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DUALS TO MOHANTY AND SAHOO'S ESTIMATORS

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1. INTRODUCTION

The use of an auxiliary variable x in the estimation of the finite population mean $\bar{Y}$ of the study variate y is a common occurrence in practice. A good illustration of this is the ratio method of estimation. It is known that ratio estimator attains minimum variance when the regression of y on x passes through the origin. Mohanty and Das (1971) was first to introduce the use of transformation of the auxiliary variate x in sample surveys to reduce the bias and mean squared error (MSE). Later, Reddy (1974), Reddy and Rao (1977), Srivenkataramana (1978), Chaudhuri and Adhikari (1979) and others have carried out a great deal of work in this direction. It is to be noted that most of these methods use the knowledge of unknown parameters $R=(\bar{Y}/\bar{X})$ and β (population regression coefficient of y on x) in the process of suggested transformation and hence have limited applicability.

This led Mohanty and Sahoo (1995) to suggest two linear transformations using known minimum and maximum values of the auxiliary variable x.

Consider a finite population $U=(U_1,U_2,U_3,...,U_N)$ of size N. The variate of interest y and the auxiliary variate x positively correlated to y, assume real non-negative values (y_i,x_i) on the unit $U_i(i=1,2,.....,N)$. Let $(\bar{Y},\bar{X})$ be the population means of y and x respectively. Assume that a simple random sample of size n is drawn without replacement (SRSWOR) from the population U. Then the traditional ratio estimator for the population mean $\bar{Y}$ is defined by

$$\bar{y}_R=(\bar{y}/\bar{x})\bar{X}\,, \tag{1}$$

where $\bar{y}=\sum_{i=1}^{n} y_i/n, \bar{x}=\sum_{i=1}^{n} x_i/n$ are the sample means of y and x respectively, and $\bar{X}$ is the known population mean of the auxiliary variate x.

Employing the transformation $x_i^*=(1+g)\bar{X}-gx_i\,, i=1,2,...,N$; with $g=n/(N-n)$; Srivenkataramana (1980) and Bandyopadhyay (1980) suggested a dual to ratio estimator for $\bar{Y}$ as

$$\bar{y}_a = \bar{y}(\bar{x}^* / \bar{X}) \tag{2}$$

where $\bar{x}^* = \{(1+g)\bar{X} - g\bar{x}\}$ such that $E(\bar{x}^*) = \bar{X}$.

Assuming that x_m (the minimum) and x_M (the maximum) of the auxiliary variate x are known, Mohanty and Sahoo (1995) suggested the linear transformations

$$z_i = \frac{x_i + x_m}{x_M + x_m}, \tag{3}$$

and

$$u_i = \frac{x_i + x_M}{x_M + x_m}, \tag{4}$$

$i = 1,2,.....N$ and consequently, suggested two ratio-type estimators for $\bar{Y}$ as

$$t_{1R} = \left(\frac{\bar{y}}{\bar{z}}\right)\bar{Z} \tag{5}$$

$$t_{2R} = \left(\frac{\bar{y}}{\bar{u}}\right)\bar{U} \tag{6}$$

where $\bar{z} = \sum_{i=1}^{n} z_i / n$ *and* $\bar{u} = \sum_{i=1}^{n} u_i / n$ such that $E(\bar{z}) = \bar{Z}$ and $E(\bar{u}) = \bar{U}$.

Employing Taylor's expansion under usual assumptions, the biases and mean squared errors (MSEs) of $\bar{y}_R, \bar{y}_a, t_{1R}$ and t_{2R} to terms of order $O(n^{-1})$ are obtained as follows.

$$B(\bar{y}_R) = (\theta/\bar{X})(RS_x^2 - \rho S_y S_x) \tag{7}$$

$$B(\bar{y}_a) = -(\theta/\bar{X})\, g\rho S_y S_x \tag{8}$$

$$B(t_{1R}) = \{\theta/(\bar{X}C_1)\}(R_1 S_x^2 - \rho S_y S_x) \tag{9}$$

$$B(t_{2R}) = \{\theta/(\bar{X}C_2)\}(R_2 S_x^2 - \rho S_y S_x) \tag{10}$$

$$MSE(\bar{y}_R) = \theta(S_y^2 + R^2 S_x^2 - 2R\rho S_y S_x) \tag{11}$$

$$MSE(\bar{y}_a) = \theta(S_y^2 + g^2 R^2 S_x^2 - 2gR\rho S_y S_x) \tag{12}$$

$$MSE(t_{1R}) = \theta(S_y^2 + R_1^2 S_x^2 - 2R_1 \rho S_y S_x) \tag{13}$$

$$MSE(t_{2R}) = \theta(S_y^2 + R_2^2 S_x^2 - 2R_2 \rho S_y S_x) \tag{14}$$

and the variance of $\bar{y}$ under SRSWOR is given by

$$Var(\bar{y}) = \theta S_y^2 \tag{15}$$

where

$$\rho = \frac{S_{xy}}{S_x S_y},\ S_y^2 = \sum_{i=1}^{N}(y_i - \bar{Y})^2/(N-1),\ S_x^2 = \sum_{i=1}^{N}(x_i - \bar{X})^2/(N-1),$$
$$R = \bar{Y}/\bar{X},\ S_{xy} = \sum_{i=1}^{N}(x_i - \bar{X})(x_i - \bar{X})/(N-1),\ R_1 = R/C_1,$$
$$R_2 = R/C_2,\ C_1 = 1 + \frac{x_m}{\bar{X}},\ C_2 = 1 + \frac{x_M}{\bar{X}},\ \theta = \left(\frac{1}{n} - \frac{1}{N}\right).$$

In this paper motivated by Srivenkataramana (1980) and Bandyopadhyaya (1980) we have suggested duals to Mohanty and Sahoo's (1995) estimators t_{1R} and t_{2R} and discussed their properties. Numerical illustration is given in the support of the present study.

