right)\frac{s_{z_{n'}}^2}{s_{z_n}^2},$$

$$k_6 = \left[s^2_{y_m} + b_2\left(s^{*2}_{x_n} - s^2_{x_m}\right)\right]\exp\left(\frac{s^2_{z_{n'}} - s^2_{z_n}}{s^2_{z_{n'}} + s^2_{z_n}}\right),$$

$$k_7 = s^2_{y_m}\exp\left(\frac{s^{*2}_{x_n} - s^2_{x_m}}{s^{*2}_{x_n} + s^2_{x_m}}\right)\exp\left(\frac{s^2_{z_{n'}} - s^2_{z_n}}{s^2_{z_{n'}} + s^2_{z_n}}\right),\ k_8 = \frac{s^2_{y_m}}{s^2_{x_m}}\left[s^{*2}_{x_n} + b_3\left(s^2_{z_{n'}} - s^2_{z_n}\right)\right],$$

$$k_9 = s^2_{y_m} + b_4\left[\frac{s^{*2}_{x_n}}{s^2_{z_n}}s^2_{z_{n'}} + s^2_{x_m}\right],\ k_{10} = s^2_{y_m} + b_5\left[\left(s^{*2}_{x_n} - s^2_{x_m}\right) + b_4\left(s^2_{z_n} - s^2_{z_{n'}}\right)\right],$$

$$k_{11} = s^2_{y_m}\frac{s^2_{x_m}}{\left[s^{*2}_{x_n} + b_6\left(s^2_{z_{n'}} - s^2_{z_n}\right)\right]},\ k_{12} = s^2_{y_m} + b_7\left[s^{*2}_{x_n}\frac{s^2_{z_n}}{s^2_{z_{n'}}} - s^2_{x_m}\right],$$

where β_i are scalar constants so that the MSE$'s$ of the estimators defined above could be minimized.

4.3. *Properties of suggested estimator T*

The bias and MSE$'s$ of the presented classes of estimators T_u and T_m are derived up to the first order of approximation under the assumption that distribution of results should approach a normal curve, and applying the following transformations:

$$s^{*2}_{y_u} = S^2_y(1+e_1),\ s^{*2}_{z_u} = S^2_z(1+e_2),\ s^2_{z_{u'}} = S^2_z(1+e_3),\ s^2_{y_m} = S^2_y(1+e_4),$$

$$s^{*2}_{x_n} = S^2_x(1+e_5),\ s^2_{x_m} = S^2_x(1+e_6),\ s^2_{z_{n'}} = S^2_z(1+e_7),\ s^2_{z_n} = S^2_z(1+e_8),$$

$$s^2_{y_u} = S^2_y(1+e_9),\ s^2_{z_u} = S^2_z(1+e_{10}),\ s^2_{x_n} = S^2_x(1+e_{11}),\ s^2_{x_m} = S^2_z(1+e_{12}),$$

such that $E(e_k) = 0$.
Thus, we have following expectations:

$$E(e_1)^2 = f^*_2(\lambda_{040}-1),\quad E(e_2)^2 = f^*_2(\lambda_{004}-1),\quad E(e_3)^2 = f'_2(\lambda_{004}-1),$$

$$E(e_4)^2 = f_1(\lambda_{040}-1),\quad E(e_5)^2 = f^*_1(\lambda_{400}-1),\quad E(e_6)^2 = f_1(\lambda_{400}-1),$$

$$E(e_7)^2 = f'(\lambda_{004}-1),\quad E(e_8)^2 = f(\lambda_{004}-1),\quad E(e_9)^2 = f_2(\lambda_{040}-1),$$

$$E(e_{10})^2 = f_2(\lambda_{004}-1),\quad E(e_{11})^2 = f(\lambda_{400}-1),\quad E(e_1e_2) = f^*_2(\lambda_{22}-1),$$

$$E(e_2e_3) = f'_2(\lambda_{004}-1),\quad E(e_1e_3) = f'_2(\lambda_{022}-1),\quad E(e_1e_9) = f_2(\lambda_{040}-1),$$

$$E(e_2e_9) = f_2(\lambda_{022}-1),\quad E(e_3e_9) = f'_2(\lambda_{022}-1),\quad E(e_6e_7) = f'(\lambda_{202}-1),$$

$$E(e_5e_6) = f^*_1(\lambda_{400}-1),\quad E(e_5e_4) = f^*_1(\lambda_{220}-1),\quad E(e_4e_6) = f_1(\lambda_{220}-1),$$

$$E(e_7e_8) = f'(\lambda_{004}-1),\quad E(e_7e_4) = f'(\lambda_{022}-1),\quad E(e_8e_5) = f(\lambda_{202}-1),$$

$$E(e_8e_4) = f(\lambda_{022}-1),\quad E(e_6e_8) = f(\lambda_{202}-1),\quad E(e_3e_{10}) = f'_2(\lambda_{004}-1),$$

$$E(e_3e_9)=f_2'(\lambda_{022}-1),\quad E(e_{10}e_9)=f_2(\lambda_{022}-1),\quad E(e_{11}e_7)=f'(\lambda_{202}-1),$$
$$E(e_{11}e_8)=f(\lambda_{202}-1),\quad E(e_{11}e_4)=f(\lambda_{220}-1),\quad E\left(e_{12}{}^2\right)=f_1(\lambda_{004}-1),$$
$$E(e_{12}e_4)=f_1(\lambda_{022}-1),\quad E(e_{12}e_6)=f_1(\lambda_{202}-1),\quad E(e_7e_5)=f'(\lambda_{202}-1),$$
$$E(e_{12}e_{11})=f(\lambda_{202}-1)\text{ and }E(e_1e_{10})=f_2(\lambda_{022}-1),$$

