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ON A CLASS OF TIME SERIES MODEL WITH DOUBLE LINDLEY DISTRIBUTION AS MARGINALS

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1. INTRODUCTION

The non-Gaussian time series models are increasingly important in real life situations as time series data cannot be always modeled by Gaussian distributions. Nowadays, the applications of non-Gaussian models can be seen in various fields such as economics, physics, climate sciences, and many others. In most of the real-life time series situations, the data are having skewed behavior, which substantiates the relevance of non-Gaussian modeling. Non-Gaussian time series models were introduced and discussed by many authors in the light of the non-normality of the many datasets in financial, actuarial, economic and other fields. See for example, Gaver and Lewis (1980), Lawrance and Lewis (1981), Sim (1986), Sim (1994), Ristic and Popovic (2002), Balakrishna and Nampoothiri (2003), etc. Further works related to non-Gaussian time series are extreme value autoregressive model by Balakrishna and Shiji (2014), autoregressive process with Birnbaum-Saunder's distribution as marginal by Rahul *et al.* (2018) and approximated beta distribution as marginal by Popovic (2010). In this paper, we use the non-Gaussian heavy tailed double Lindley distribution, recently introduced by the authors as the marginal of an additive autoregressive process. A brief outline of this distribution is presented in the next paragraph.

Lindley distribution defined on $[0, \infty)$ was introduced by Lindley (1958). An extensive study of its properties and application was done by Ghitany *et al.* (2008). Later many authors discussed different types of generalizations of the Lindley distribution. A generalization with a three-parameter Lindley distribution was given by Zakerzadeh and Dolati (2009). Further extensions of this distribution can be seen in Mahmoudi and Zakerzadeh (2010), Nadarajah *et al.* (2011), Ghitany *et al.* (2013), Elbatal *et al.* (2013),

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Bhati *et al.* (2015), Asgharzadeh *et al.* (2016), Maya and Irshad (2017), Asgharzadeh *et al.* (2018), and Ekhosuehi and Opone (2018). A discrete version of the Lindley distribution was discussed by Deniz and Ojeda (2011). Using the concept of mixing, Shibu and Irshad (2016) studied an extended version of the new generalized Lindley distribution, through which more flexibility can be obtained for the modelling of lifetime data. More recently, Irshad *et al.* (2020) considered binomial mixture Lindley distribution. Another work seen recently is a Marshall-Olkin extension of this distribution by Algarnia (2021). As an extension of the Lindley distribution, a new distribution on the real line was introduced by Nitha and Krishnarani (2017), called double Lindley distribution. It is a useful probability distribution for modelling heavy-tailed symmetric data sets that commonly arise in financial and actuarial data. Later, Satheeshkumar and Rosmi (2019) also studied this generalization of the Lindley distribution in which reliability properties of the same have been investigated. The double Lindley distribution with parameter θ, hereafter denoted as DLD (θ), is derived as a mixture of a Laplace random variable with mean zero, variance $2\theta^2$, and a two-sided gamma random variable with shape parameter 2 and scale parameter θ.

That is, the probability density function (pdf) of $DLD(\theta)$ is,

$$f(x) = \beta f_1(x) + (1-\beta) f_2(x), \tag{1}$$

where $\beta = \frac{\theta}{1+\theta}$, $f_1(x) = \frac{\theta}{2} e^{-\theta|x|}$, the pdf of a Laplace random variable with mean zero and variance $2\theta^2$ and $f_2(x) = \frac{\theta^2}{2}|x|e^{-\theta|x|}$, the pdf of a two sided gamma random variable with shape parameter 2 and scale parameter θ.

Clearly using (1) the pdf of a DLD random variable X with scale parameter θ, now takes the form

$$f(x) = \frac{\theta^2}{2(\theta+1)}(1+|x|)e^{-\theta|x|}, -\infty < x < \infty, \theta > 0 \tag{2}$$

and its characteristic function is given by

$$\phi_X(t) = \frac{\theta^2}{(1+\theta)(\theta^2+t^2)}\left[\theta + \frac{\theta^2 - t^2}{\theta^2+t^2}\right]. \tag{3}$$

Mean and variance of DLD(θ) are,

$$E(X) = 0 \text{ and } \mathrm{Var(X)} = \frac{2(\theta+3)}{\theta^2(1+\theta)}.$$

From the application point of view, Mazucheli and Achcar (2011) discussed Lindley distribution, as a competing risk lifetime model. The autoregressive model with Lindley distribution as marginal was introduced by Bakouch and Popovic (2016). But, further studies haven't been seen in the literature about the application of this distribution and its generalizations in the context of time series data analysis. So in this paper, we study the application of the double Lindley distribution in the analysis and forecasting of time series data.