2. THE SUGGESTED ESTIMATORS

We consider the following estimators for $\bar{Y}$ as

$$t_{1a} = \bar{y}(\bar{z}^*/\bar{Z}) \tag{16}$$

and

$$t_{2a} = \bar{y}(\bar{u}^*/\bar{U}) \tag{17}$$

where $\bar{u}^* = \sum_{i=1}^{n} u_i^*/n$ *and* $\bar{z}^* = \sum_{i=1}^{n} z_i^*/n$ are unbiased estimators of $\bar{U}$ *and* $\bar{Z}$ respectively,

$$u_i^* = (1+g)\bar{U} - g\,u_i,\ and\ z_i^* = (1+g)\bar{Z} - g\,z_i,\ \text{(i=1,2,.....,N).}$$

To the first degree of approximation, the biases and mean squared errors of t_{1a} and t_{2a} are respectively given by

$$B(t_{1a}) = -\{\theta/(\bar{X}C_1)\}\ g\rho S_y S_x \quad (18)$$

$$B(t_{2a}) = -\{\theta/(\bar{X}C_2)\}\ g\rho S_y S_x \quad (19)$$

$$MSE(t_{1a}) = \theta(S_y^2 + g^2 R_1^2 S_x^2 - 2gR_1\rho S_y S_x) \quad (20)$$

$$MSE(t_{2a}) = \theta(S_y^2 + g^2 R_2^2 S_x^2 - 2gR_2\rho S_y S_x) \quad (21)$$

We note that the exact formulae for the mean squared errors (MSEs) of the estimators t_{1a} and t_{2a} can be derived which is not the case for Mohanty and Sahoo's (1995) estimators t_{1R} and t_{2R}. As pointed out by Srivenkataramana (1980; p. 200) that terms terms involving $1/n^2$ and $1/n^3$ in the mean square errors can be neglected, provided N is large enough and n at least moderately large.

3. COMPARISON OF BIAS AND EFFICIENCY

3.1. *Comparison of Bias*

From (7), (8), (9), (10), (18) and (19), it follows that

(i) $|B(t_{1a})| < |B(\bar{y}_R)|$ if

$$\left|\frac{\beta}{C_1}\right| < \frac{(N-n)}{n}|R-\beta| \quad (22)$$

(ii) $|B(t_{1a})| < |B(\bar{y}_a)|$ if

$$\left|\frac{\beta}{C_1}\right| < |\beta| \quad (23)$$

which is always true.

(iii) $|B(t_{1a})| < |B(t_{1R})|$ if

$$|\beta| < \frac{(N-n)}{n}|R_1-\beta| \quad (24)$$

(iv) $|B(t_{1a})| < |B(t_{2R})|$ if

$$gC_2|\beta| < C_1|R_2-\beta| \quad (25)$$

(v) $\left|B(t_{2a})\right| < \left|B(\bar{y}_{\mathrm{R}})\right|$ if

$$g\left|\beta\right| < C_2\left|\mathrm{R}-\beta\right| \tag{26}$$

(vi) $\left|B(t_{2a})\right| < \left|B(\bar{y}_a)\right|$ if

$$\left|\frac{\beta}{C_2}\right| < \left|\beta\right| \tag{27}$$

which is always true.

(vii) $\left|B(t_{2a})\right| < \left|B(t_{1\mathrm{R}})\right|$ if

$$gC_1\left|\beta\right| < C_2\left|\mathrm{R}_1-\beta\right| \tag{28}$$

(viii) $\left|B(t_{2a})\right| < \left|B(t_{2\mathrm{R}})\right|$ if

$$\left|\beta\right| < \frac{(N-n)}{n}\left|\mathrm{R}_2-\beta\right| \tag{29}$$

(xi) $\left|B(t_{2a})\right| < \left|B(t_{1a})\right|$ if

$$x_m < X_M \tag{30}$$

which is always true.

3.2. *Comparison of Efficiency*

From (11), (12), (13), (14), (15), (20) and (21) it follows that

(i) $MSE(t_{1a}) < V(\bar{y})$ if

$$K > \frac{g}{2C_1} \tag{31}$$

(ii) $MSE(t_{1a}) < MSE(\bar{y}_{\mathrm{R}})$ if

$$K < \frac{1}{2}\left(1+\frac{g}{C_1}\right), C_1 > g \tag{32}$$

(iii) $MSE(t_{1a}) < MSE(\bar{y}_a)$ if

$$K < \frac{g(1+C_1)}{2C_1} \tag{33}$$

(iv) $MSE(t_{1a}) < MSE(t_{1R})$ if

$$K < \frac{(1+g)}{2C_1}, \quad f < \frac{1}{2} \tag{34}$$

(v) $MSE(t_{1a}) < MSE(t_{2R})$ if

$$\textit{either} \quad K < \frac{(C_1 + gC_2)}{2C_1C_2}, \quad f < \frac{C_1}{C_1 + C_2}$$

$$\textit{or} \quad K > \frac{(C_1 + gC_2)}{2C_1C_2}, \quad f > \frac{C_1}{C_1 + C_2} \tag{35}$$

(vi) $MSE(t_{2a}) < V(\bar{y})$ if

$$K > \frac{g}{2C_2} \tag{36}$$

(vii) $MSE(t_{2a}) < MSE(\bar{y}_R)$ if

$$K < \frac{1}{2}\left(1 + \frac{g}{C_2}\right), C_2 > g \tag{37}$$

(viii) $MSE(t_{2a}) < MSE(\bar{y}_a)$ if

$$K < \frac{g(1+C_2)}{2C_2} \tag{38}$$

(ix) $MSE(t_{2a}) < MSE(t_{1R})$ if

$$\textit{either} \quad K < \frac{1}{2}\left(\frac{1}{C_1} + \frac{g}{C_2}\right), \quad C_2 > gC_1$$

$$\textit{or} \quad K > \frac{1}{2}\left(\frac{1}{C_1} + \frac{g}{C_2}\right), \quad C_2 < gC_1 \tag{39}$$

(x) $MSE(t_{2a}) < MSE(t_{2R})$ if

$$K < \frac{(1+g)}{2C_2}, \quad f < \frac{1}{2} \tag{40}$$

(xi) $MSE(t_{2a}) < MSE(t_{1a})$ if

$$K < \frac{g(C_1 + C_2)}{2C_1C_2}, \tag{41}$$

where

$$K = (\rho C_y / C_x) = \frac{\beta}{\mathrm{R}}$$

Combining (31), (32), (33), (34), and (35) it is observed that

(i) t_{1a} is preferred over $\bar{y}$ and $\bar{y}_{\mathrm{R}}$, when

$$\frac{g}{2C_1} < K < \frac{1}{2}\left(1 + \frac{g}{C_1}\right),\ C_1 > g \tag{42}$$