where $\lambda_{st} = \mu_{rst}/\left(\mu_{200}^{r/2}\mu_{020}^{r/2}\mu_{002}^{r/2}\right)$, $\mu_{rst} = E\left[\left(x_i-\bar{X}\right)^r\left(y_i-\bar{Y}\right)^s\left(z_i-\bar{Z}\right)^t\right]$ with $(r,s,t)\geq 0$ are integers. Furthermore we have that

$$f_2^*=\left(\tfrac{1}{uq_2+2p_2}-\tfrac{1}{N}\right),\quad f_2=\left(\tfrac{1}{u}-\tfrac{1}{N}\right),\quad f_2'=\left(\tfrac{1}{u'}-\tfrac{1}{N}\right),\quad f_1^*=\left(\tfrac{1}{nq_1+2p_1}-\tfrac{1}{N}\right),$$
$$f=\left(\tfrac{1}{n}-\tfrac{1}{N}\right)f_1=\left(\tfrac{1}{m}-\tfrac{1}{N}\right)\text{and}f'=\left(\tfrac{1}{n'}-\tfrac{1}{N}\right).$$

Now, in order to express the classes of estimators T_u in terms e's we expand $F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)$ about the point S_y^2,S_z^2,S_z^2 bar in a third order Taylor's series expansions, and the following results are obtained

$$\begin{aligned}F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)=&F\left(S_y^2,S_z^2,S_z^2\right)+\left(s_{y_u}^{*2}-S_y^2\right)I_1+\left(s_{z_u}^2-S_z^2\right)I_2+\left(s_{z_u{}'}^2-S_z^2\right)I_3\\&+\frac{1}{2}\left\{\left(s_{y_u}^{*2}-S_y^2\right)^2I_{11}+\left(s_{z_u}^2-S_z^2\right)^2I_{22}+\left(s_{z_u{}'}^2-S_z^2\right)^2I_{33}\right.\\&\left.+2\left(s_{z_u}^2-S_z^2\right)\left(s_{y_u}^{*2}-S_y^2\right)I_{12}+2\left(s_{y_u}^{*2}-S_y^2\right)\left(S_{z_{u'}}^2-S_z^2\right)I_{13}\right\}\\&\times\frac{1}{6}\left\{\left(s_{y_u}^{*2}-S_y^2\right)\frac{\partial}{\partial s_{y_u}^{*2}}+\left(s_{z_u}^2-S_z^2\right)\frac{\partial}{\partial s_{z_u}^2}\right.\\&\left.+\left(s_{z_{u'}}^2-S_z^2\right)\frac{\partial}{\partial s_{z_u}^2}\right\}^3F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u'}^2\right)+\ldots,\end{aligned}$$

where

$$I_1=\left.\frac{\partial F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)}{\partial s_{y_u}^{*2}}\right|_{\left(S_y^2,S_z^2,S_z^2\right)},I_2=\left.\frac{\partial F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)}{\partial s_{z_u}^2}\right|_{\left(S_y^2,S_z^2,S_z^2\right)},$$
$$I_3=\left.\frac{\partial F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)}{\partial s_{z_u{}'}^2}\right|_{\left(S_y^2,S_z^2,S_z^2\right)}$$

and $(I_{11},I_{22},I_{33},I_{12},I_{13},I_{23})$ are the second-order partial derivatives of $F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u{}'}^2\right)$ at the point $\left(S_y^2,S_z^2,S_z^2\right)$ and $s_{y_u}^{*2'} = S_y^2+\theta\left(s_{y_u}^{*2}-S_y^2\right)$, $s_{z_u}^{2'} = S_z^2+\theta\left(s_{z_u}^2-S_z^2\right)$, $s_{z_u{}'}^{2'} = S_z^2+\theta\left(s_{z_u{}'}^2-S_z^2\right)$ for $0<\theta<1$. Under the conditions described above, regarding $F\left(s_{y_u}^{*2},s_{z_u}^2,s_{z_u'}^2\right)$ in the Equations (3), it is noted that

$$F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right) = S_y^2 \Rightarrow I_1 = 1,$$

and

$$I_{11} = \left.\frac{\partial^2}{\partial\left(s_{y_u}^{*2}\right)^2} F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)\right|_{\left(S_y^2, S_z^2, S_z^2\right)} = 0.$$

By imposing the constraint as

$$I_2 = -I_3,$$

the expression of suggested estimators is shown as

$$\begin{aligned} T_u =& F\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right) = S_y^2(1+e_1) + S_z^2(e_{10} - e_3) I_2 \\ &+ \frac{1}{2}\left\{\left(S_z^4\right)\left({e_{10}}^2 I_{22} + e_3^2 I_{33} + 2e_{10}e_3 I_{23}\right) + 2S_y^2 S_z^2 \left(e_1 e_{10} I_{12} + e_1 e_3 I_{13}\right)\right\}. \end{aligned} \tag{6}$$

