A CONVERSATION WITH ADELCHI AZZALINI AND NARAYANASWAMY BALAKRISHNAN

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On November 6, 2017, the Department of Statistical Sciences of the University of Bologna organized a workshop to celebrate the life and works of Antonella Capitanio, one year after her premature death. The event also represented the inauguration of the academic year for the PhD program in Statistical Sciences. In the opening of the event, the following conversation of Angela Montanari with Adelchi Azzalini and Narayanaswamy Balakrishnan took place.

1. CONVERSATION

Angela Montanari (A.M.): While thinking of how we could organize things during this conversation, I thought that it would have been nice to guide this conversation by using three main keywords. The first one is passion. I think that Antonella did everything she did with great passion. This doesn't mean that she did anything. I mean, I think that there was no way to have Antonella do something she didn't like to do. So the first thing she tried to do, when asked to do something that she didn't like, was to escape, but in case she could't escape she tried to work on herself until she liked the thing she was asked to do. And after liking it, she started doing it with great passion and commitment. And I think that all the research topics Antonella dealt with, were dealt with strong passion. She could not do research on a topic she didn't like. So the first thing I would like to talk about with Adelchi and Balakrishnan is how, according to their experience, Antonella fell in love with the research topics she worked on. I think she did a great job in dealing with the family of skew distributions and I think that this was not a thing she studied because she had to, but she studied because she really liked it from the beginning.

Adelchi Azzalini (A.A.): I can tell the exact date when our interaction started; it was on March 25, 1996. The occasion was a workshop held in Padua on March 25-26, 1996, to commemorate the figure of Oliviero Lessi. Oliviero, who happened to have

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been Antonella's PhD supervisor, had an appointment in Bologna while he was based in Padua. So he had come to Bologna to give a lecture and, on the way driving back to Padua, he had a fatal car accident on March 24, 1995. That workshop, one year later, had been organized to commemorate Oliviero. Antonella had regarded Oliviero as her primary reference point for research after completing the PhD with him, and therefore she was at that time searching for new research topics and new collaborators. We started talking in that occasion. She had read some earlier work that I had done with Alessandra Dalla Valle on the multivariate skew-normal distribution. I believe she was interested in the theme because it allowed her to work in the way she preferred, very precise and mathematically oriented. And that was, in my perception, in best agreement with her own aptitude.

Antonella has also worked in other areas of statistics, but clearly the context where she was more comfortable and more productive was mathematical statistics and applied probability. This was the perfect place for her.

From the day which I have indicated, we started cooperating. She used to come to Padua once in a while, we spent a day together and then we continued cooperation by email and by telephone. It was a very intense period. Right away, it was obvious, as it has already been remarked, that she was a very meticulous person and very passionate. She could spend endless hours in fine tuning certain results; it didn't matter how much time it took, the only concern was the result to achieve, in a fully satisfactory form.

Narayanaswamy Balakrishnan (N.B.): [A picture is shown, see Figure 1] When you get to my age, when you are old, time is short. Yesterday, we were talking about which year was this picture taken. For me, it looks like it was just a couple of years ago. But Bruno [Bruno Scarpa, ed.] corrected me, he said that it was actually in 2008. This was a workshop with many of the faces you see here – Reinaldo, Marc, Bruno [Reinaldo Arellano Valle, Marc Genton, Bruno Scarpa, ed.] – we were in Bertinoro and that was my first meeting with Antonella, whom you can see next to Adelchi. I used to joke with her that people change as they age. But, Antonella was time-invariant, she always looked the same. That was my first meeting with Antonella – at the Bertinoro workshop – but before, we exchanged several emails about work, reviews, etc. Then, we spent one evening discussing, and she was talking to me about her previous academic life. That was before "skew distributions", which was a common interest of ours. That is how we started discussing. She worked on distributions of quadratic forms, which was one another common interest of ours. That was my early interest. We discussed quite a bit and she explained how she came into skew distributions work after listening to one of Adelchi's presentations, and that was in 1996. What struck me most is - I didn't get to see her diligence and careful work until later on – her humility, how simple and humble she was. When you try to give credit, she would try to divert it towards someone else rather than taking credit for it. We then established a good collaboration, but I don't miss her as a collaborator, rather I miss her as a friend.