(ii) t_{1a} is preferred over $\bar{y}$ and $\bar{y}_a$, when

$$\frac{g}{2C_1} < K < \frac{g(1 + C_1)}{2C_1} \tag{43}$$

(iii) t_{1a} is preferred over $\bar{y}$ and $t_{1\mathrm{R}}$, when

$$\frac{g}{2C_1} < K < \frac{(1 + g)}{2C_1} \tag{44}$$

(iv) t_{1a} is preferred over $\bar{y}$ and $t_{2\mathrm{R}}$, when

$$\textit{either} \quad \frac{g}{2C_1} < K < \frac{(C_1 + gC_2)}{2C_1C_2},\ f < \frac{C_1}{C_1 + C_2} \tag{45}$$

$$\textit{or} \quad K > \frac{1}{2}\left(\frac{1}{C_2} + \frac{g}{C_1}\right),\ f > \frac{C_1}{C_1 + C_2} \tag{46}$$

Further combining (42), (43) and (44) it is seen that t_{1a} is preferred over $\bar{y}$, $\bar{y}_{\mathrm{R}}$, $\bar{y}_a$ and $t_{1\mathrm{R}}$, when

$$\left.\begin{array}{ll} \textit{either} & \dfrac{g}{2C_1} < K < \dfrac{g(1 + C_1)}{2C_1},\quad gC_1 < 1 \\ \textit{or} & \dfrac{g}{2C_1} < K < \dfrac{(1 + g)}{2C_1},\quad gC_1 > 1 \end{array}\right\} \tag{47}$$

Again combining (45) and (47) we find that the estimator t_{1a} is better than $\overline{y}$, $\overline{y}_R$, $\overline{y}_a$, t_{1R} and t_{2R} if

$$\left.\begin{array}{ll} \textit{either} & \dfrac{g}{2C_1} < K < \dfrac{(C_1 + gC_2)}{2C_1C_2}, gC_1 > 1 \\ \textit{or} & \dfrac{g}{2C_1} < K < \dfrac{g(1+C_1)}{2C_1}, \;\; gC_1 < 1 \end{array}\right\} \tag{48}$$

Now combining (36), (37), (38), (39), (40), and (41) we find that

(i) t_{2a} is preferred over $\overline{y}$ and $\overline{y}_R$, when

$$\frac{g}{2C_2} < K < \frac{1}{2}\left(1 + \frac{g}{C_2}\right) \tag{49}$$

(ii) t_{2a} is preferred over $\overline{y}$ and $\overline{y}_a$, when

$$\frac{g}{2C_2} < K < \frac{g(1+C_2)}{2C_2} \tag{50}$$

(iii) t_{2a} is preferred over $\overline{y}$ and t_{1R}, when

$$\textit{either} \;\; \frac{g}{2C_2} < K < \frac{1}{2}\left(\frac{1}{C_1} + \frac{g}{C_2}\right), \;\; C_2 > gC_1 \tag{51}$$

$$\textit{or} \;\; K > \frac{1}{2}\left(\frac{1}{C_1} + \frac{g}{C_2}\right), \;\; C_2 < gC_1 \tag{52}$$

(iv) t_{2a} is preferred over $\overline{y}$ and t_{2R}, when

$$\frac{g}{2C_2} < K < \frac{(1+g)}{2C_2} \tag{53}$$

(v) t_{2a} is preferred over $\overline{y}$ and t_{1a}, when

$$\frac{g}{2C_2} < K < \frac{(C_1 + C_2)g}{2C_1C_2} \tag{54}$$

Further combining (49), (50), (53) and (54) it is observed that t_{2a} is better than $\overline{y}$, $\overline{y}_R$, $\overline{y}_a$, t_{2R} and t_{1a} when

$$\frac{g}{2C_2} < K < \frac{g(1+C_2)}{2C_2} \tag{55}$$

Again, combining (51) and (55) we find that estimator t_{2a} is more efficient than $\bar{y}$, $\bar{y}_{\text{R}}$, $\bar{y}_a$, $t_{1\text{R}}$, $t_{2\text{R}}$ and t_{1a} if

$$\textit{either} \qquad \frac{g}{2C_2} < K < \frac{g(1+C_2)}{2C_2},\ gC_2 < 1 \tag{56}$$

$$\textit{or} \qquad \frac{g}{2C_2} < K < \frac{(1+g)}{2C_2}, \quad \frac{1}{C_2} < g < \frac{C_2}{C_1} \tag{57}$$

4. UNBIASED ESTIMATION

It is observed that the estimators t_{1a} and t_{2a} are biased. In some applications biasedness is disadvantageous. This led us to investigate unbiased estimators of $\bar{Y}$.

If there is no correlation between the study variate y and the auxiliary variate x, then $B(t_{ja}) = 0,\ j = 1,2;$ and hence the estimators t_{1a} and t_{2a} are unbiased. But this situation is not good since there will be an unacceptable increase in variance relative to the usual unbiased estimator $\bar{y}$. Owing to this we consider the following alternatives.

4.1. *Unbiased Product Estimators For Interpenetrating Subsample Design*

Consider the case of interpenetrating sub sample discussed by Murthy (1964). Let y_i and x_i be unbiased estimates of the population totals Y and X respectively based on the ith independent interpenetrating subsample, i =1,2,.....,n. Consider now the following estimators

$$t_{1a}^{(1)} = \left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} z_i^*\right)\Big/\bar{Z} \tag{58}$$

$$= \bar{y}\bar{z}^*/\bar{Z}$$

$$t_{1a}^{(2)} = \sum_{i=1}^{n} y_i z_i^* / (n\bar{Z}) \tag{59}$$

$$t_{2a}^{(1)} = \left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} u_i^*\right)\Big/\bar{U} \tag{60}$$

$$=\bar{y}\bar{u}^{*}/\bar{U}$$

$$t_{2a}^{(2)}=\sum_{i=1}^{n}y_{i}u_{i}^{*}/(n\bar{U}) \tag{61}$$

as estimators for the population mean $\bar{Y}$. Following Murthy (1964), we can show that

$$B(t_{1a}^{(2)})=nB(t_{1a}^{(1)}) \tag{62}$$

and

$$B(t_{2a}^{(2)})=nB(t_{2a}^{(1)}) \tag{63}$$

and hence that

$$t_{3a}=(nt_{1a}^{(1)}-t_{1a}^{(2)})/(n-1) \tag{64}$$

and

$$t_{4a}=(nt_{2a}^{(1)}-t_{2a}^{(2)})/(n-1) \tag{65}$$

are unbiased estimators of the population mean $\bar{Y}$.