The rest of the paper is organised as follows. In Section 2, we construct a first order autoregressive process with double Lindley distribution as marginal and study the probability distribution of the innovation process. In Section 3, we discuss the properties of the double Lindley autoregressive process. The parameters are estimated using the Gaussian method in Section 4. Estimation procedures are illustrated using simulated data in Section 5. In Section 6, the application of the model is discussed using the share price data of Bharath Petroleum Corporation Ltd., and the conclusion is given in the last Section.

2. First order Autoregressive model with Double Lindley as Marginal (DLDAR(1))

Consider a first order autoregressive process of the form,

$$X_n = aX_{n-1} + \epsilon_n, \tag{4}$$

where $|a| < 1$ and $\{X_n, n \geq 1\}$ is a stationary process with double Lindley distribution as marginals and $\{\epsilon_n\}$ is a sequence of independent and identically distributed (i.i.d) random variables, ϵ_n is independent of $X_i, (i < n)$. The practical applications and further studies of this process depend upon the distribution of the innovation sequence $\{\epsilon_n\}$. One of the common procedures to identify the innovation sequence is using the characteristic function.
Let $\phi_{\epsilon_n}(t)$ be the characteristic function of $\{\epsilon_n, n \geq 1\}$ and $\phi_{X_n}(t)$ be that of the $\{X_n\}$. Then, since $\{X_n\}$ is a stationary sequence, we get $\phi_{\epsilon_n}(t)$ as follows,

$$\phi_{\epsilon_n}(t) = \frac{\phi_{X_n}(t)}{\phi_{X_n}(at)}. \tag{5}$$

Since X_n is following the double Lindley distribution, its characteristic function is given by

$$\phi_{X_n}(t) = \frac{\theta^2}{(1+\theta)(\theta^2+t^2)}[\theta + \frac{\theta^2 - t^2}{\theta^2 + t^2}]. \tag{6}$$

Substituting this in equation (5), we obtain,

$$\phi_{\epsilon_n}(t) = \frac{\frac{\theta^2}{(1+\theta)(\theta^2+t^2)}\left[\theta + \frac{\theta^2-t^2}{\theta^2+t^2}\right]}{\frac{\theta^2}{(1+\theta)(\theta^2+a^2t^2)}\left[\theta + \frac{\theta^2-a^2t^2}{\theta^2+a^2t^2}\right]}. \tag{7}$$

From the above characteristic function of the innovation sequence, the distribution of the same is to be identified. But, because of its intricate structure, the usual inversion formula is not suitable in this case. So we have used the partial fraction decomposition method and considered two cases of the values of θ, $\theta = 1$ and $\theta \neq 1$.

The case of $\theta = 1$ which yields a moderately simple structure for the innovation is particularly considered. When $\theta = 1$ the above characteristic function takes the form

$$\phi_{\epsilon_n}(t) = \frac{(1+a^2t^2)[1+\frac{1-t^2}{1+t^2}]}{(1+t^2)[1+\frac{1-a^2t^2}{1+a^2t^2}]} \tag{8}$$

$$= \frac{(1+a^2t^2)^2}{(1+t^2)^2}. \tag{9}$$

Using the partial fraction decomposition method it can be written as,

$$\frac{(1+a^2t^2)^2}{(1+t^2)^2} = \left[\frac{A}{(1+t^2)^2} + \frac{B}{1+t^2} + C\right], \tag{10}$$

where $A = (1-a^2)^2$, $B = 2a^2(1-a^2)$ and $C = a^4$.

Hence,

$$\phi_{\epsilon_n}(t) = a^2 + (1-a^2)\left[\frac{(1-a^2)}{(1+t^2)^2} + \frac{2a^2}{1+t^2} - a^2\right]. \tag{11}$$

Then the random variable ϵ_n can be written as

$$\epsilon_n = \begin{cases} 0, & \text{with probability } a^4; \\ L_{1n}, & \text{with probability } 2a^2(1-a^2); \\ L_{2n} + L_{3n}, & \text{with probability } (1-a^2)^2, \end{cases} \tag{12}$$

where L_{in}, i = 1, 2, 3 are independent standard Laplace distributed random variables.