A.M.: Thank you. Now I think we can move to the second question I thought. This

is the opening event for our PhD programme and I think that maybe the PhD students don't know many things about skew distributions and they might fall in love with them just like Antonella did if you explain them what they are in the same way maybe you explained them to Antonella when you met her.

Figure 1 – "Workshop on Skew Symmetric Probability Distributions", Bertinoro, 2008. Among others, the participants in the picture are: Glòria Mateu-Figueras, Antonio Canale, Simona Pacillo, Narayanaswamy Balakrishnan, Arthur Pewsey, Bruno Scarpa, Márcia D'Elia Branco, Reinaldo Arellano Valle, Claudio Agostinelli (bottom row, kneeled, left to right), M. Chris Jones, Adelchi Azzalini, Antonella Capitanio, Anna Clara Monti, Heleno Bolfarine, Elena Stanghellini, Barry Arnold, Marc Genton, Andrea Tancredi, Nicola Loperfido, Chris J. Adcock (back row, standing).

A.A.: To say very briefly, the so-called area of 'skew-symmetric distributions' is an approach to construct and examine parametric families of distributions that enjoy a high degree of flexibility. Do not be misled by the term 'skew' because that is one feature connected to this formulation, and it is the more obvious one in a sense, but it is a type of construction which has a wider scope and a wider impact than simply dealing with skewness. If people — especially research students, post doctoral researchers, junior people — are interested in mathematical statistics having connections with applied probability, that is an interesting area to work in.

It has already being remarked several times that Antonella was very meticulous, very

precise and much into mathematical aspects. These are, of course, fundamental qualities. But even more important is another aspect: in the first place, she was concerned with high quality research. She was not concerned with the number of pages which were going to be printed. This is very different from the current style in many scientific environments, where there is a push by various actors, such as the PhD supervisors, to produce papers in a mechanical way, take the easy route and produce a paper, then make a little variation to produce another paper which differs from the previous one by six words, and go on. This was absolutely not what Antonella was interested in. Neither she was interested in hiding difficulties or problems in the paper. Often, there are publications where the numerical work is only included to highlight the good side of the construction, but the problematic aspects are hidden. Again, this was not what Antonella was pursuing. There was a strong trait of intellectual honesty. She was interested in producing quality research, in delivering the correct message, not to mislead the readers, without hiding problems or difficulties or issues with her research. In essence, once again: he was interested in high profile research. Since you asked me what the students could learn, I think this is the foremost thing that students ought to learn.

Figure 2 – Reinaldo Arellano-Valle, Antonella Capitanio, and Adelchi Azzalini (left to right).

N.B.: I think Adelchi has already pointed out the special characteristics that Antonella possessed and brought to the field. The question was what sort of lessons should

students take and how do they develop an interest in the subject. I am going to take a little technical angle on this. Perhaps it is not the right way, but at least I feel that's one direction that I looked at which students can view easily. I am by no means an expert on skewed models and skew-symmetric distributions. [A picture is shown, see Figure 2] That's a picture from the workshop in Bertinoro. I think we were having a dinner in one of the restaurants later. You see that Marc [Marc Genton, ed.] is missing here, but we can edit the picture by adding Marc in it – I think these four would probably in my view be considered as the most illustrious, well-revered experts on skewed models, on skew-symmetric distributions. Clearly, I should not be in that picture. I look - for the benefit of students - at skew-symmetric distributions slightly in a different angle.

If you go back in the literature, look at time-wise how various things developed. You can start in the 50s and 60s. There were some people who had written on non-normality. Non-normality was one of the themes, and it was basically to highlight what happens to statistical procedures based on normality when normality assumption is violated. I think, if I remember correctly, the earliest work in this direction goes back to Edgeworth. He wrote a very controversial paper entitled "Normality is a myth, there never was, and never will be a normal distribution in practice" ["Normality is a myth; there never was, and never will be, a normal distribution", Geary (1947), ed.]. This was a highly controversial statement at that time. So, many people started to follow this. One of the papers that motivated me when I was a student, long time ago before many of you were born, was a paper in Biometrika by Geary. He had written many papers on using Edgeworth's approximations to create non-normality. So you can create skewness or more kurtosis using the normal curve. Then, you would study what happens to the *t*-test, what happens to the *F*-test, what happens to the distribution of the sample correlation coefficient, etc. And this was in the 60s. Another person who played a significant role in this direction is A.K. Gayen, who also interestingly considered alternatives to bivariate normality through Edgeworth series.