To express T_m in terms of e's, we expand $G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)$ about the point $S_y^2, S_x^2, S_x^2, S_z^2, S_z^2$ bar in third order Taylor's series expansions and following results are obtained

$$\begin{aligned} T_m =& S_y^2 + \frac{\partial T_m}{\partial y_m}\left(y_m - S_y^2\right) + \frac{\partial T_m}{\partial x_m}\left(x_m - S_x^2\right) + \frac{\partial T_m}{\partial x_n}\left(x_n - S_x^2\right) + \frac{\partial T_m}{\partial z_n}\left(z_n - S_z^2\right) \\ &+ \frac{\partial T_m}{\partial z_{n'}}\left(z_{n'} - S_z^2\right) + \frac{1}{2}\left\{\frac{\partial^2 T_m}{\partial^2 y_m}\left(y_m - S_y^2\right)^2 + \frac{\partial^2 T_m}{\partial^2 x_m}\left(x_m - S_x^2\right)^2\right. \\ &+ \frac{\partial^2 T_m}{\partial^2 x_n}\left(x_n - S_x^2\right)^2 + \frac{\partial^2 T_m}{\partial^2 z_n}\left(z_n - S_z^2\right)^2 + \frac{\partial^2 T_m}{\partial^2 z_n}\left(z_{n'} - S_z^2\right)^2 \\ &+ 2\frac{\partial T_m}{\partial y_m \partial x_m}\left(y_m - S_y^2\right)\left(x_m - S_x^2\right) + 2\frac{\partial T_m}{\partial y_m \partial x_n}\left(y_m - S_y^2\right)\left(x_n - S_x^2\right) \\ &\left.+ 2\frac{\partial T_m}{\partial y_m \partial z_n}\left(y_m - S_y^2\right)\left(z_n - S_z^2\right) + +2\frac{\partial T_m}{\partial y_m \partial z_{n'}}\left(y_m - S_y^2\right)\left(z_{n'} - S_z^2\right)\right\} \\ &+ \ldots, \end{aligned}$$

$$\begin{aligned} T_m =& S_y^2 + K_1\left(y_m - S_y^2\right) + K_2\left(x_m - S_x^2\right) + K_3\left(x_n - S_x^2\right) + K_4\left(z_n - S_z^2\right) \\ &+ K_5\left(z_{n'} - S_z^2\right) + \frac{1}{2}\left\{K_{11}\left(y_m - S_y^2\right)^2 + K_{22}\left(x_m - S_x^2\right)^2 + K_{33}\left(x_n - S_x^2\right)^2\right. \\ &+ K_{44}\left(z_n - S_z^2\right)^2 + K_{55}\left(z_{n'} - S_z^2\right)^2 + 2K_{12}\left(y_m - S_y^2\right)\left(x_m - S_x^2\right) \\ &+ 2K_{13}\left(y_m - S_y^2\right)\left(x_n - S_x^2\right) + 2K_{14}\left(y_m - S_y^2\right)\left(z_n - S_z^2\right) \\ &\left.+ 2K_{15}\left(y_m - S_y^2\right)\left(z_{n'} - S_z^2\right)\right\} + \ldots, \end{aligned}$$

$$\begin{aligned}T_m =&G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{n'}^2\right) = S_y^2(1+e_4) + S_x^2(e_6-e_5)K_2 + S_z^2(e_8-e_7)K_4\\&+\frac{1}{2}\left\{\left(S_x^4\right)\left(e_6^2K_{22}+e_5^2K_{33}+2e_6e_5K_{23}\right)+\left(S_z^4\right)\left(e_8^2K_{44}+e_7^2K_{55}+2e_8e_7K_{45}\right)\right.\\&+2S_y^2S_x^2(e_4e_6K_{12}+e_4e_5K_{13})+2S_y^2S_z^2(e_4e_8K_{14}+e_4e_7K_{15})\\&\left.+2S_x^2S_z^2(e_6e_8K_{24}+e_6e_7K_{25}+e_5e_8K_{34}+e_5e_7K_{35})\right\},\end{aligned}\tag{7}$$

where

$$K_2 = \left.\frac{\partial G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)}{\partial s_{x_m}^2}\right|_{\left(S_y^2,S_x^2,S_x^2,S_z^2,S_z^2\right)},$$

$$K_3 = \left.\frac{\partial G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)}{\partial s_{x_n}^{*2}}\right|_{\left(S_y^2,S_x^2,S_x^2,S_z^2,S_z^2\right)},$$

$$K_4 = \left.\frac{\partial G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)}{\partial s_{z_n}^2}\right|_{\left(S_y^2,S_x^2,S_x^2,S_z^2,S_z^2\right)},$$

$$K_5 = \left.\frac{\partial G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)}{\partial s_z}\right|_{\left(S_y^2,S_x^2,S_x^2,S_z^2,S_z^2\right)},$$

where $K_2 = -K_3$ and $K_4 = -K_5$ and $(K_{22}, K_{33}, K_{44}, K_{55}, \ldots, K_{35})$ are the second order partial derivatives of order $G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_n'}^2\right)$ at the point $\left(S_y^2, S_x^2, S_x^2, S_z^2, S_z^2\right)$. The bias and MSE up to the first-order approximation of the estimator T, respectively, are

$$B(T) = \varphi B(T_u) + (1-\varphi)B(T_m) \tag{8}$$

and

$$\mathrm{MSE}(T) = \varphi^2\mathrm{MSE}(T_u) + (1-\varphi)^2\mathrm{MSE}(T_m), \tag{9}$$

where

$$\begin{aligned}B(T_u) =& \frac{1}{2}\left\{\left(S_z^4\right)(\lambda_{004}-1)\left(f_2I_{22}+f_2'I_{33}+2f_2'I_{23}\right)\right.\\&\left.+2S_y^2S_z^2(\lambda_{022}-1)\left(f_2I_{12}+f_2'I_{13}\right)\right\},\end{aligned}\tag{10}$$