Using this, we can represent the random variable X_n as,

$$X_n = \begin{cases} aX_{n-1}, & \text{with probability } a^4; \\ aX_{n-1} + L_{1n}, & \text{with probability } 2a^2(1-a^2); \\ aX_{n-1} + L_{2n} + L_{3n}, & \text{with probability } (1-a^2)^2. \end{cases} \tag{13}$$

The sample path of the process (4) having DLD(θ) as marginal with $\theta = 1$ and for different values of a are plotted in Figure 1.

Next, we consider the case where $\theta \neq 1$. Here, we recall the characteristic function (7) of ϵ_n, and express it in the form,

$$\phi_{\epsilon_n}(t) = a^2 + (1-a^2)\left[A\frac{\theta^4}{(\theta^2+t^2)^2} + B\frac{\theta^2}{\theta^2+t^2} - C\frac{\theta^2(1+\theta)}{\theta^2(\theta+1)+(\theta-1)a^2t^2}\right]. \tag{14}$$

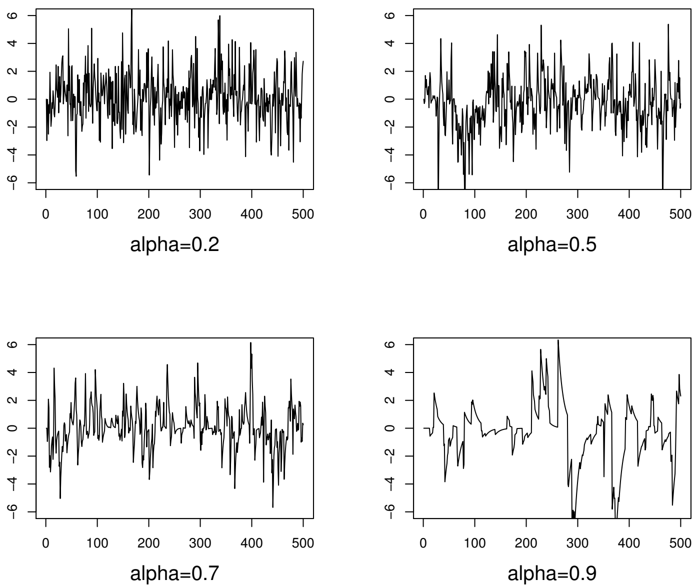

Figure 1 – Sample path of DLDAR(1) process for different values of a and $\theta = 1$.

Similarly, as in the above discussed case, we can derive the values of A, B and C as,

$$A = \frac{2(1-2a^2\theta - a^4 + \theta + a^4\theta)}{(a^2\theta - \theta - a^2 - 1)^2}, \tag{15}$$

$$B = \frac{(a^4\theta^2 - 4a^4\theta + 3a^4 - 2a^2\theta^2 + 4a^2\theta + 6a^2 + \theta^2 - 1)}{(a^2\theta - \theta - a^2 - 1)^2}, \tag{16}$$

$$C = \frac{4a^2}{(a^2\theta - \theta - a^2 - 1)^2}. \tag{17}$$

Therefore, we can represent the variable X_n as a generalized mixture of standard Laplace distributed random variables, which is given by,

$$X_n = aX_{n-1} + \begin{cases} 0, & \text{with probability } a^2; \\ L', & \text{with probability } (1-a^2); \end{cases} \tag{18}$$

where L' is the generalised mixture of Laplace random variables with characteristic function of the form,

$$\phi_{L'}(t) = [A\phi_{L'_{1n}+L'_{2n}}(t) + B\phi_{L'_{3n}}(t) - C\phi_{L'_{4n}}(t)]. \tag{19}$$