Then in the 70s, there were a lot of papers coming on outlier models and robustness. These include the works of Barnett, Lewis, Murphy, etc. [Barnett and Lewis (1978), ed.], and they created non-normality through outliers and discussed robustness of estimators and procedures. Then came "contamination models", "mixture models", which were mainly due to John Tukey who wrote a paper in Sankhyā around 1967. They attacked a problem constructing two normals and mixtures of two normals and talked about how the efficiency of normality procedures varied. Now, this brings in the 80s and 90s the skew-symmetric models.

There are two ways to look at it. One of them will be to say that this provides some alternative means to carry out statistical inference compared to the classical method of approach that people used. Then you may ask: "Why is this different? How is this different compared to the three or four approaches that came before?" In my view, I think they were more a verification or an examination of a procedure. What happens to the test for correlation when normality is not satisfied? But in the skew-symmetric distribution, thanks to the four folks, you have a lot of procedures, not only you can examine their effects, but also you have procedures, fundamental methods by which you

can analyze data. This, I think, is the main motivation that the students can take. The students should see that this is not fundamentally a different way to attack a problem, but a more flexible, a more congenial way to analyze data. I think that's probably the take-away for students, PhD students and young researchers.

A.M.: What Adelchi said brings me to my second keyword, which indeed are two keywords together. I though about care and precision. I think that Antonella did everything with great care. And it was not because she was paranoid about care, but it was because she thought that doing things with care was a sign of respect towards the people that were to use the things she did. This applies to her teaching, this applies to the way she behaved, to the way she dressed. She was always very elegant, but this was not because she wanted to attract attention, it was a sign of respect towards the people who were dealing with her. So I think that, about her work, she was never completely satisfied about her results and she always did her best to improve those results. She used to work hard in order to reach all the goals she had fixed to herself. So my question about this second keyword is: "What are in your opinion the most interesting results that Antonella obtained in her work or that you obtained while working with her?"

A.A.: First of all, I must thank 'Bala' [N. Balakrishnan, ed.] for his kind words about our work. Of course, this theme is a cooperative work of a number of people, and surely not everything has been done by these four people, I mean these three in the picture and Marc Genton. There is a large community of people that contributed to it. But I do agree that some of the work produced by these four people had some significant impact on the community activity. Of course, this is a very broad question that could take a long time to discuss, but I pinpoint a single element that in my opinion was the turning point of the popularity of these methods. Before the publication of the 1999 paper with Antonella, the theme comprised some papers had been published either individually by myself, or in cooperation with Alessandra Dalla Valle, there were three papers altogether, and these had received very little consideration in the literature; they were almost invisible.

The turning point was in 1999 with the work of Azzalini and Capitanio (1999) published in the Journal of the Royal Statistical Society Series B [a slide with Azzalini and Capitanio (1999, p. 599) is shown]. Near the end, on page 599, there is a short section, of one page only, which however contains something which generated a lot of attention, because it creates a bridge between the construction in the earlier part of the paper, concerning the skew-normal distribution, and the world of elliptical distributions, that is, a very vast theme that had been considered by a large community over many years. This page connects the two domains. Technically, there is Lemma 1 leading to Corollary 2, which in practice says: if Equation (21) includes the density function f of elliptical type and, in the second component, the factor $W(y)$ is a linear function, then you get a proper density function for any choice of the factor G which is the symmetric-type distribution function. This specific construction did not obtain by itself much attention but, as I said, it allowed people to see a new area to explore and from that point on there was a

kind of explosion. Until 1999, the references to previous work on this theme were perhaps two or three. After our 1999 paper, those references turned into a multitude. This happened because people started looking at the previously-ignored earlier work. For instance, the 1996 paper with Alessandra Dalla Valle started receiving a lot of attention [Azzalini and Dalla Valle (1996), ed.].