$$\begin{aligned}B(T_m) =& \frac{1}{2}\left\{\left(S_x^4\right)(\lambda_{400}-1)(f_1K_{22}+f_1^*K_{33}+2f_1^*K_{23})\right.\\&+\left(S_z^4\right)(\lambda_{004}-1)\left(fK_{44}+f'K_{55}+2f'K_{45}\right)\\&+2S_y^2S_x^2(\lambda_{220}-1)(f_1K_{12}+f_1^*K_{13})\\&+2S_y^2S_z^2(\lambda_{022}-1)\left(fK_{14}+f'K_{15}\right)\\&\left.+2S_x^2S_z^2(\lambda_{202}-1)\left(fK_{24}+f'K_{25}+fK_{34}+fK_{35}\right)\right\},\end{aligned}\tag{11}$$

$$\begin{aligned}\mathrm{MSE}(T_u) = E\left(T_u - S_y^2\right)^2 = S_y^4 f_2^* (\lambda_{040} - 1) + S_z^4 \left(f_2 - f_2'\right)(\lambda_{004} - 1) I_2^2 \\ + 2S_y^2 S_z^2 \left(f_2 - f_2'\right)(\lambda_{022} - 1) I_2\end{aligned} \tag{12}$$

and

$$\begin{aligned}\mathrm{MSE}(T_m) = E\left(T_m - S_y^2\right)^2 = S_y^4 f_1 (\lambda_{040} - 1) + S_x^4 (f_1 - f_1^*)(\lambda_{400} - 1) K_2^2 \\ + S_z^4 \left(f - f'\right)(\lambda_{004} - 1) K_4^2 + 2S_x^2 S_y^2 (f_1 - f_1^*)(\lambda_{220} - 1) K_2 \\ + 2S_y^2 S_z^2 \left(f - f'\right)(\lambda_{022} - 1) K_4.\end{aligned} \tag{13}$$

The two samples are considered as non-overlapping; thus, $C(T_u, T_m)$ will be ignored.

4.4. *Minimum MSE of the classes of estimators T*

Let's consider

$$I_2 = \frac{-S_y^2(\lambda_{022} - 1)}{S_z^2(\lambda_{004} - 1)}, \quad K_2 = \frac{-S_y^2(\lambda_{220} - 1)}{S_x^2(\lambda_{400} - 1)}, \quad K_4 = \frac{-S_y^2(\lambda_{022} - 1)}{S_z^2(\lambda_{004} - 1)}. \tag{14}$$

Under above conditions the presented classes of estimators T_u and T_m are minimum. The MSE of the suggested estimator T is

$$\mathrm{MSE}(T)_{Min} = \varphi^2 \mathrm{MSE}(T_u)_{Min} + (1 - \varphi)^2 \mathrm{MSE}(T_m)_{Min} \tag{15}$$

$$\mathrm{MSE}(T_u)_{Min} = S_y^4 \left\{ f_2^* (\lambda_{040} - 1) - \left(f_2 - f_2'\right) \frac{1}{(\lambda_{004} - 1)} (\lambda_{022} - 1)^2 \right\}, \tag{16}$$

and

$$\begin{aligned}\mathrm{MSE}(T_m)_{Min} = S_y^4 \Bigg\{ f_1 (\lambda_{040} - 1) - (f_1 - f_1^*) \frac{1}{(\lambda_{400} - 1)} (\lambda_{220} - 1)^2 - \left(f - f'\right) \\ \frac{1}{(\lambda_{004} - 1)} (\lambda_{022} - 1)^2 \Bigg\}.\end{aligned} \tag{17}$$

Since the minimum MSE of the estimator T in Equation (15) is a function of the unknown scalar (constant) φ, it is minimum with respect to φ, and afterwards the optimum value of φ is obtained as

$$\varphi_{opt} = \frac{\mathrm{MSE}(T_m)_{Min}}{\mathrm{MSE}(T_u)_{Min} + \mathrm{MSE}(T_m)_{Min}} \tag{18}$$

Now putting the value of φ_{opt} in Equation (15), we have the optimum MSE of the estimator T as

$$\mathrm{MSE}(T)_{opt} = \frac{\mathrm{MSE}(T_u)_{Min} \cdot \mathrm{MSE}(T_m)_{Min}}{\mathrm{MSE}(T_u)_{Min} + \mathrm{MSE}(T_m)_{Min}} \tag{19}$$

4.5. Some special propositions

Here are two propositions regarding random non-response to test the performance of the estimators suggested above.

PROPOSITION 1. *When random non-response occurs only on the first occasion, the classes of estimators of population variance S_y^2 on the current occasion may be formulated as*

$$T^* = \varphi^* T_u^* + (1-\varphi^*)\, T_m, \tag{20}$$

where $T_u^* = F^*\left(s_{y_u}^{*2}, s_{z_u}^2, s_{z_u'}^2\right)$, $T_m = G\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{z_{n'}}^2\right)$ *and* $\varphi^*\,(0 \le \varphi^* \le 1)$ *is an unknown scalar (constant).*

Properties of classes of estimators T^*

$$B(T^*) = \frac{1}{2}\left\{\left(S_z^4\right)(\lambda_{004}-1)\left(f_2 I_{22} + f_2' I_{33} + 2 f_2 I_{23}\right)\right.$$
$$\left. + 2 S_y^2 S_z^2 (\lambda_{022}-1)\left(f_2 I_{12} + f_2' I_{13}\right)\right\}.$$