The conditions for t_{3a} and t_{4a} to be more efficient than $t_{1a}^{(1)}$ and $t_{2a}^{(1)}$ respectively are similar to those given in Murthy and Nanjamma (1959) in the case of obtaining an almost unbiased ratio estimator.

Motivated by the approach illustrated in Rao (1981), we consider more generally, the estimator

$$T_{1P}=\delta t_{1a}^{(1)}+\{1-E(f(\delta))\}t_{1a}^{(2)} \tag{66}$$

where δ is random and $f(\delta)$ is a function of δ.

Then T_{1P} is unbiased for $\bar{Y}$ if

$$E(T_{1P})=\bar{Y}$$

i.e. if $E[\delta t_{1a}^{(1)}-E(f(\delta))t_{1a}^{(2)}]=E(\bar{y}-t_{1a}^{(2)})$ (67)

for which

$$\delta=\frac{\bar{Z}}{\bar{z}^{*}} \text{ and } f(\delta)=\frac{\delta\bar{z}^{*}}{\bar{Z}}$$

is a solution.

We further write (67) as

$$E[\delta\, t_{1a}^{(1)} - E(f(\delta))\,t_{1a}^{(2)}] = E[\overline{y} - t_{1a}^{(2)} + \alpha\, t_{1a}^{(1)} - \alpha\, t_{1a}^{(1)}] \tag{68}$$

where α is a scalar,
From the relation

$$B(t_{1a}^{(1)}) = \frac{1}{n} B(t_{1a}^{(2)})$$

$$\Rightarrow E[t_{1a}^{(1)} - \overline{Y}] = \frac{1}{n} B(t_{1a}^{(2)})$$

$$\Rightarrow E(t_{1a}^{(1)}) = \overline{Y} + \frac{1}{n} B(t_{1a}^{(2)})$$

$$= \overline{Y} + \frac{1}{n} E[t_{1a}^{(2)} - \overline{Y}]$$

$$= (1 - \frac{1}{n})\overline{Y} + \frac{1}{n} E(t_{1a}^{(2)})$$

$$= \left(\frac{n-1}{n}\right)\overline{Y} + \frac{1}{n} E(t_{1a}^{(2)})$$

i.e.

$$E(t_{1a}^{(1)}) = C\overline{Y} + (1-C)E(t_{1a}^{(2)}) \tag{69}$$

where C = (n-1)/n.
Putting (69) in (67) we have

$$E[\delta\, t_{1a}^{(1)} - E(f(\delta))\,t_{1a}^{(2)}] = E[\overline{y} - t_{1a}^{(2)} + \alpha\, t_{1a}^{(1)} - \alpha\{C\,\overline{y} + (1-C)\,t_{1a}^{(2)}\}]$$

$$= E\left[(1-\alpha C)\overline{y} + \alpha\,\overline{y}\,\frac{\overline{z}^*}{\overline{Z}} - \{1 + \alpha(1-C)\}\,t_{1a}^{(2)}\right]$$

$$= E\left[\left\{\alpha + (1-\alpha C)\frac{\overline{Z}}{\overline{z}^*}\right\} t_{1a}^{(1)} - \{1 + \alpha(1-C)\}\,t_{1a}^{(2)}\right]$$

which gives a solution

$$\delta = \alpha + (1-\alpha C)\frac{\overline{Z}}{\overline{z}^*} \quad \text{and} \quad f(\delta) = \delta\frac{\overline{z}^*}{\overline{Z}}$$

for which

$$E(f(\delta)) = \{\alpha + (1-\alpha C)\} = \{1 + (1-C)\alpha\}$$

Thus putting $\delta = \left\{\alpha + (1-\alpha)\frac{\bar{Z}}{\bar{z}^*}\right\}$ and $E(f(\delta)) = \{1 + (1-C)\alpha\}$ in (66) we get a general class of estimators of population mean $\bar{Y}$ as

$$T_{1P} = \left[\left\{\alpha + (1-\alpha C)\frac{\bar{Z}}{\bar{z}^*}\right\} t_{1a}^{(1)} - \alpha(1-C)\, t_{1a}^{(2)}\right]$$

or

$$T_{1P} = [(1-\alpha C)\bar{y} + \alpha\, t_{1a}^{(1)} - (1-C)\alpha\, t_{1a}^{(2)}] \tag{70}$$

Remark 4.1. For $\alpha = 0$, T_{1P} reduces to the usual unbiased estimator $\bar{y}$ while $\alpha = C^{-1}$ gives the estimator t_{3a} in (64) when $\alpha = (1-C)^{-1}$. We get another estimator

$$\hat{T}_{1P}^{(1)} = (2-n)\bar{y} + n\, t_{1a}^{(1)} - t_{1a}^{(2)} \tag{71}$$

Many other unbiased estimators can be generated from T_{1P} at (70) just by putting suitable value of α.

Remark 4.2. Proceeding in a similar way we obtain another class of estimators of $\bar{Y}$ as

$$T_{2P} = [(1-\alpha_1 C)\bar{y} + \alpha_1\, t_{2a}^{(1)} - (1-C)\alpha_1\, t_{2a}^{(2)}] \tag{72}$$

where α_1 is a suitable chosen scalar, for $\alpha_1 = 0$, T_{2P} boils down to the estimator $\bar{y}$ while for $\alpha_1 = C^{-1}$ it reduces to the estimator

$$T_{2P}^{(1)} = \frac{1}{C} t_{2a}^{(1)} - \frac{(1-C)}{C} t_{2a}^{(1)}$$

$$= t_{4a} \tag{73}$$

when $\alpha_1 = (1-C)^{-1}$ we get another unbiased estimator of $\bar{Y}$ as

$$T_{2P}^{(2)} = [(2-n)\bar{y} + n\, t_{2a}^{(1)} - t_{2a}^{(2)}] \tag{74}$$

many other unbiased estimators can be generated from (72) just by putting suitable value of α_1.