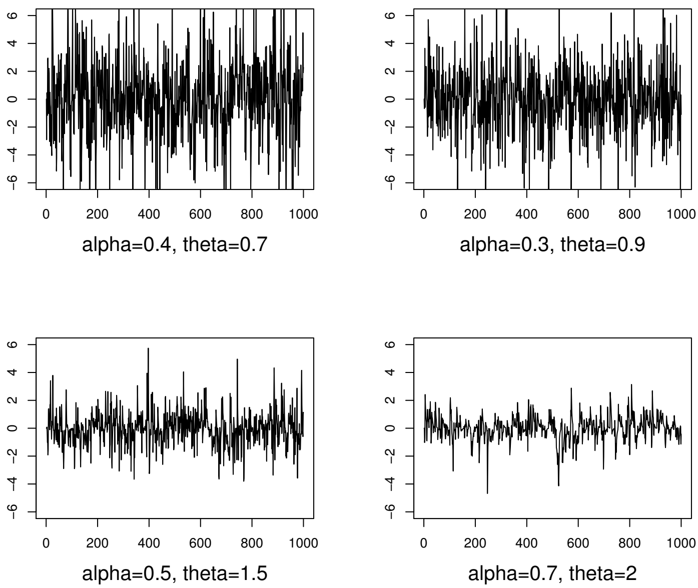

Figure 2 – Sample path of $DLDAR(1)$ process for different values of a and θ

Here L'_{in}, i = 1, 2, 3, 4 are independent Laplace distributed random variables with the characteristic functions,

$$\phi_{L'_{1n}+L'_{2n}}(t)=\frac{\theta^4}{(\theta^2+t^2)^2},\ \phi_{L'_{3n}}(t)=\frac{\theta^2}{\theta^2+t^2},\ \text{and}\ \phi_{L'_{4n}}(t)=\frac{\theta^2(1+\theta)}{\theta^2(\theta+1)+(\theta-1)a^2t^2}.$$

Note that, when θ=1 (18) has the form (13).

Now we define the DLDAR(1) model as follows.

DEFINITION 1. *A Markovian sequence* $\{X_n\}$,

$$X_n = aX_{n-1} + \epsilon_n, \tag{20}$$

where $|a| < 1$ *is said to be a first order double Lindley autoregressive process, if the sequence* $\{X_n\}$ *has a DLD(θ) distribution as marginal and* $\{\epsilon_n\}$ *has a mixture distribution with characteristic function (14).*

In the next section, we study the analytical properties of the DLDAR(1) process. The sample path behaviour of the process for different values of a and θ are depicted in Figure 2.

3. ANALYTICAL PROPERTIES OF THE DLDAR(1) PROCESS

The conditional statistical measures of DLDAR(1) process are derived by following the same steps as in Bakouch and Popovic (2016). From the equation (4) and because of the weak stationarity of the sequence, we can directly write that,

$$E(\epsilon_n) = (1-a)E(X_n) = 0.$$

The one step ahead conditional mean can be written as,

$$E(X_n|X_{n-1} = x_{n-1}) = ax_{n-1} + E(\epsilon_n). \tag{21}$$

Therefore, the expression for (k+1) step ahead conditional mean is given by,

$$E(X_{n+k}|X_{n-1} = x_{n-1}) = a^{k+1}x_{n-1}. \tag{22}$$

When $k \to \infty$,

$$E(X_{n+k}|X_{n-1} = x_{n-1}) \to 0, \tag{23}$$

which is the unconditional mean of the process.
Unconditional variance of the model can be identified by letting $k \to \infty$ in the expression,

$$\text{Var}(X_{n+k}|X_{n-1} = x_{n-1}) = (1-a^{2(k+1)})\frac{2(\theta+3)}{\theta^2(1+\theta)}. \tag{24}$$

When $k \to \infty$ we get,

$$\text{Var}(X_{n+k}|X_{n-1} = x_{n-1}) \to \frac{2(\theta+3)}{\theta^2(1+\theta)}, \tag{25}$$

which is equal to the unconditional variance of the process.
The general expression for the conditional characteristic function of the process is given by,

$$\phi_{X_{n+k}|X_{n-1}}(t) = e^{ita^{k+1}x_{n-1}}\phi_\epsilon\left(\frac{1-a^{k+1}}{1-a}t\right). \tag{26}$$

Therefore in the case of DLDAR(1) process, the above expression becomes

$$e^{ita^{k+1}x_{n-1}}\left[\frac{\theta^2(1-a^2)+a^2(1-a^{(k+1)})^2t^2}{(1-a^2)\theta^2+(1-a^{(k+1)})^2t^2}\right]^2 \times \left[\frac{\theta^2(1+\theta)(1-a)^2+(\theta-1)(1-a^{(k+1)})^2t^2}{\theta^2(1-a)^2(1+\theta)+(\theta-1)a^2(1-a^{(k+1)})^2t^2}\right]. \tag{27}$$