The bias of T_m is shown above in Equation (11).
The MSE of estimator T^* is obtained as

$$\mathrm{MSE}(T^*) = \varphi^* \mathrm{MSE}(T_u^*) + (1-\varphi^*)\,\mathrm{MSE}(T_m), \tag{21}$$

where

$$\begin{aligned} \mathrm{MSE}(T_u^*) =& S_y^4 f_2^* (\lambda_{040}-1) + S_z^4 \left(f_2 - f_2'\right)(\lambda_{004}-1) I_2^2 \\ &+ 2 S_y^2 S_z^2 \left(f_2 - f_2'\right)(\lambda_{022}-1) I_2. \end{aligned} \tag{22}$$

Also $\mathrm{MSE}(T_m)$ is shown in Equation (13).
The minimum MSE of estimator T^* is derived as

$$\mathrm{MSE}(T^*)_{Min} = \varphi^{*2} \mathrm{MSE}(T_u^*)_{Min} + (1-\varphi^*)^2\, \mathrm{MSE}(T_m)_{Min}, \tag{23}$$

where

$$\mathrm{MSE}(T_u^*)_{Min} = S_y^4 \left\{ f_2^* (\lambda_{040}-1) - \left(f_2 - f_2'\right) \frac{1}{(\lambda_{004}-1)} (\lambda_{022}-1)^2 \right\} \tag{24}$$

and the minimum MSE of T_m is shown in Equation (17).
Further,

$$\varphi^*_{opt} = \frac{\mathrm{MSE}(T_m)_{Min}}{\mathrm{MSE}(T_u^*)_{Min} + \mathrm{MSE}(T_m)_{Min}}. \tag{25}$$

Thus, the minimum MSE of the estimator T^* is defined as

$$\mathrm{MSE}(T^*)_{opt} = \frac{\mathrm{MSE}(T_u^*)_{Min} \cdot \mathrm{MSE}(T_m)_{Min}}{\mathrm{MSE}(T_u^*)_{Min} + \mathrm{MSE}(T_m)_{Min}}. \tag{26}$$

PROPOSITION 2. *When random non-response occurs only on the second (current) occasion, the classes of estimators of population variance S_y^2 on the current occasion may be formulated as*

$$T^{**} = \varphi^{**} T_u + (1-\varphi^{**}) T_m^{**}, \tag{27}$$

where $T_m^{**} = F^{**}\left(s_{y_m}^2, s_{x_m}^2, s_{x_n}^{*2}, s_{z_n}^2, s_{n'}^2\right)$ $T = \varphi T_u + (1-\varphi) T_m$ *and,* φ^{**} $(0 \le \varphi^{**} \le 1)$ *is an unknown scalar (constant).*

Properties of classes of estimators T^{**}

The bias of the estimators T_m^{**} is derived as

$$\begin{aligned} B(T_m^{**}) =& \frac{1}{2}\left\{\left(S_x^2\right)^2(\lambda_{400}-1)(f_1K_{22}+f_1^*K_{33}+2f_1^*K_{23})\right. \\ &+\left(S_z^2\right)^2(\lambda_{004}-1)\left(fK_{44}+f'K_{55}+2f'K_{45}\right) \\ &+2S_y^2S_x^2(\lambda_{220}-1)(f_1K_{12}+f_1^*K_{13}) \\ &+2S_y^2S_z^2(\lambda_{022}-1)\left(fK_{14}+f'K_{15}\right) \\ &\left.+2S_x^2S_z^2(\lambda_{202}-1)\left(fK_{24}+f'K_{25}+fK_{34}+f'K_{35}\right)\right\}. \end{aligned} \tag{28}$$

The MSE of estimators T^{**} up to the first-order approximation is derived as

$$\mathrm{MSE}(T^{**}) = \varphi^{**}\mathrm{MSE}(T_u) + (1-\varphi^{**})\mathrm{MSE}(T_m^{**}), \tag{29}$$

where $\mathrm{MSE}(T_u)$ is given in Equation (12) and

$$\begin{aligned} \mathrm{MSE}(T_m^{**}) = E\left(T_m - S_y^2\right)^2 =& \left(S_y^2\right)^2 f_1(\lambda_{040}-1) \\ &+\left(S_x^2\right)^2(f_1-f_1^*)(\lambda_{400}-1)K_2^2 \\ &+\left(S_z^2\right)^2\left(f-f'\right)(\lambda_{004}-1)K_4^2 \\ &+2S_x^2S_y^2(f_1-f_1^*)(\lambda_{220}-1)K_2 \\ &+2S_y^2S_z^2\left(f-f'\right)(\lambda_{022}-1)K_4. \end{aligned} \tag{30}$$

Using the values of derivatives given in Equations (14), the MSE of the classes of estimators T^{**} is derived as

$$\mathrm{MSE}(T^{**})_{Min} = \varphi^{**2}\mathrm{MSE}(T_u)_{Min} + (1-\varphi^{**})^2\,\mathrm{MSE}(T_m^{**})_{Min} \tag{31}$$

and

$$\begin{aligned} \mathrm{MSE}(T_m^{**})_{Min} =& S_y^4\left\{f_1(\lambda_{040}-1)+(f_1-f_1^*)\frac{1}{(\lambda_{400}-1)}(\lambda_{220}-1)^2\right. \\ &\left.-\left(f-f'\right)\frac{1}{(\lambda_{004}-1)}(\lambda_{022}-1)^2\right\}. \end{aligned} \tag{32}$$