4.2. *Optimum Estimator in the Class* T_{1P}

From (70) we have

$$Var(T_{1P}) = (1-\alpha C)^2 Var(\overline{y}) + \alpha^2 Var(t_{1a}^{(1)}) + (1-C)^2 \alpha^2 Var(t_{1a}^{(2)})$$
$$+2\alpha(1-\alpha C)Cov(\overline{y}, t_{1a}^{(1)}) - 2\alpha(1-C)(1-\alpha C)Cov(\overline{y}, t_{a}^{(2)}) - 2\alpha^2(1-C)Cov(t_{1a}^{(1)}, t_{1a}^{(2)})$$
$$= Var(\overline{y}) + \alpha^2[C^2 Var(\overline{y}) + Var(t_{1a}^{(1)}) + (1-C)^2 Var(t_{1a}^{(2)})$$
$$-2C\,Cov(\overline{y}, t_{1a}^{(1)}) + 2C(1-C)Cov(\overline{y}, t_{1a}^{(2)}) - 2(1-C)Cov(t_{1a}^{(1)}, t_{1a}^{(2)})]$$
$$-2\alpha[C\,Var(\overline{y}) - Cov(\overline{y}, t_{1a}^{(1)}) + 2(1-C)Cov(\overline{y}, t_{1a}^{(2)})]$$
$$= Var(\overline{y}) + \alpha^2 B - 2\alpha A \tag{75}$$

where

$$A = [C\,Var(\overline{y}) - Cov(\overline{y}, t_{1a}^{(1)}) + 2(1-C)Cov(\overline{y}, t_{1a}^{(2)})]$$

$$B = [C^2 Var(\overline{y}) + Var(t_{1a}^{(1)}) + (1-C)^2 Var(t_{1a}^{(2)}) - 2C\,Cov(\overline{y}, t_{1a}^{(1)})$$
$$+ 2C(1-C)Cov(\overline{y}, t_{1a}^{(2)}) - 2(1-C)Cov(t_{1a}^{(1)}, t_{1a}^{(2)})]$$

The variance of T_{1P} at (75) is minimized for

$$\alpha_{opt} = \frac{A}{B}$$
$$= \frac{Cov(\overline{y}, t)}{Var(t)} \tag{76}$$

where

$$t = (C\,\overline{y} - t^*),\; t^* = [t_{1a}^{(1)} - (1-C)t_{1a}^{(2)}].$$

Thus the resulting minimum variance of T_{1P} is given by

$$\min.Var(T_{1P}) = Var(\overline{y})(1-\rho^{*2}) \tag{77}$$

where $\rho^* = \dfrac{Cov(\overline{y}, t)}{\sqrt{Var(\overline{y})Var(t)}}$ is the correlation between $\overline{y}$ and t.

It follows from (77) that $\min.Var(T_{1P}) < Var(\overline{y})$.
Further from (20) and (77) that $\min.Var(T_{1P}) < Var(t_{1a}^{(1)})$

If

$$Var(\bar{y})(1-\rho^{*2}) < Var(\bar{y}) + g^2 R_1^2 Var(\bar{x}) - 2gR_1\rho\sigma_{(\bar{y})}\sigma_{(\bar{x})}$$
$$(gR_1\sigma_{(\bar{x})} - \rho\sigma_{(\bar{y})})^2 + Var(\bar{y})(\rho^{*2} - \rho^2) > 0 \tag{78}$$

Thus a sufficient condition for the estimator $T_{1P(opt)}$ (i.e. Optimum estimator in the class T_{1P}) to be more efficient than $t_{1a}^{(1)}$ is that $\rho^{*2} > \rho^2$. It may be noted here that the sufficient conditions can be examined in practice for their estimators which are obtained based on the subsamples [see Rao(1983)].

Remark 4.3. Similar studies can be carried out for the class of unbiased estimators T_{2P} in (72).
The variance of T_{2P} is given by

$$Var(T_{2P}) = Var(\bar{y}) + \alpha_1^2 B^* - 2\alpha_1 A^* \tag{79}$$

where

$$A^* = Cov(\bar{y}, t_1), \qquad B^* = Var(t_1), \qquad t_1 = (C\,\bar{y} - t_1^*)$$

and

$$t_1^* = [t_{2a}^{(1)} - (1-C)t_{2a}^{(2)}].$$

The variance of T_{2P} at (79) is minimized for

$$\alpha_1 = \frac{Cov(\bar{y}, t_1)}{Var(t_1)} = \frac{A^*}{B^*} = \alpha_{1opt} \quad (say) \tag{80}$$

Thus the resulting minimum variance of T_{2P} is given by

$$\min.Var(T_{2P}) = Var(\bar{y})(1-\rho_1^{*2}) \tag{81}$$

$\rho_1^* = \dfrac{Cov(\bar{y}, t_1)}{\sqrt{Var(\bar{y})Var(t_1)}}$ is the correlation coefficient between $\bar{y}$ and t_1.

The 'Optimum' estimator $T_{2P(opt)}$ is more efficient than $t_{2a}^{(1)}$ if

$$(gR_2\sigma_{(\bar{x})} - \rho\sigma_{(\bar{y})})^2 + Var(\bar{y})(\rho_1^{*2} - \rho^2) > 0 \tag{82}$$

which is always true if $\rho_1^{*2} > \rho^2$.

It follows from (81) that the 'Optimum' estimator $T_{2P(opt)}$ is better than usual unbiased estimator $\overline{y}$.

4.3. *Unbiased Product Estimators for SRSWOR Design*

For the case of simple random sampling without replacement (SRSWOR), let y_i and x_i denote respectively the y and x values of the i[th] sample unit, i=1,2,....,n. Then we consider the following estimators of $\overline{Y}$ as

$$t_{1a}^{(1)} = \overline{yz}^* / \overline{Z}, \quad t_{1a}^{(2)} = \frac{1}{n\overline{Z}}\sum_{i=1}^{n} y_i z_i^*$$

$$t_{2a}^{(1)} = \overline{yu}^* / \overline{U}, \quad t_{2a}^{(2)} = \frac{1}{n\overline{U}}\sum_{i=1}^{n} y_i u_i^*$$

and

$$B(t_{ja}^{(2)}) = (1-C^*)^{-1} B(t_{ja}^{(1)}), \quad j = 1,2 \tag{83}$$

where $C^* = N(n-1)/\{n(N-1)\}$

Following the procedure as outlined in section 4.2, we get the following class of unbiased estimators of $\overline{Y}$ as

$$T_{jP}^* = [(1-C^*\gamma_j)\overline{y} + \gamma_j\, t_{ja}^{(1)} - (1-C^*)\gamma_j t_{ja}^{(2)}] \tag{84}$$

where $\gamma_j (j=1,2)$ is the suitable chosen scalar.