The joint characteristic function of (X_{n-1}, X_n) is given by,

$$\phi_{X_{n-1},X_n}(t_1,t_2) = \phi_{X_{n-1}}(t_1 + a t_2)[\phi_{\epsilon_n}(t_2)] \tag{28}$$

$$= \frac{\theta^2}{(1+\theta)(\theta^2+(t_1+at_2)^2)}\left(\theta + \frac{\theta^2-(t_1+at_2)^2}{\theta^2+(t_1+at_2)^2}\right)\times$$
$$\left[\left(\frac{\theta^2+a^2(t_2)^2}{\theta^2+(t_2)^2}\right)^2\left[\frac{\theta^2(1+\theta)+(\theta-1)(t_2)^2}{\theta^2(1+\theta)+(\theta-1)a^2(t_2)^2}\right]\right]. \tag{29}$$

Hence it follows that $\phi_{X_{n-1},X_n}(0,0) = 1$.
Also we get the autocovariance function of lag k,

$$\gamma(k) = a^k \frac{2(\theta+3)}{\theta^2(1+\theta)}, \tag{30}$$

and the autocorrelation function,

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = a^k. \tag{31}$$

In the next section we discuss the estimation of the unknown parameters of the process.

4. Estimation of the parameters

Since the mean function $E(X_t) = 0$, the conditional least square method of estimation cannot be employed in the DLDAR(1) model. The maximum likelihood method also does not yield an explicit solution as the log-likelihood function has a cumbersome form. So we use another method, called the Gaussian estimation method used by Bakouch and Popovic (2016), since our model DLDAR(1) is a generalized version of the Lindley autoregressive model discussed by them.

4.1. *Gaussian estimation method*

Whittle (1961) introduced this method where the author used normal likelihood as the basis function for the estimation and then for the analysis of correlated binomial data, it was used by Crowder (1985). Although this method is an approximation, it suits our model producing good and accurate estimates of the parameters. To apply this method of estimation, we need conditional expectation and variance. The conditional maximum likelihood function is given by

$$L(a,\theta) = f(x_1)\prod_{t=2}^{n} f(x_t|x_{t-1}). \tag{32}$$

Here $f(x_t|x_{t-1})$ and $f(x_1)$ respectively are the conditional and marginal probability functions of $\{x_t\}$. We assume normal distribution for both $f(x_1)$ and $f(x_t|x_{t-1})$. Then the log-likelihood function can be written as

$$log(L(a,\theta)) = nlog(\frac{1}{\sqrt{2\pi}}) - \frac{1}{2}\sum_{t=2}^{n}\left(log\sigma^2_{x_{t-1}} + \frac{(x_t - m_{x_{t-1}})^2}{\sigma^2_{x_{t-1}}}\right), \quad (33)$$

where $m_{x_{t-1}} = E(X_t|X_{t-1}) = ax_{t-1}$ and $\sigma^2_{x_{t-1}} = Var(X_t|X_{t-1}) = (1-a^2)\frac{2(\theta+3)}{\theta^2(1+\theta)}$. So, the Gaussian log-likelihood function corresponding to $DLDAR(1)$ process becomes,

$$\begin{aligned} log(L(a,\theta)) =& nlog\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{1}{2}\sum_{t=2}^{n}\Bigg[log\left((1-a^2)\frac{2(\theta+3)}{\theta^2(1+\theta)}\right) \\ &+ \left(\frac{\theta^2(1+\theta)}{2(1-a^2)(\theta+3)}\right)(x_t - ax_{t-1})^2\Bigg]. \end{aligned} \quad (34)$$

Clearly, here the likelihood has only a global optimum. The common method for obtaining the estimates involves, finding the partial derivatives of (34), with respect to a and θ, equating to zero and solving the resulting two equations. But direct solutions for the parameters a and θ are not possible. Therefore we have used a numerical method for calculating the estimates of the parameters. We use the R function `nlminb()` for this purpose by applying the Nelder-Mead method. For checking the performance of the model estimation, we use simulation techniques and mean squared error is used for the comparison of the accuracy of the procedure.