If you ask me what Antonella's most prominent traits in research work, an important qualitative aspect is the pursue of high-quality work, as we have already underlined. On the more technical side, I was fascinated by her mastery of linear algebra. This skill was often most useful, but even decisive at certain points. Among these, her deep understanding of linear algebra was crucial in elaborating the idea of canonical form of skew-normal distributions; I still have a copy of her 1997 hand-written note with the core formulation. That idea appeared in our 1999, but later on she developed it substantially, leading to her 2012 work on `arXiv.org` (Capitanio, 2012).

N.B.: I have to first start with an admission that I have not collaborated so extensively and for so long with Antonella. But, our collaboration started in the summer of 2008. Antonella visited McMaster [McMaster University, ed.] for one month I think and Bruno Scarpa – who is here – also visited at the same time. I enjoyed their collaboration very much. In fact, I collaborated with Antonella separately and also I had some collaboration with Bruno and Antonella together. I realized quickly that the way Antonella worked and the way I worked were diametrically opposite. We don't work in the same way. We have a nice big room next to my office – room 207. In this room, you have blackboards all around and I like to scribble on the boards whenever I talk and I will never erase it, I will go from one board to another. There are 9 or 10 boards in this room. At the end of the discussion, we would start with some issues and I would say: "Well, that's that. I think it is done." And Antonella would say: "No, I need to think about this." She was very diligent, that's the way I would put it. She did not like to rush, she liked to take time, she liked to think, and to reflect. We used to discuss and then 2 or 3 days later she would come and say: "I like the way we were thinking" and then she would have some arguments.

I remember distinctly one work which is a very minor piece of work that we did. We were asked by the Journal of the Korean Statistical Society, they had a discussion paper of Chris Jones and they asked me to discuss this paper (Balakrishnan and Capitanio, 2008). So I gave it to Antonella and we started reading together, and we constructed some arguments to prove the properties. I think Adelchi put it very nicely that she likes to work things through no matter how much of a hassle it is. I had a premonition somehow that the tail behavior of this model of Jones is different on the right compared to the left. I kept telling her that it should be possible for us to show that the tail behavior of this model is different on either side. She constructed a very elegant argument purely on her own. I realized how careful she was, and how diligent she was in doing the details. So, that was part of her personality.

A.M.: I think that Antonella never stopped doing research. There is a paper which

is still unpublished but which is already receiving great attention and it is published on ArXiv [Capitanio (2012), ed.]. I think Adelchi has something to say about this paper.

A.A.: That paper on `arXiv.org` is, I would say, very typical of her. She did entirely on her own, and it fully reflects her way of working. It is precise, deep, and concise. Everything in there is inserted because it is necessary. The paper constitutes an important achievement which has received much less attention than it deserves. I see people using what essentially are her results without quoting her work. It would be good if indeed this paper takes a published form in a journal, because it does deserve it. The story of the paper reflects the nature of Antonella. She submitted it to a journal that rejected publication with comments that she recognized as sensible. She amended the paper and soon resubmitted it to another journal. There was a rejection again, but this time on the basis of unfriendly comments which she did not accept. So, she stopped pursuing publication because she felt kind of offended, I would say. For her, it was important to have obtained the results, not to add another line to the list of publications. It is good that Statistica is going to publish this piece of work.

N.B.: I was going to add a comment. When she was in McMaster, we discussed a lot about this unpublished work. I was not aware of the history because this unpublished work was mentioned in the workshop in Bertinoro by a few. I also know from Bertinoro workshop that somebody else had quoted the same result without giving due credit to Antonella. Only one important comment about this: one evening we went to a pub – I think Bruno was with us – we ordered some beer and nachos and we were eating, and she was explaining about this harrowing experience she had with the paper. The next day, when she came to the office, I told her we should try to put it for a good use, what can we do with this result, this is something to do with the canonical structure. We discussed – the three of us – how to use it effectively for developing an inferential result which we did complete, thanks to Antonella and Bruno Scarpa, and it did get published subsequently (Balakrishnan *et al.*, 2014). This unpublished work from Antonella is referred there. This paper did get published only due to the insistence of Bruno Scarpa, otherwise this would have been one more unpublished work.