Remark 4.4. Notice that $\gamma_j = 0 \;\; j = 1,2;$ gives the conventional unbiased estimator $\overline{y}$ and $\gamma_j = (C^*)^{-1}$ yields the estimator

$$T_{ja}^{*(1)} = \frac{1}{C^*} t_{ja}^{(1)} - \frac{(1-C^*)}{C^*} t_{ja}^{(2)} \;\; ; \; j = 1,2 \; ; \tag{85}$$

For j=1, we have

$$T_{1P}^{(1)} = \frac{n(N-1)}{N(n-1)} \overline{y}\frac{\overline{z}^*}{\overline{Z}} - \frac{(N-n)}{N(n-1)}\left(\frac{1}{n}\sum_{i=1}^{n} y_i z_i^* / \overline{Z}\right)$$

$$= \overline{y}\frac{\overline{z}^*}{\overline{Z}} - \left(\frac{1}{n} - \frac{1}{N}\right)\frac{s_{z^*y}}{\overline{Z}} \tag{86}$$

and for j=2, we have

$$T_{2P}^{(2)} = \frac{n(N-1)}{N(n-1)}\bar{y}\frac{\bar{u}^*}{\bar{U}} - \frac{(N-n)}{N(n-1)}\left(\frac{1}{n}\sum_{i=1}^{n} y_i\bar{u}_i^* / \bar{U}\right)$$

$$= \bar{y}\frac{\bar{u}^*}{\bar{U}} - \left(\frac{1}{n}-\frac{1}{N}\right)\frac{s_{u^*y}}{\bar{U}} \tag{87}$$

where

$$s_{z^*y} = \frac{1}{(n-1)}\sum_{i=1}^{n}(z_i^* - \bar{z}^*)(y_i - \bar{y})$$

and

$$s_{u^*y} = \frac{1}{(n-1)}\sum_{i=1}^{n}(u_i^* - \bar{u}^*)(y_i - \bar{y})$$

Many other unbiased estimators of $\bar{Y}$ can be obtained from (84) just by putting suitable values of scalars $\gamma_j\,(j=1,2)$.

As in the case of interpenetrating subsample, it is easy to obtain the optimum value of $\gamma_j\,(j=1,2)$.

4.4. *Quenouille's Jackknife Method*

In this method we take a sample n = 2m and split it at random into two sub samples of m units each. Let $\bar{y}_i, \bar{x}_i$, (i =1,2) be unbiased estimators of $\bar{Y}$ and $\bar{X}$ based on the subsamples and $\bar{y}, \bar{x}$ those based on the entire sample. Take

$$\bar{z}_i^* = (1+g)\bar{Z} - g\bar{z}_i \;, \bar{u}_i^* = (1+g)\bar{U} - g\bar{u}_i;$$

$$\bar{z}^* = (1+g)\bar{Z} - g\bar{z} \;, \bar{u}^* = (1+g)\bar{U} - g\bar{u};$$

$$\bar{z}_i = \frac{\bar{x}_i + x_m}{x_M + x_m}, \bar{u}_i = \frac{\bar{x}_i + x_M}{x_M + x_m};\; (i=1,2)$$

Consider the product–type estimators

$$t_{1a}^{(i)} = \bar{y}_i\bar{z}_i^* \big/ \bar{Z}, \quad t_{1a} = \bar{y}\bar{z}^* \big/ \bar{Z}$$

$$t_{2a}^{(i)} = \bar{y}_i\bar{u}_i^* \big/ \bar{U}, \quad t_{2a} = \bar{y}\bar{u}^* \big/ \bar{U}$$

Then it can be easily shown that

$$t_{5a} = \frac{(2N-n)}{N} t_{1a} - \frac{(N-n)}{2N}(t_{1a}^{(1)} + t_{1a}^{(2)}) \tag{88}$$

and

$$t_{6a} = \frac{(2N-n)}{N} t_{2a} - \frac{(N-n)}{2N}(t_{2a}^{(1)} + t_{2a}^{(2)}) \tag{89}$$

Further it can be shown to the first degree of approximation that

$$MSE(t_{5a}) = MSE(t_{1a}) \tag{90}$$

$$MSE(t_{6a}) = MSE(t_{2a}) \tag{91}$$

Since $t_{5a}\,(t_{6a})$ is unbiased while $t_{1a}\,(t_{2a})$ is not the former $t_{5a}\,(t_{6a})$ is to be preferred to the latter $t_{1a}\,(t_{2a})$.

Now we define a class of product–type estimators for $\overline{Y}$ as

$$t_w^{(j)} = \omega_{1j}\overline{y} + \omega_{2j} t_{ja} + \omega_{3j} t_{ja}^{*};\ (j = 1,2) \tag{92}$$

where $\omega_{ij}\,'s\ (i = 1,2,3\ ;\ j = 1,2)$ are suitable chosen constants such that

$$\omega_{1j} + \omega_{2j} + \omega_{3j} = 1\ ,\ j = 1,2 \tag{93}$$

and

$$t_{ja}^{*} = \frac{1}{2}\sum_{l=1}^{2} t_{ja}^{(l)} \tag{94}$$

we have

$$\left.\begin{aligned}
B(t_{1a}) &= -\frac{(N-n)}{nN}\left(\frac{g}{\bar{X}C_1}\right)\rho S_y S_x \\
B(t_{2a}) &= -\frac{(N-n)}{nN}\left(\frac{g}{\bar{X}C_2}\right)\rho S_y S_x \\
B(t_{1a}^{*}) &= -\frac{(2N-n)}{nN}\left(\frac{g}{\bar{X}C_1}\right)\rho S_y S_x \\
B(t_{2a}^{*}) &= -\frac{(2N-n)}{nN}\left(\frac{g}{\bar{X}C_2}\right)\rho S_y S_x
\end{aligned}\right\} \tag{95}$$