The asymptotic distribution and properties of the Gaussian estimators are stated next. This may be proved using the multidimensional Taylor series expansion around the point $(\hat{a},\hat{\theta})$ and assuming the regularity conditions for the partial derivatives with respect to the true values of the parameters. So we write the asymptotic distribution and properties of these estimates without proof for conciseness. The asymptotic distribution of $\sqrt(n)\Big((\hat{a},\hat{\theta}) - (a,\theta)\Big)$ is bivariate normal with mean vector $(0,0)'$ and covariance matrix $(I(a,\theta))^{-1}$ where,

$$I(a,\theta) = -E\begin{bmatrix} \frac{\partial^2}{\partial a^2}L(a,\theta) & \frac{\partial^2}{\partial a\partial\theta}L(a,\theta) \\ \frac{\partial^2}{\partial\theta\partial a}L(a,\theta) & \frac{\partial^2}{\partial\theta^2}L(a,\theta) \end{bmatrix}$$

and the maximum likelihood estimator $(\hat{a},\hat{\theta})$ is consistent.

5. Simulation study

The efficacy of the estimation method is verified in this section using simulation techniques. We simulated 100 samples of sizes 100, 500, 1000, 5000 and 10000 for different values of the parameters a and θ. The particular values considered for this purpose are (1) a=0.3 and θ=1.5 (2) a=0.5 and θ=2.5 (3) a=0.7 and θ=3.5. The plots of the sample path, autocorrelation function (ACF), and partial autocorrelation function (PACF) in one situation where a=0.3 and θ=1.5 is given in Figure 3. The estimates of the parameters and the corresponding mean square error (MSE) values are found. The results are presented in Table 1. It is evident that the Gaussian estimators are approaching the true parameter values when the sample size increases.

TABLE 1
Estimated values of a, θ and corresponding mean square error (MSE).

$a = 0.3, \theta = 1.5$				
Sample size	$\hat{a}$	$\hat{\theta}$	MSE($\hat{a}$)	MSE($\hat{\theta}$)
100	0.28	1.58	0.09	0.15
500	0.30	1.53	0.04	0.07
1000	0.30	1.53	0.03	0.05
5000	0.30	1.52	0.01	0.02
10000	0.30	1.52	0.01	0.02
$a = 0.5, \theta = 2.5$				
100	0.48	2.64	0.08	0.28
500	0.49	2.60	0.04	0.13
1000	0.50	2.57	0.03	0.09
5000	0.50	2.56	0.01	0.04
10000	0.50	2.51	0.01	0.03
$a = 0.7, \theta = 3.5$				
100	0.68	3.94	0.07	0.68
500	0.70	3.71	0.03	0.28
1000	0.69	3.69	0.02	0.20
5000	0.70	3.65	0.00	0.10
10000	0.70	3.64	0.01	0.06

In the next Section, we illustrate the application of the $DLDAR(1)$ process with a real data set.

6. Data Analysis

The significance and usability of the proposed model are established by applying it in a practical situation. We have considered the percentage difference of Bharath petroleum Corporation Ltd. (BPCL) share price index data (https:// www. moneycontrol.com) for the period from 23-10-2017 to 19-10-2018. The Kolmogorov-Smirnov test and the