A.M.: Thank you very much. What Balakrishnan said leads me directly to the last question of the second keyword, which is for benefit of our PhD students: "Which aspects regarding skew-symmetric distributions are still understudied and would benefit from further research in your opinion? Or is there any area that would take advantage from being contaminated by the skew distribution perspective?"

A.A.: The answer is yes, there are many themes within the broad area of skew-symmetric distributions which can be examined and explored. But we must again go back to what I said before – I know that I'm repeating what I already said but I want to stress it because I think it is fundamental. If there is a message that students should bring home from here is how to pursue research. And Antonella did it in the right way. Young

people are under pressure of publishing. This is natural because this is how the system works. However, one has to match up the pressure to publish with the quality of the work to be published. I want to say: one should consider publication but (s)he should not be obsessed with the number of publications. There are lots of published papers which are totally useless. We all know that. If I have to send a message to the students, I would say: do consider the importance of publishing your work, but you must be convinced that it is worth publication. Maybe you are right, maybe you are wrong, but you must be convinced that it is worth publication. If you are not convinced that it is of that level, leave it, or at least improve it. If your advisor pushes you to publish things which are some sort of remixing of existing results, with no new content, etcetera, then my best advise is to change advisor.

N.B.: I think that much of what Adelchi said applies to any area of statistics, not necessarily to skew-symmetric distributions. I am not an expert on skew-symmetric models, but I do keep myself aware of what is going on in the literature. There seems to be still a considerable amount of activity in the area and its applications. Lot of application, now in the field of finance, is coming up. The advice that Adelchi gave to young researchers to check the worthiness of publications to the worthiness of a study is very important. There must be still a lot of room to contribute to this area and I think that the talks that are going to be given following this presentation are going to indicate what are the possible avenues for further research and what are the areas of research that one can contribute to.

A.M.: Still regarding Antonella's research, I remember that one of the last things we talked about was an invitation that she had obtained from Iran to participate to the meeting and present a talk in the Meeting of the Iranian Statistical Society. She was very sad she couldn't go. But we applied for a grant and at the beginning she hoped that well, having money, she might have the chance of going there, which was never possible. I think that Adelchi knows something about the story of this invitation to Iran.

A.A.: Both of us had been invited to the Meeting of the Iranian Statistical Society for a plenary lecture, and Antonella was very keen on going. In her typical style, she immediately started reading, in addition to scientific matters, all about Iran, travel opportunities, and things to visit. Unfortunately, she soon felt that her health would not allow her. This happened several months before the meeting, and she decided to write to the organizers and say "I am sorry, but I won't come". That was very respectful of the organization, because she could have waited to see how her conditions evolved, but this choice could have later caused problems to the organization. This story says, again, how respectful she was of colleagues.

A.M.: Thank you. So now we move to the last keyword, that is smile, as I think that Antonella had a deep sense of humor and an innate attitude towards establishing friendly relationships. Our conversations always ended with the smile from both of us.

So, I would like to conclude this conversation by asking you to recollect and share with us some smiling episodes that you might have experienced during the many occasions you had to live with Antonella.

A.A.: That is certainly true, she had a very light attitude sense. She was cheerful. The picture I propose [a picture is shown, see Figure 3] is something that one day she drew on a little whiteboard for quick notes; she pictured herself, for no specific reason, probably to express her mood of the moment. I think that it reflects much of her: a cheerful smile as well as her huge hair, very typical; so I took a snapshot before the whiteboard was erased. Also, it has been remarked already that she was always very careful in dressing up properly, but she was similarly very careful at avoiding to impress people. She only wanted to look right. She was absolutely far away from overdoing it. Again, this is the same attitude she had at work. The work has to be right, the dress has to be right, and everything has to be right, and this was smart.
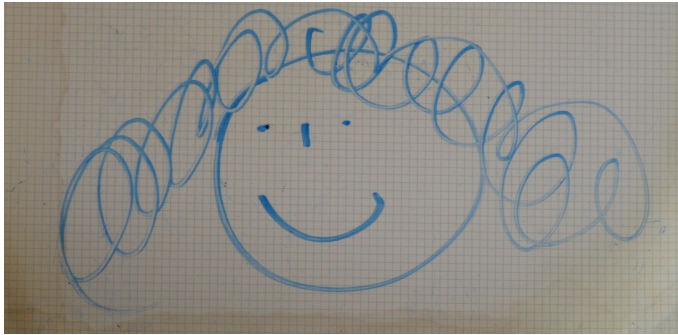

Figure 3 – Antonella's self-portrait.