It follows from (95) that

$$\frac{B(t_{ja})}{B(t_{ja}^*)} = \frac{(N-n)}{(2N-n)} \;;\; j=1,2\;; \tag{96}$$

The class of estimators $t_\omega^{(j)}$ is unbiased if

$$\omega_{2j}B(t_{ja}) + \omega_{3j}B(t_{ja}^*) = 0 \tag{97}$$

Thus from (96) and (97) we get

$$\begin{aligned}\omega_{3j} &= -\omega_{2j}\frac{B(t_{ja})}{B(t_{ja}^*)} \\ &= -\omega_{2j}\frac{(N-n)}{(2N-n)}\end{aligned} \tag{98}$$

With $\omega_{2j} = \omega^{(j)}$ (a constant) and from (92), (93) and (98) we have

$$t_\omega^{(j)} = [(1-C\omega^{(j)})\bar{y} + \omega^{(j)}t_{ja} - \omega^{(j)}(1-C)t_{ja}^*] \quad (j=1,2) \tag{99}$$

Thus for j=1, we get

$$\begin{aligned}t_\omega^{(1)} &= [(1-C\omega^{(1)})\bar{y} + \omega^{(1)}t_{1a} - \omega^{(1)}(1-C)t_{1a}^*] \\ &= \left[(1-C\omega^{(1)})\bar{y} + \omega^{(1)}\bar{y}\frac{\bar{z}^*}{\bar{Z}} - \omega^{(1)}(1-C)\left\{\frac{1}{2\bar{Z}}\sum_{l=1}^{2}\bar{y}_j\bar{z}_j^*\right\}\right]\end{aligned}$$

a class of unbiased product–type estimators of $\bar{Y}$.
For j=2, we get another class of unbiased product–type estimators for $\bar{Y}$ as

$$\begin{aligned}t_\omega^{(2)} &= [(1-C\omega^{(2)})\bar{y} + \omega^{(2)}t_{2a} - \omega^{(2)}(1-C)t_{2a}^*] \\ &= \left[(1-C\omega^{(2)})\bar{y} + \omega^{(2)}\bar{y}\frac{\bar{u}^*}{\bar{U}} - \omega^{(2)}(1-C)\left\{\frac{1}{2\bar{U}}\sum_{l=1}^{2}\bar{y}_j\bar{u}_j^*\right\}\right]\end{aligned}$$

Remark 4.5. For $\omega^{(j)} = 0$, $t_\omega^{(j)}$ reduce to the usual unbiased estimator $\bar{y}$ while for $\omega^{(j)} = C^{-1}$ it boils down to the estimator $t_{5a}(j=1)$ and $t_{6a}(j=2)$.

4.5. *Optimum Estimator in the Class* $t_{\omega}^{(j)}$

From (99) we have

$$\begin{aligned}Var(t_{\omega}^{(j)}) = [&Var(\overline{y}) + \omega^{(j)^2}\{C^2 Var(\overline{y}) + Var(t_{ja}) + (1-C)^2 Var(t_{ja}^*)\\ &-2C\,Cov(\overline{y},t_{ja}) + 2C(1-C)Cov(\overline{y},t_{ja}^*) - 2C(1-C)Cov(t_{ja},t_{ja}^*)\}\\ &-2\omega^{(j)}\{C\,Var(\overline{y}) - Cov(\overline{y},t_{ja}) + (1-C)Cov(\overline{y},t_{ja}^*)\}]\\ &j=1,2.\end{aligned} \qquad (100)$$

To the first degree of approximation, it is easy to see that

$$\left.\begin{aligned}&Var(t_{ja}) = Var(t_{ja}^*) = Cov(t_{ja},t_{ja}^*) = \theta[S_y^2 + g^2 \mathrm{R}_j^2 S_x^2 - 2g\mathrm{R}_j\,\rho S_y S_x]\\ &Cov(\overline{y},t_{ja}) = Cov(\overline{y},t_{ja}^*) = \theta(S_y^2 - g\mathrm{R}_j \rho S_y S_x)\\ &Var(\overline{y}) = \theta S_y^2\end{aligned}\right\} \qquad (101)$$

$$j=1,2 \qquad \mathrm{R}_j = \mathrm{R}/C_j$$

Putting (101) in (100) we get the variance of $t_{\omega}^{(j)}; j=1,2;$ to terms of order n^{-1} as

$$Var(t_{\omega}^{(j)}) = \theta[S_y^2 + \omega^2 g^2 C^2 \mathrm{R}_j^2 S_x^2 - 2\omega C g \mathrm{R}_j\,\rho S_y S_x] \qquad (102)$$

which is minimized for

$$\omega^{(j)} = \left(\frac{C_j}{Cg}\right)\frac{\beta}{\mathrm{R}} = \omega_{opt}^{(j)} \qquad (103)$$

$$j=1,2.$$

Thus the resulting minimum variance of $t_{\omega}^{(j)}$ is given by

$$\min.Var(t_{\omega}^{(j)}) = \theta S_y^2(1-\rho^2) \qquad (104)$$

which is equal to the approximate variance of the usual biased regression estimator

$$\overline{y}_{lr} = \overline{y} + \hat{\beta}(\overline{X} - \overline{x}) \qquad (105)$$

where $\hat{\beta}$ is the sample regression coefficient of y on x.

Substituting the value of $\omega_{opt}^{(j)}$ for $\omega^{(j)}$ in (99) we get the asymptotically optimum unbiased product-type estimator in the class (4.42) as

$$t_{\omega(0)}^{(j)} = \left[\left\{ 1 - \left(\frac{C_j}{g} \right) \frac{\beta}{R} \right\} \bar{y} + \left(\frac{C_1}{gC} \right) \frac{\beta}{R} t_{ja} - \left(\frac{C_1}{gC} \right) \frac{\beta}{R} (1 - C) t_{ja}^{*} \right], \; j = 1, 2. \tag{106}$$

with the variance as given in (104).

5. EMPIRICAL STUDY

To illustrate the performance of the suggested estimators t_{1a} and t_{2a} over $\bar{y}, \bar{y}_R, \bar{y}_a, t_{1R}$, and t_{2R}, we have considered four populations whose descriptions are given in the Table 1.