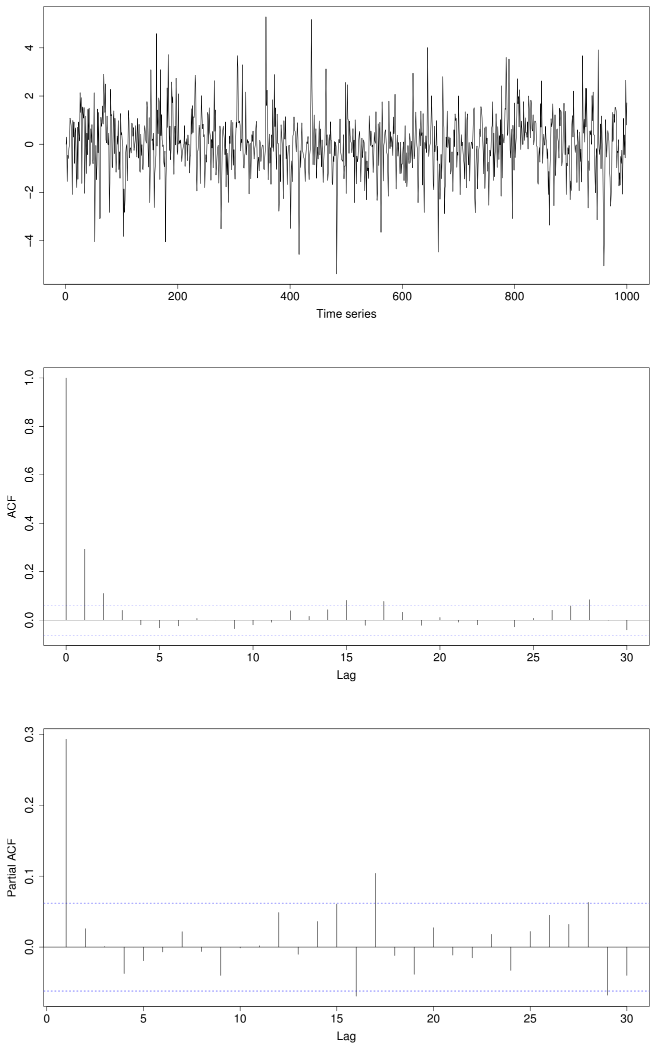

Figure 3 – Simulated time series, ACF, PACF plots corresponding to $a = 0.3$, $\theta = 1.5$.

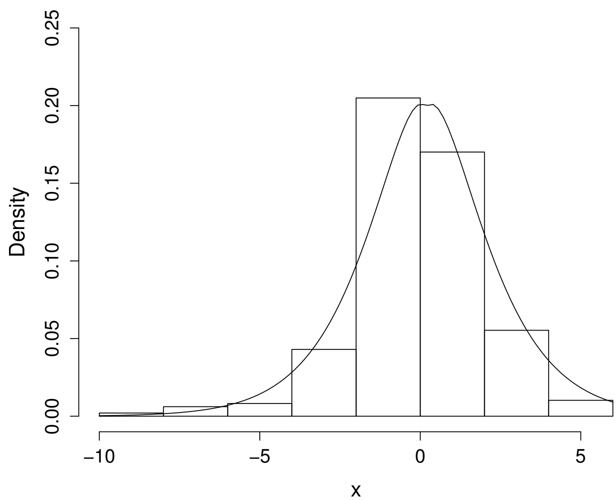

Figure 4 – Fitted Double Lindley distribution for the BPCL data.

corresponding p-value (0.10) validate that DLD(θ) distribution is suitable for fitting the differenced data. Hence we modeled the differenced data using DLDAR(1) (see Figure 4). From the plots of time series, ACF and PACF presented in Figure 5, it is visible that the ACF decays exponentially and PACF is significant at lag 1. Hence we use AR(1) model for this differenced data. The Gaussian estimation method is performed and the values of the parameters are obtained as $\hat{a} = 0.77$ and $\hat{\theta} = 0.18$. Akaike information criteria (AIC), Bayesian information criteria (BIC), root mean square error (RMSE) and mean absolute percentage error (MAPE) values are used for the comparison with other models. Comparison of DLDAR(1) with Gaussian AR(1) model is done and the values used for comparison are presented in Table 2. Although the values in Table 2 convey that DLDAR(1) model is equally competitive with the Gaussian AR(1) model, all these selection criteria values are slightly lesser in the DLDAR(1) model than the Gaussian model. Hence we choose DLDAR(1) as a better model. Residual analysis has been conducted and prediction of future values has also been done. The ACF and PACF of the residuals are within the limits (see Figure 6) and hence they are random. The predicted values are given in Figure 7.

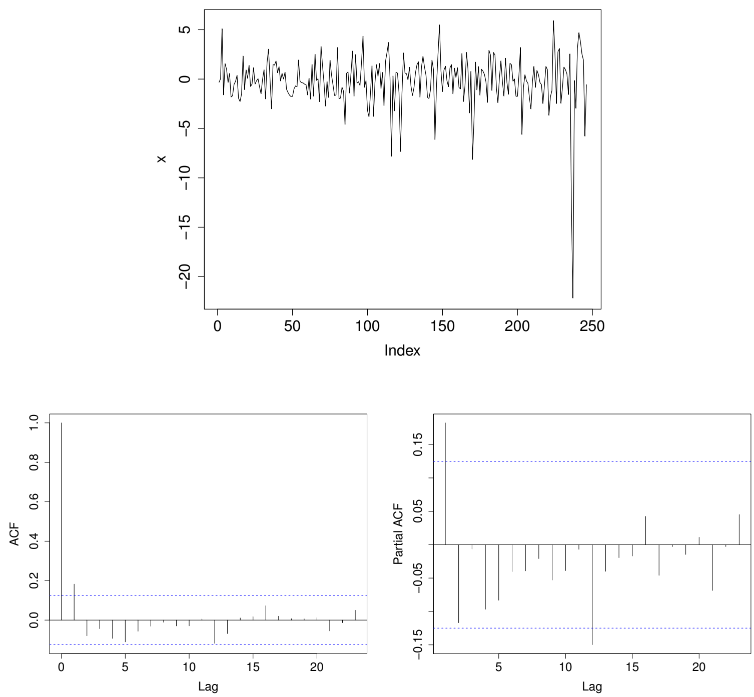

Figure 5 – ACF, PACF plots of the percentage difference of BPCL share price index data.