N.B.: This was a picture taken in a Chinese restaurant in Hamilton where we used to go almost every week, Bruno would remember (see Figure 4). This was a group of visitors I had in 2008 summer. I want to point out two things. Bruno is sitting in the corner there, and on the other side is Antonella. The lady who is sitting to Antonella's right side is a former student of mine, Dr. Katherine Davies. You will notice her in all the pictures that were taken that summer and Antonella was always with Katherine Davies. They both became very good friends, very close friends. In fact, they used to talk on Skype every two weeks. Whenever Katherine said anything, Antonella will always laugh, with a big laugh. One time, I heard them sitting in the next room laughing. When we went for lunch and I asked Antonella "What did she say that is so funny you are always laughing?" And she said to me: "I do not understand what she says". I think Bruno also had the same problem, because Katherine speaks very fast and she speaks in a Canadian English. So Antonella said: "I always find her funny, I cannot understand what she says". My secretary of the department said two things. When I told them that I am coming to Bologna, they said that they can only remember two things about

Antonella: one is the smile and the other one is the smell. She always smelled nice [...].

Figure 4 – People in the picture are: Erhard Cramer, Marco Burkschat, George Iliopoulos, Peng Zhao, Bruno Scarpa, Anna Dembinska, H.K. Tony Ng, Donghoon Han, Katherine Davies, Antonella Capitanio, Sarah Balakrishnan, Narayanaswamy Balakrishnan.

I want to take this time to just reflect on a few things. To my knowledge, she had not traveled too much before. But, she told me that she had two dreams in her life. One was to travel to Chile, to go up North and go down South, and she told me that she wanted to see the variability, the north and the south. And the other dream was to go to India. She used to tell me that was one of her childhood dreams. I always told her that one time, when I go, she will go with me. I used to joke around a lot with her, she liked to laugh.

[A picture is shown, see Figure 5] I want to show this picture because I remember when she was in Hamilton, it was summer time. I used to walk around with a t-shirt and shorts. But she was always cold. Every room she walked in, she would say "I'm very cold". So, jokingly, I told her one day that I have to give her a spiced tea, and the spiced tea would make her warm. She immediately said it is really true, and I told her: "That's what my mother used to say". I don't know whether it is true because I don't drink tea. So, I said I will make her a tea. I brought to the office a spiced tea and I made her the spiced tea. She drank it and then told me that she was sick the night before and she felt

better after this tea. I doubt it, but I think it's psychological. But anyway, it has a long story because the night before she flew back to Bologna – I think Bruno will remember - she called me on Skype saying: Bala I am not well. And I asked her: "Do you want to postpone the travel back to Bologna?" because I was driving her to the airport the next day. She said: "No, but I want some spiced tea". So I brought her some tea packages for her to travel back with. She wrote me after she reached Bologna saying that the travel was better because of the tea. So, I sent this package to her home when she was not well, and she used to tell me on Skype that, whenever I call, that she drank a tea and made her feel much warmer inside. She had a very light sort of attitude towards life and a very gentle personality, and that brought a very good sense of humor about things. This was one of those occasions I still remember that when I go to India and my mother brings it, it brings back fun memories of Antonella. Thank you very much.

Figure 5 – Masala Tea.

also due to Adelchi Azzalini and Narayanaswamy Balakrishnan for their feedback on the conversation.

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SUMMARY

On November 6, 2017, the Department of Statistical Sciences of the University of Bologna organized a workshop to celebrate the life and works of Antonella Capitanio, one year after her premature death. The event also represented the inauguration of the academic year for the PhD program in Statistical Sciences. In this paper, the conversation that Angela Montanari had with Adelchi Azzalini and Narayanaswamy Balakrishnan on this occasion is reported.

Keywords: Antonella Capitanio; Skew-symmetric distributions; Skew-normal distribution.