TABLE 1

Description of populations

Population	Source	N	n	Y	X	ρ	C_x	C_y
I	Panse and Sukhatme (1967) p.118	25	10	Parental plot mean (mm)	Parental plant value (mm)	0.53 (1.83) {0.98}	0.07 (2.15) {24.95}	0.03 (0.26) {24.37}
II	Panse and Sukhatme (1967) p.118 (1-20)	20	8	Parental plot mean (mm)	Parental plant value (mm)	0.56 (1.83) {0.97}	0.07 (2.15) {25.09}	0.04 (0.29) {24.37}
III	Panse and Sukhatme (1967) p.118 (1-10)	10	4	Porgeny mean (mm)	Parental plant value (mm)	0.44 (1.92) {0.92}	0.07 (2.13) {25.48}	0.05 (0.31) {23.50}
IV	Sampford (1962) p.61 (1-9)	9	3	Acreage under oats in 1957	Acreage of crops and grass in 1947	0.07 (1.86) {0.25}	0.10 (2.12) {58}	0.29 (0.19) {14.78}

Where ρ is the correlation coefficient between x and y, and $C_x = S_x / \bar{X}$ and $C_y = S_y / \bar{Y}$ are the coefficients of variation of x and y respectively.

(Figures in (.) indicate values of C_1, C_2 and K respectively and in {.} show the values of R, $\bar{X}$ and $\bar{Y}$ respectively. (***See***. Mohanty and Sahoo (1995))).

We have computed the absolute biases of the estimators $\bar{y}_R, \bar{y}_a, t_{1R}, t_{2R}, t_{1a}$ and t_{2a} and presented in Table-5.2. Percent relative efficiency (%) of these estimators ($\bar{y}_R, \bar{y}_a, t_{1R}, t_{2R}, t_{1a}$ and t_{2a}) have also been computed and compiled in Table-5.3. Formulae for absolute biases and percent relative efficiencies are given below.

$$\left| B(\bar{y}_R) \right| = \left| (\theta / \bar{X})(R S_x^2 - \rho S_y S_x) \right| \tag{107}$$

$$\left| B(\bar{y}_a) \right| = \left| (\theta / \bar{X}) \, g \rho S_y S_x \right| \tag{108}$$

$$\left|B(t_{1R})\right| = \left|\{\theta/(\bar{X} C_1)\}(R_1 S_x^2 - \rho S_y S_x)\right| \tag{109}$$

$$\left|B(t_{2R})\right| = \left|\{\theta/(\bar{X} C_2)\}(R_2 S_x^2 - \rho S_y S_x)\right| \tag{110}$$

$$\left|B(t_{1a})\right| = \left|\{\theta/(\bar{X} C_1)\}\ g\rho S_y S_x\right| \tag{111}$$

$$\left|B(t_{2a})\right| = \left|\{\theta/(\bar{X} C_2)\}\ g\rho S_y S_x\right| \tag{112}$$

$$PRE(\bar{y}_R, \bar{y}) = \frac{S_y^2}{(S_y^2 + R^2 S_x^2 - 2R\rho S_y S_x)} \times 100 \tag{113}$$

$$PRE(\bar{y}_a, \bar{y}) = \frac{S_y^2}{(S_y^2 + g^2 R^2 S_x^2 - 2gR\rho S_y S_x)} \times 100 \tag{114}$$

$$PRE(t_{1R}, \bar{y}) = \frac{S_y^2}{(S_y^2 + R_1^2 S_x^2 - 2R_1\rho S_y S_x)} \times 100 \tag{115}$$

$$PRE(t_{2R}, \bar{y}) = \frac{S_y^2}{(S_y^2 + R_2^2 S_x^2 - 2R_2\rho S_y S_x)} \times 100 \tag{116}$$

$$PRE(t_{1a}, \bar{y}) = \frac{S_y^2}{(S_y^2 + g^2 R_1^2 S_x^2 - 2gR_1\rho S_y S_x)} \times 100 \tag{117}$$

$$PRE(t_{2a}, \bar{y}) = \frac{S_y^2}{(S_y^2 + g^2 R_2^2 S_x^2 - 2gR_2\rho S_y S_x)} \times 100 \tag{118}$$

TABLE 2

Absolute biases of $\bar{y}_R$, $\bar{y}_a$, t_{1R}, t_{2R}, t_{1a} *and* t_{2a}

Estimator	Absolute Biases			
	Population I	Population II	Population III	Population IV
$\bar{y}_R$	0.0054	0.0072	0.0122	0.0282
$\bar{y}_a$	0.0013	0.0020	0.0036	0.0034
t_{1R}	0.0011	0.0014	0.0020	0.0065
t_{2R}	0.0007	0.0008	0.0014	0.0046
t_{1a}	0.0007	0.0011	0.0019	0.0018
t_{2a}	0.0006	0.0008	0.0017	0.0016

TABLE 3

Percent relative efficiencies (PRE's) of $\bar{y}, \bar{y}_R, \bar{y}_a, t_{1R}, t_{2R}, t_{1a}$ *and* t_{2a}

Estimator	Percent Relative Efficiencies			
	Population I	Population II	Population III	Population IV
$\bar{y}$	100.00	100.00	100.00	100.00
$\bar{y}_R$	33.62	39.27	55.15	92.90
$\bar{y}_a$	71.34	82.77	92.67	99.29
t_{1R}	94.69	107.82	110.63	98.99
t_{2R}	112.01	125.30	115.91	99.49
t_{1a}	130.62	141.79	123.44	100.39
t_{2a}	136.69	145.62	124.00	100.44

It is observed from Table-2 that the estimator t_{2a} has least bias followed by t_{1a} and t_{2R} for population I while in population II it is at par with t_{2R} and has less bias than $\bar{y}_R$, $\bar{y}_a$, t_{1R} and t_{1a}. However in population III, the estimator t_{2a} has less bias than $\bar{y}_R$, $\bar{y}_a$, t_{1R} and t_{1a} but has marginally more bias than t_{2R}. Table 3 exhibits that the estimator t_{2a} has largest efficiency for all the population data sets I-IV followed by t_{1a}. Thus the proposed estimators t_{1a} and t_{2a} are to be preferred in practice.

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SUMMARY

Duals to Mohanty and Sahoo's estimators

This paper proposes duals to Mohanty and Sahoo's (1995) estimators and analyzes their properties. Unbiased estimators have also been obtained for interpenetrating subsample design and by using Jackknife technique given by Quenouille (1956). An empirical study is carried out to demonstrate the performances of the suggested estimators over other estimators.