Further, we have

$$\varphi^{**}_{\text{opt}} = \frac{\text{MSE}(T^{**}_m)_{Min}}{\text{MSE}(T_u)_{Min} + \text{MSE}(T^{**}_m)_{Min}} \tag{33}$$

and

$$\text{MSE}(T^{**})_{opt} = \frac{\text{MSE}(T_u)_{Min} \cdot \text{MSE}(T^{**}_m)_{Min}}{\text{MSE}(T_u)_{Mn} + \text{MSE}(T^{**}_m)_{Min}}. \tag{34}$$

4.6. *Efficiency comparisons*

We check the performance of the suggested classes of estimators T, T^* and T^{**} w.r.t the estimator τ, which does not involve the auxiliary information and is proposed for the complete response scenario. The estimator τ is defined as:

$$\tau = \psi\tau_u + (1-\psi)\tau_m, \tag{35}$$

where $\tau_u = s^2_{y_u}, \tau_m = S^2_{y_m}\left(\frac{s^2_{x_n}}{s^2_{x_m}}\right)$ and $\psi(0 \leq \psi \leq 1)$ is an unknown constant. The mean squared error of the estimator τ is formulated as:

$$M(\tau)_{Min} = \frac{V(\tau_u)M(\tau_m)}{V(\tau_u) + M(\tau_m)}, \tag{36}$$

where

$$V(\tau_u) = S^4_y[f_2(\lambda_{040} - 1)] \tag{37}$$

and

$$M(\tau_m) = S^4_y[f_1 * (\lambda_{400} + \lambda_{040} - 2\lambda_{220}) + f^*(2\lambda_{220} - \lambda_{400})]. \tag{38}$$

Thus, the $PRE's$ of the suggested estimators are as follows:

$$E = \left[\frac{M(\tau)_{Min}}{M(T)_{Opt}}\right] \times 100, E^* = \left[\frac{M(\tau)_{Min}}{M(T^*)_{Opt}}\right] \times 100, E^{**} = \left[\frac{M(\tau)_{Min}}{M(T^{**})_{Opt.}}\right] \times 100. \tag{39}$$

5. Numerical study

For empirical study a numerical comparison is done to investigate by employing the following two data sets taken from earlier studies in the literature. Data taken from Murthy (1967) called population 1 here, and data considered from Sukhatme and Sukhatme (1970), called population 2.

Population 1, data source: Murthy (1967), page-399.
Let y, x and z be the area under wheat in 1964,1963 and 1961, respectively. The data statistics are given as:

$$N = 34, \lambda_{004} = 2.8082, \lambda_{400} = 2.9122, \lambda_{040} = 3.7256,$$

$$\lambda_{022} = 2.9789, \lambda_{220} = 3.1045, \lambda_{202} = 2.7389$$

Population 2, data source: Sukhatme and Sukhatme (1970), page-185.
Let y, x and z are the area under wheat in 1937,1936 and 1931, respectively.
The data statistics are given as:

$$N = 34, \lambda_{040} = 3.5469, \lambda_{400} = 3.3815, \lambda_{oo4} = 2.7425,$$

$$\lambda_{20} = 2.5068, \lambda_{02} = 2.6868, \lambda_{202} = 2.0652.$$

5.1. *Simulation Study*

By using the statistical software R we carried out simulation studies relevant to our theoretical results. For our purpose data is generated from Normal (Gaussian) distributions, with given parameters for the study and the auxiliary variables. The population parameters for the data generated are given below:

Population 3
$N = 50, \lambda_{040} = 3.0128, \lambda_{400} = 2.6061, \lambda_{004} = 2.5541, \lambda_{220} = 2.7023, \lambda_{02} = 2.6395, \lambda_{202} =$ 2.5195.

5.2. *Interpretations of Empirical results*

One can note from Table 1 and 2, that

(i) $PRE's$ increase with the decreasing values of n, n', m, and u for constant values of the non-response probabilities p_1 and p_2,

(ii) $PRE's$ decrease with increasing values of p_2 for constant values of p_1, n, n', m, and u',

(iii) $PRE's$ decrease with increasing values of p_1 for constant values of p_2, n, n', m, and u',

(iv) $PRE's$ increase with decreasing values of p_2 for constant values of p_1, n, n', m, and u',

(v) $PRE's$ increase with decreasing values of p_1 for constant values of p_2, n, n', m, and u'.

Moreover, note that from Tables 3,4,5 and 6, it is obvious that the $PRE's$ follow the same sequence as that of note from Table 1 and 2.

From Tables 7 we observe that our proposed classes of estimators are more effective than the conventional one in absence of non-response also.

TABLE 1
Percentage Relative Efficiencies (E) of the proposed estimator when non-response occurs on both occasions.