TABLE 2
AIC, BIC, RMSE and MAPE values of both DLDAR(1) and Gaussian AR(1) models.

Model	a	θ	μ	σ	AIC	BIC	RMSE	MAPE
DLDAR(1)	0.77	0.18	-	-	1173.32	1178.01	3.16	2.49
Gaussian AR	0.18	-	-0.24	2.6	1173.89	1178.99	3.29	4.10

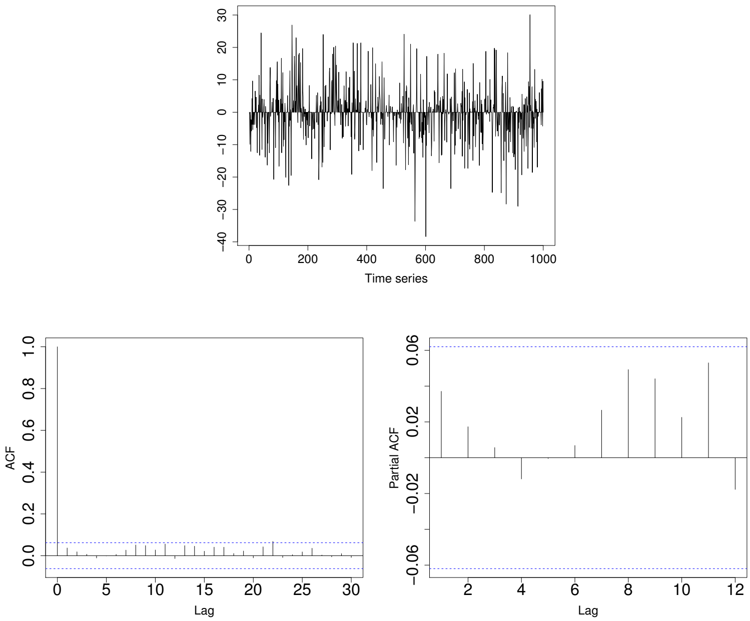

Figure 6 – Plots of residuals series and its ACF and PACF.

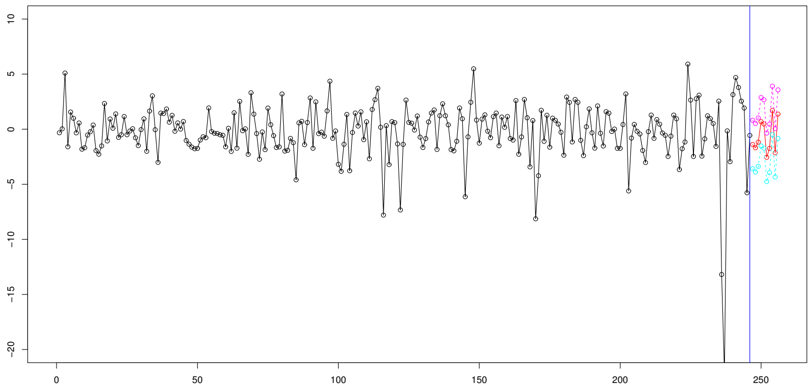

Figure 7 – Plot of the predicted values of BPCL data.

7. Conclusion

This paper has mainly focused on the analysis of time series data having non-Gaussian behavior. In particular, first order autoregressive process with double Lindley marginals is constructed, and a short account of its important properties is given. The key part of any modeling procedure is to establish the accuracy of the model and then the prediction of the future values using the model. These are done with respect to real data, which emphasizes the importance of double Lindley distribution in time series analysis.

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SUMMARY

An autoregressive process of order one with double Lindley distribution as marginal is introduced. A mixture distribution is obtained for the innovation process. Analytical properties of the process are discussed. The parameters of the process are estimated and simulation studies are done. Practical application of the process is discussed with the help of a real data set.

Keywords: Double Lindley distribution; Autoregressive process; Non-normal time series models; Stationarity.