Population→			I				II			
n=20, n'=27, u'=15			P_1				P_1			
$m\downarrow$		$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
11	9	0.05	179.53	165.48	153.08	142.06	148.39	146.23	143.96	141.57
		0.10	175.36	161.31	148.91	137.89	143.75	141.58	139.31	136.93
		0.15	171.57	157.52	145.12	134.10	139.45	137.29	135.02	132.63
		0.20	168.12	154.07	141.68	130.65	135.47	133.30	131.03	128.65
13	7	0.05	197.48	179.18	163.44	149.75	162.81	159.25	155.56	151.75
		0.10	193.63	175.32	159.58	145.89	159.12	155.56	151.88	148.06
		0.15	190.16	171.86	156.11	142.42	155.72	152.16	148.47	152.16
		0.20	187.04	168.73	152.99	139.30	152.57	149.00	145.32	141.51
15	5	0.05	212.98	190.17	170.99	150.63	177.05	171.50	165.86	160.12
		0.10	209.85	187.04	167.86	151.51	174.61	169.05	163.41	157.68
		0.15	207.05	184.24	165.06	148.71	172.34	166.79	161.14	155.41
		0.20	204.53	181.73	162.55	146.19	170.23	164.68	159.03	153.30
n=13, n'=24, u'=8										
$m\downarrow$		$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
6	7	0.05	174.47	163.96	154.24	145.21	131.72	130.48	129.17	127.78
		0.10	172.16	161.66	151.94	142.90	128.83	127.60	126.29	124.90
		0.15	169.93	159.43	149.70	140.67	126.03	124.79	123.48	122.09
		0.20	167.76	157.26	147.53	138.50	123.30	122.06	120.75	119.36
8	5	0.05	215.27	198.72	183.89	170.51	163.37	160.63	157.78	154.79
		0.10	212.96	196.42	181.58	168.20	160.96	158.22	155.36	152.37
		0.15	210.76	194.21	179.37	166.00	158.62	155.89	153.03	150.04
		0.20	208.65	192.10	177.27	163.89	156.38	153.64	150.78	147.80
10	3	0.05	255.27	231.73	211.17	193.06	198.51	193.18	187.72	182.13
		0.10	253.76	230.22	209.66	191.55	197.32	191.98	186.52	180.93
		0.15	252.31	228.77	208.21	190.10	196.16	190.82	185.36	179.77
		0.20	250.92	227.38	206.83	188.71	195.03	189.70	184.23	178.64
n=10, n'=22, u'=7										
$m\downarrow$		$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
4	6	0.05	171.73	163.65	155.98	148.71	128.59	127.78	126.92	126.00
		0.10	169.29	161.21	153.54	146.27	125.59	124.78	123.92	123.00
		0.15	166.92	158.83	151.17	143.89	122.67	121.86	121.00	120.08
		0.20	164.60	156.52	148.85	141.58	119.81	119.00	118.14	117.23
6	4	0.05	227.33	212.12	198.25	185.56	168.52	166.15	163.68	161.10
		0.10	225.03	209.81	195.94	183.26	166.22	166.86	161.39	158.81
		0.15	222.81	207.60	193.73	181.04	164.00	161.64	159.17	156.59
		0.20	220.69	205.47	191.60	178.92	161.85	159.48	157.01	154.43
8	2	0.05	283.88	259.86	238.61	219.68	216.33	210.81	210.17	199.40
		0.10	283.88	259.86	238.61	219.68	216.33	210.81	210.17	199.40
		0.15	283.88	259.86	238.61	219.68	216.33	210.81	210.17	199.40
		0.20	283.88	259.86	238.61	219.68	216.33	210.81	210.17	199.40

TABLE 2
Percentage Relative Efficiencies of the proposed estimator (E) when non-response occurs on both occasions (Simulated Data).

Population			III			
n=20, n'=27, u'=15			P_1			
$m\downarrow$	$u\downarrow$	P_2	0.05	0.1	0.15	0.2
11	9	0.05	157.29	146.60	136.71	127.54
		0.10	155.31	144.62	134.73	125.56
		0.15	153.42	142.74	132.85	123.68
		0.20	151.63	140.94	131.06	121.89
13	7	0.05	171.97	159.09	147.30	136.46
		0.10	170.25	157.37	145.58	134.73
		0.15	168.62	155.79	143.95	133.10
		0.20	167.07	154.59	142.40	131.56
15	5	0.05	185.89	170.73	156.97	144.41
		0.10	184.59	169.45	155.69	143.13
		0.15	183.38	168.23	154.47	141.92
		0.20	182.23	167.08	153.32	146.77
$n=13$, $n'=24$, $u'=8$			P_1			
$m\downarrow$	$u\downarrow$	P_2	0.05	0.1	0.15	0.2
6	7	0.05	167.57	156.77	146.73	137.36
		0.10	166.17	155.38	145.33	135.96
		0.15	164.80	154.00	143.96	134.59
		0.20	163.45	152.66	142.61	133.25
8	5	0.05	201.61	186.40	172.52	159.80
		0.10	200.37	185.17	171.29	158.57
		0.15	199.17	183.97	170.09	157.37
		0.20	198.00	182.80	168.92	156.20
10	3	0.05	236.56	216.48	198.39	182.00
		0.10	235.90	215.82	197.73	181.35
		0.15	235.27	215.18	197.09	180.71
		0.20	234.64	214.56	196.47	180.09
$n=10$, $n'=22$, $u'=7$			P_1			
$m\downarrow$	$u\downarrow$	P_2	0.05	0.1	0.15	0.2
4	6	0.05	168.43	158.76	149.69	141.15
		0.10	166.99	157.32	148.24	139.71
		0.15	165.57	155.90	146.82	138.29
		0.20	164.17	154.50	145.43	136.89
6	4	0.05	217.98	202.14	187.66	174.37
		0.10	216.82	200.98	186.50	173.21
		0.15	215.69	199.84	185.36	172.08
		0.20	214.59	198.74	184.26	170.97
8	2	0.05	270.88	247.82	227.12	208.43
		0.10	270.88	247.82	227.12	208.43
		0.15	270.88	247.82	227.12	208.43
		0.20	270.88	247.82	227.12	208.43

TABLE 3
Percentage Relative Efficiencies (E^*) of the estimator T^* when non-response occurs only on the second occasion.

Population→		I				II			
$n = 20$, $n' = 27$, $u' = 15$		P_1				P_1			
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
11	9	183.20	169.15	156.75	145.73	152.39	150.23	147.96	145.57
13	7	200.91	182.60	166.86	153.17	165.99	162.42	158.73	154.93
15	5	215.76	192.95	173.77	157.42	179.15	173.60	167.95	162.22
$n = 13$, $n' = 24$, $u' = 8$									
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
6	7	176.36	165.86	156.13	147.10	134.08	132.85	131.54	130.15
8	5	217.20	200.65	185.81	172.44	165.37	162.63	159.77	156.79
10	3	256.53	232.10	212.44	194.33	199.50	194.16	188.70	183.11
$n = 10$, $n' = 22$, $u' = 7$									
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
4	6	173.73	165.65	157.98	150.71	131.03	130.23	129.36	128.45
6	4	229.25	214.03	200.16	187.48	170.40	168.03	165.56	162.98
8	2	283.88	259.86	238.61	219.68	216.33	210.81	205.17	199.40

TABLE 4
Percentage Relative Efficiencies (E^*) of the estimator T^* when non-response occurs only on the second occasion (simulated data)

Population→		III			
$n = 20$, $n' = 27$, $u' = 15$		P_2			
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
11	9	194.09	189.73	185.70	181.97
13	7	213.74	209.16	205.01	201.22
15	5	231.79	227.43	223.53	220.02
$n = 13$, $n' = 24$, $u' = 8$					
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
6	7	187.07	184.74	182.46	180.24
8	5	239.66	237.12	234.69	232.36
10	3	284.96	283.00	281.13	279.34
$n = 10$,$n' = 22$, $u' = 7$					
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
4	6	198.22	195.84	193.51	191.23
6	4	260.81	258.28	255.85	253.51
8	2	329.36	329.37	329.37	329.37

TABLE 5
Percentage Relative Efficiencies (E^{**}) of the estimator T^{**} when non-response occurs only on the first occasion.

Population→		I				II			
$n=20$, $n'=27$, $u'=15$		P_1				P_2			
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
11	9	183.20	169.15	156.75	145.73	152.39	150.23	147.95	145.57
13	7	200.91	182.60	166.86	153.17	165.98	162.42	158.74	154.93
15	5	215.76	192.95	173.77	157.42	179.15	173.59	167.95	162.22
$n=13$, $n'=24$, $u'=8$									
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
6	7	176.36	165.86	156.13	147.10	134.08	132.85	131.54	130.15
8	5	217.20	200.65	185.81	172.44	165.37	162.63	159.77	156.79
10	3	256.53	232.10	212.44	194.33	199.50	194.16	188.70	183.11
$n=10$, $n'=22$, $u'=7$									
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
4	6	173.73	165.65	157.98	150.71	131.04	130.23	129.36	128.45
6	4	229.25	214.03	200.16	187.48	170.40	168.04	165.54	162.98
8	2	283.88	259.86	238.61	219.68	216.33	210.81	205.17	199.40

TABLE 6
Percentage relative efficiencies (E^{**}) of estimator T^{**} when non-response occurs only on the first occasion (Simulated Data).

Population→		III			
$n=20$, $n'=27$, $u'=15$		P_1			
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
11	9	159.38	148.69	138.80	129.63
13	7	173.79	160.91	149.12	138.28
15	5	187.22	172.08	158.32	145.76
$n=13$, $n'=24$, $u'=8$					
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
6	7	168.99	158.20	148.15	138.79
8	5	202.87	187.67	173.79	161.07
10	3	237.23	217.15	199.06	182.67
$n=10$, $n'=22$, $u'=7$					
$m\downarrow$	$u\downarrow$	0.05	0.1	0.15	0.2
4	6	169.90	160.23	151.15	142.61
6	4	219.17	203.33	188.84	175.56
8	2	270.88	247.82	227.12	208.43

TABLE 7
Percentage Relative Efficiencies (E) of proposed estimator, when there is no non response on both occasions.

Population→		I	II	III
$n = 20$, $n' = 27$, $u' = 15$				
$m \downarrow$	$u \downarrow$			
11	9	179.66	129.98	170.97
13	7	202.39	146.97	187.91
15	5	224.24	165.63	203.97
$n = 13$, $n' = 24$, $u' = 8$				
$m \downarrow$	$u \downarrow$			
6	7	174.35	116.69	180.63
8	5	219.74	148.04	219.59
10	3	266.03	185.34	259.65
$n = 10$, $n' = 22$, $u' = 7$				
$m \downarrow$	$u \downarrow$			
4	6	166.63	110.76	180.21
6	4	266.62	149.70	236.59
8	2	289.96	200.07	296.72

6. Conclusion

From the above discussion, we may conclude that the suggested class of estimators T, T^* and T^{**} contributes significantly to deal with the different realistic situations of random non-responses while estimating population variance on current (second) in two-occasion successive sampling using a two-phase setup. For multi-phase case readers refer to Bhatti (2012). This, the suggested classes of estimators in comparison with the estimator τ are highly rewarding in terms of increased precision of the estimates with reduced cost of the survey even when non-responding units are increasing on either of the occasions. Hence, the proposed class of estimators may be recommended for practical applications to the survey practitioners.

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